

# Digital Signatures

## Objectives

- ▶ Features of hand-written signatures in Digital World
- ▶ Ensure hardness of forgery

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  - ▶ Correctness: A correct signature should always be verified true.

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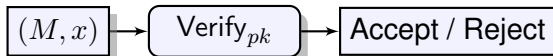
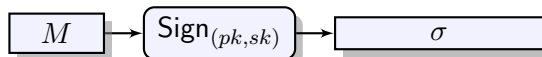
- ▶ Function: bind a statement/message to its authors.
- ▶ Verification is public. (against a prior authenticated one)
- ▶ Properties:
  - ▶ Correctness: A correct signature should always be verified true.
  - ▶ Security: Hard to forge.

# Signature Schemes

Signature Scheme (Gen, Sign, Verify)

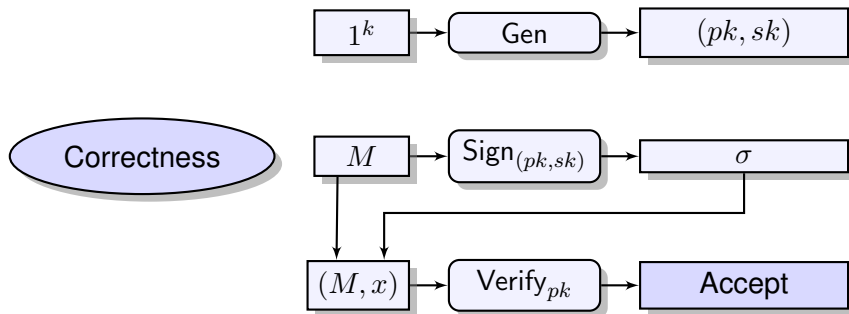
Required Properties:

- ▶ Correctness
- ▶ Unforgeability



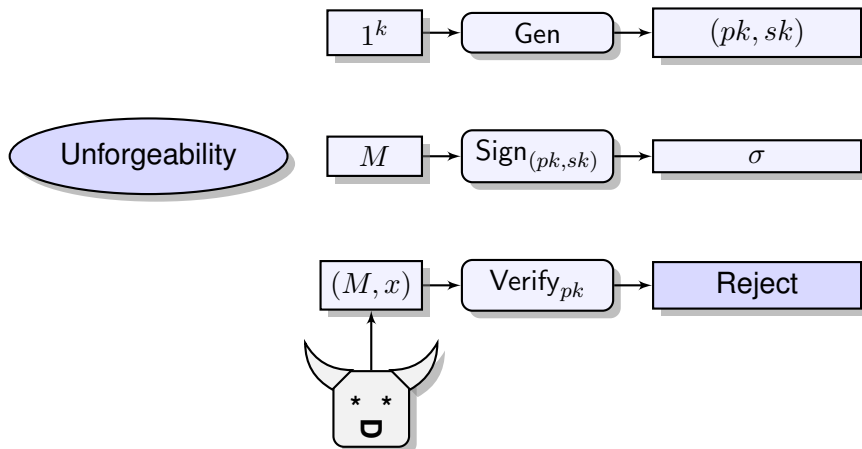
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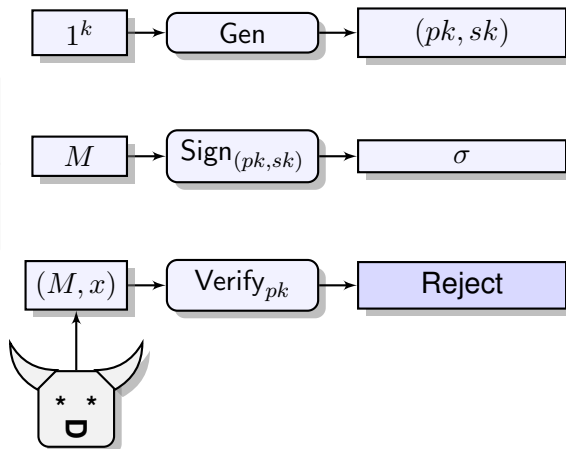


# Signature Schemes

Signature Scheme ( $\text{Gen}$ ,  $\text{Sign}$ ,  $\text{Verify}$ )

## Unforgeability:

Must output forgery for a message for which the attacker did not request the signature.



# Signature Schemes Designs: RSA Full Domain Hash

- ▶ **Public Functions** A hash function  $H : \{0, 1\}^* \rightarrow \mathbb{Z}_N^*$
- ▶ **Keygen:** Run RSA.Keygen.  $pk = (e, N)$ ,  $sk = (d, N)$ .
- ▶ **Sign:** Input:  $sk, M$ . Output  
$$\sigma = \text{RSA.Dec}(sk, H(M)) = H(M)^d \bmod N$$

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## note

A hash function takes strings of arbitrary length as input and produces a fixed length output. For cryptographic hash functions, given a  $z$ , it is very expensive to find  $x$  such that  $H(x) = z$