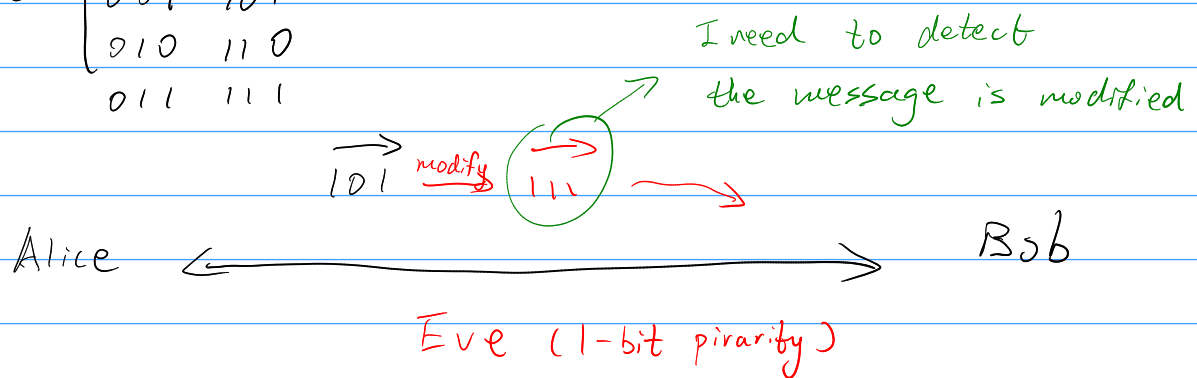


$$M = \begin{cases} 000 & 100 \\ 001 & 101 \\ 010 & 110 \\ 011 & 111 \end{cases}$$



Solution!

101  $\square \rightarrow$  tag checksum

111  $\rightarrow \square$  It is not correct.

$$\bigtriangleup \quad 1011 \quad \bigoplus_{i=0}^2 x_i$$

$$1111 \oplus 0 \quad \bigtriangleup$$

$$100 \rightarrow 1 \rightarrow 1 \oplus 1 \rightarrow 0$$

$$1 \oplus 1 \oplus 1 \rightarrow 1 \neq 0 \quad (\times)$$

$\bigcirc$   $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  Eve could change the check sum.!

uni  $\rightarrow$  asks for bids.

$S_1 \xrightarrow{1000}$

竞争最低价

$S_2 \xrightarrow{1050} 900$

如果别人价格泄露

$S_3 \xrightarrow{950}$

别人就会调低价格,

hash function

谁拿到 key 都会泄露

$S_1 \quad H(1000) = y_1 \quad f \quad 1000 \xrightarrow{\text{map}} y_1$

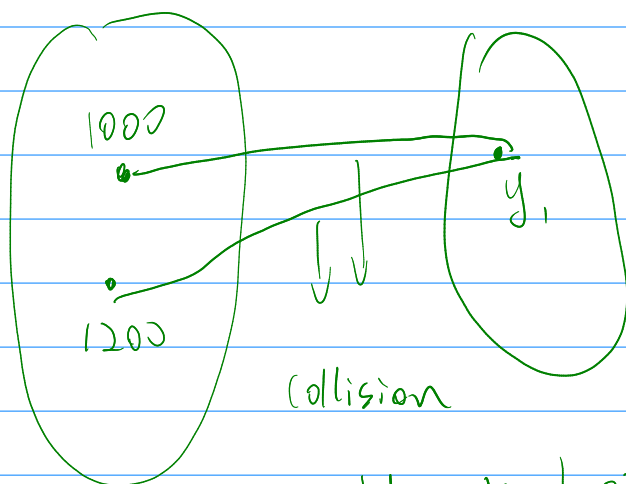
$S_2 \quad H(1050) = y_2$



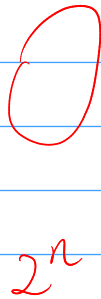
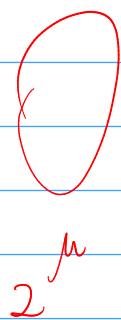
$S_3 \quad H(950) = y_3$

finding  $x$

from  $H(x)$  is hard!

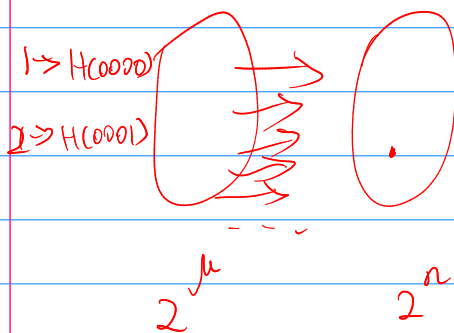


$\therefore$  could not happen like this!



$\mu \gg n$   
 $\nwarrow$   
 much larger than

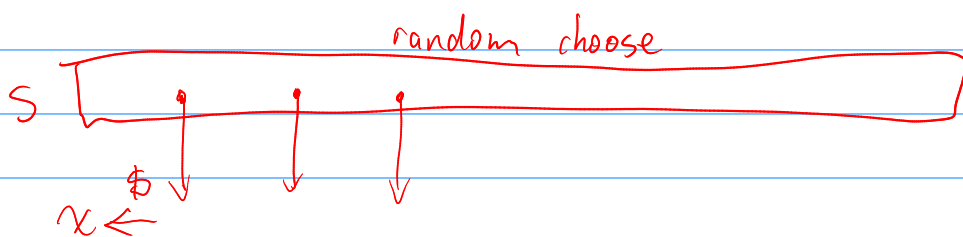
Hardness Collision Attack



Sufficiently try  $2^n$  times

collision may happen  
 after  $2^{\frac{n}{2}}$  time.

Set  $N = 2^n$



for  $i = 1$  to  $q$   
 compute  $H(i)$

? For what  $q$ , there will be a repetition?

$i = 1$     $i = 2$     $i = 3$    ...    $i = q$

$p = 0$     $p = \frac{1}{N}$     $p = \frac{2}{N}$     $p = \frac{q-1}{N}$

No collision

$p = (1 - \frac{1}{N})$     $p = (1 - \frac{2}{N})$    ...    $p = 1 - \frac{q-1}{N}$

No collision

$$P[\text{coll}_2] \times P[\text{coll}_3] \dots$$

$$= \left(1 - \frac{1}{N}\right) \times \left(1 - \frac{2}{N}\right) \times \dots \times \left(1 - \frac{q-1}{N}\right)$$

$$\approx \prod_{i=1}^{q-1} \left(1 - \frac{i}{N}\right) = \left(1 - \frac{\sum_{i=1}^{q-1} i}{N}\right)$$

$i = 1$

$$= 1 - \frac{q^2}{2^{n+1}}$$

$1+2+\dots+N$   
 $= \frac{N(N+1)}{2}$

$1+2+\dots+(q-1)$   
 $= \frac{(q-1) \cdot q}{2}$

$\frac{(q-1) \cdot q}{2}$   
 $2^n$

$\frac{(q-1) \cdot (q-1+1)}{2}$   
 $2^{n+1}$

$$1 - \left(1 - \frac{q^2}{2^{n+1}}\right)$$
$$\frac{q^2}{2^{n+1}}$$

