Lecture 3

A point to remember

- Symmetric-Key Cryptography: Sender and Receiver both know the secret key. The encryption and decryption algorithms **need not be identical**.

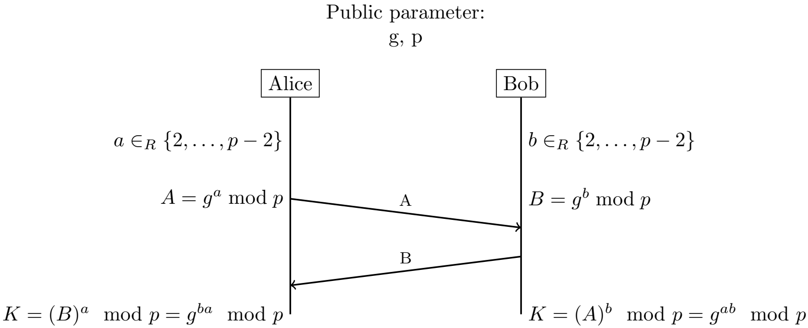
Public Key Cryptography

University of Birmingham

Outline of This Lecture

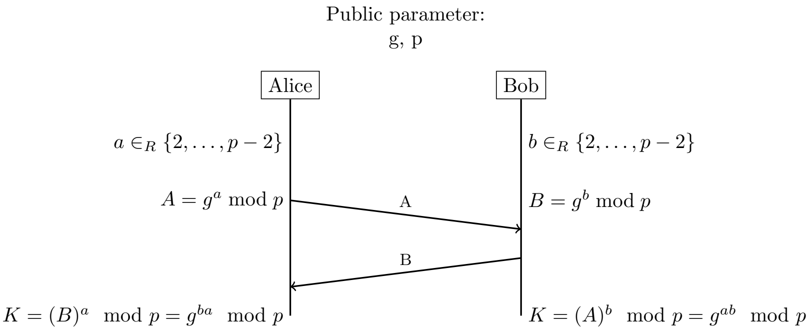
- Diffie-Hellman Key Exchange
- The Setup of Public Key Cryptography
- RSA Encryption and Signatures
- Public Key Certificates

Secure Key Exchange

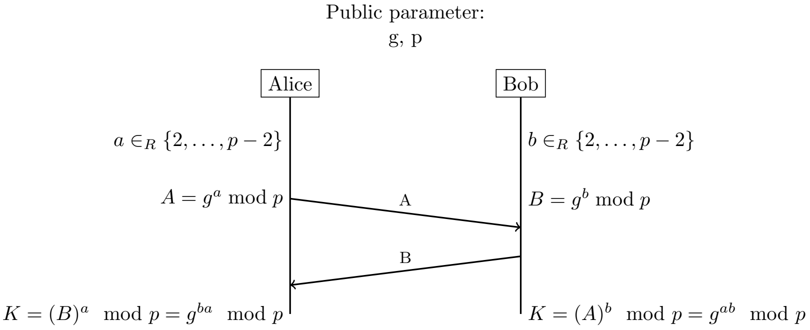


Example: Suppose $p = 13$ and $g = 7$.

Secure Key Exchange



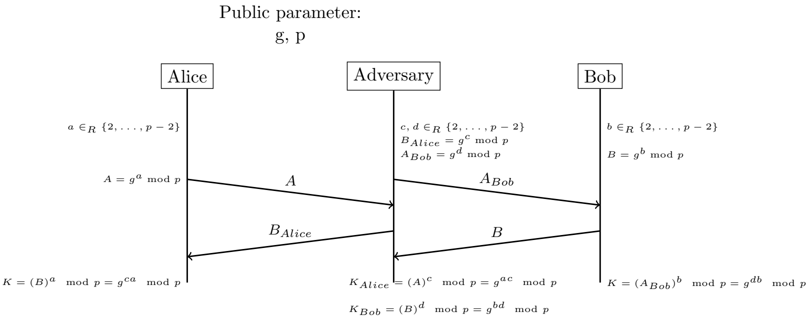
Secure Key Exchange: Idea of Public-Key Cryptography



Public Keys

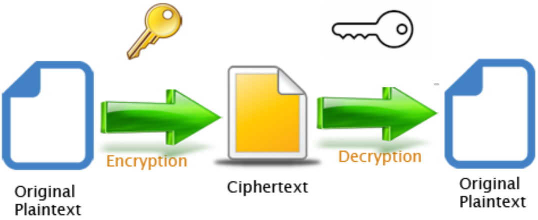
$g^a \mod p$ and $g^b \mod p$ can be called public keys. The secrets a and b are private keys.

Public Keys needs to be authenticated



Encryption and Authentication using Public Keys

Encryption using Public-Key



- Encryption uses receiver's Public-Key
- Decryption uses receiver's Private-Key

Encryption using RSA

Procedure $\mathsf{Keygen}(1^\lambda)$

Choose two random $\lambda/2$-bit primes p and q

$n = p \cdot q$

$\phi = (p-1)(q-1)$

Select e such that
$1 < e < \phi$ and $\gcd(e, \phi) = 1$

Compute d such that
$1 < d < \phi$ and $ed \equiv 1 \pmod{\phi}$

Set $PK = (e, n)$

Set $SK = (d, n)$

return (PK, SK)

Procedure $\mathsf{Encrypt}(PK, m)$

// We assume $m \in \mathbb{Z}_n^*$

$c = m^e \mod n$

return c

Procedure $\mathsf{Decrypt}(SK, c)$

$m = c^d \mod n$

return m

Encryption using RSA:Example

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$m = c^d \bmod n$

return m

- Let $p = 5, q = 11$
- $n = 55, \phi(n) = 4 \times 10 = 40$
- Suppose $e = 7$. Then $d = 23$ as $7 \times 23 = 161 \equiv 1$ mod 40
- PK=$(7, 55)$, SK=$(23, 55)$

Notes on RSA

- RSA security depends on hardness of finding d from e, n; Related to hardness of factoring of n.
- The textbook algorithms are deterministic. In practice, some random padding is used.
- Shor's quantum algorithm can solve factoring in polynomial time. However, a quantum computer of required capacity is still quite far away in the future.

Notes on RSA

- RSA security depends on hardness of finding d from e, n; Related to hardness of factoring of n.
- The textbook algorithms are deterministic. In practice, some random padding is used.
- Shor's quantum algorithm can solve factoring in polynomial time. However, a quantum computer of required capacity is still quite far away in the future.
- Wikipedia says any $m < n$ would work. Strictly speaking, it is required that gcd$(m, n) = 1$. On the other hand, finding a $m < n$ such that gcd$(m, n) > 1$ will lead to finding p or q, and breaking the system.

Digital Signatures

Signature Scheme $(\mathsf{Gen}, \mathsf{Sign}, \mathsf{Verify})$

Required Properties:

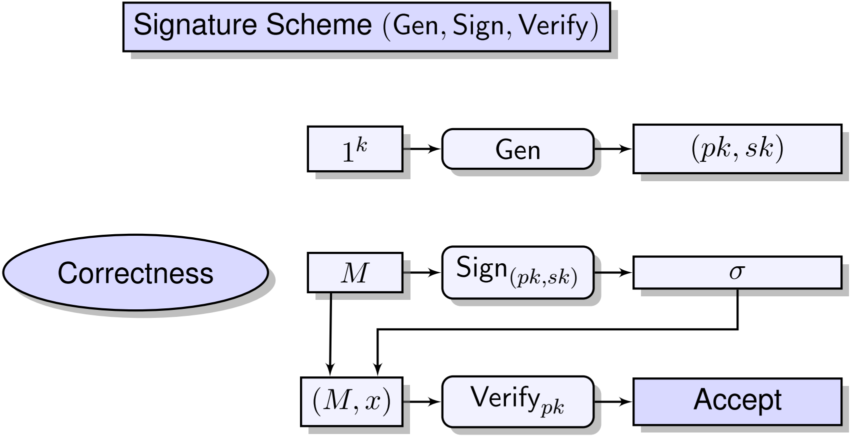
- Correctness
- Unforgeability

$1^k \rightarrow \mathsf{Gen} \rightarrow (pk, sk)$

$M \rightarrow \mathsf{Sign}_{(pk,sk)} \rightarrow \sigma$

$(M, x) \rightarrow \mathsf{Verify}_{pk} \rightarrow$ Accept / Reject

Digital Signatures



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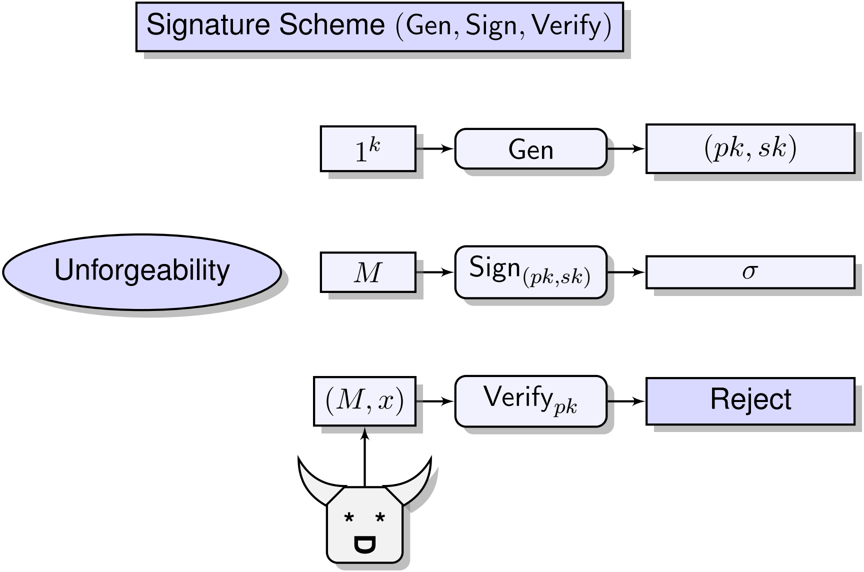

Signature using RSA

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Procedure $\mathsf{Sign}(SK, m)$

// We assume $m \in \mathbb{Z}_n^*$
$\sigma = H(m)^d \bmod n$
return σ

Procedure $\mathsf{Verify}(PK, m, \sigma)$

if $H(m) = \sigma^e \bmod n$
  return Accept
else return Reject

What is H

H is a cryptographic hash function. More on hash functions next week.

Signature using RSA

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Why use H

Possible attack without H.

Authenticating Public Keys

Certificates of Public-Key. Demo in class.