

# A probabilistic early-warning signal for noise-induced tipping in quasi-stationary regimes of metastability

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# Precursors and indicators

## Definition

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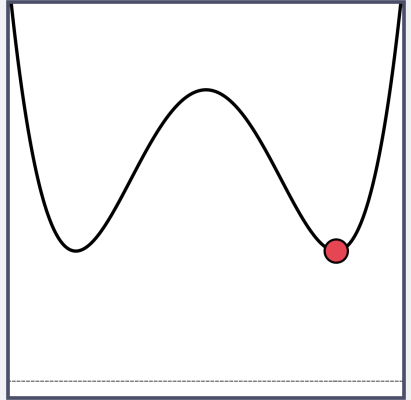
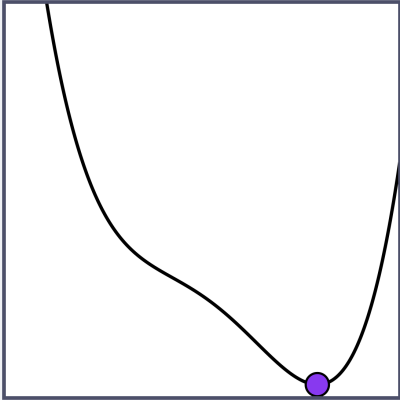
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# Measures of incoming criticality

# The stability landscape





# A numerical method

**Objective:** derive an early-warning signal with the following desirable properties:

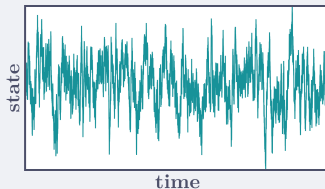
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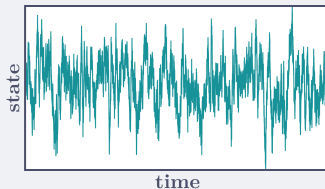
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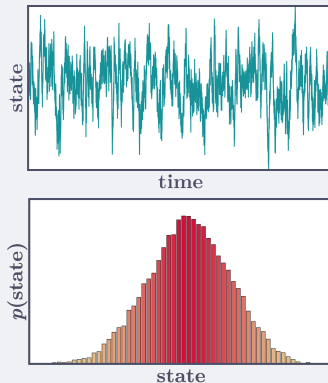
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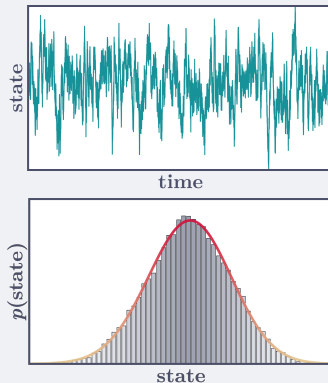
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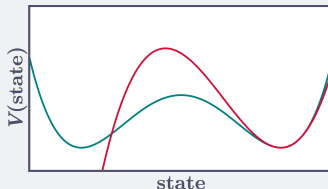
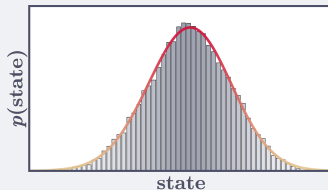
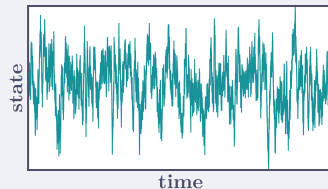
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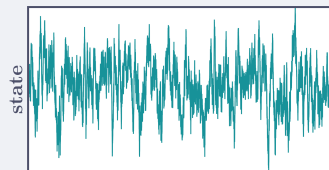
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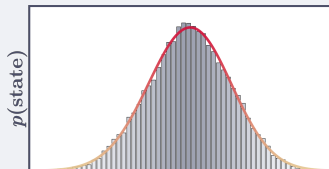
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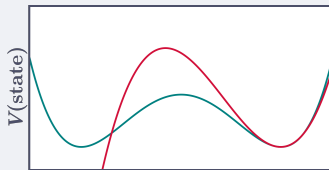
3) **Output:** probability of tipping<sup>[3]</sup>



time

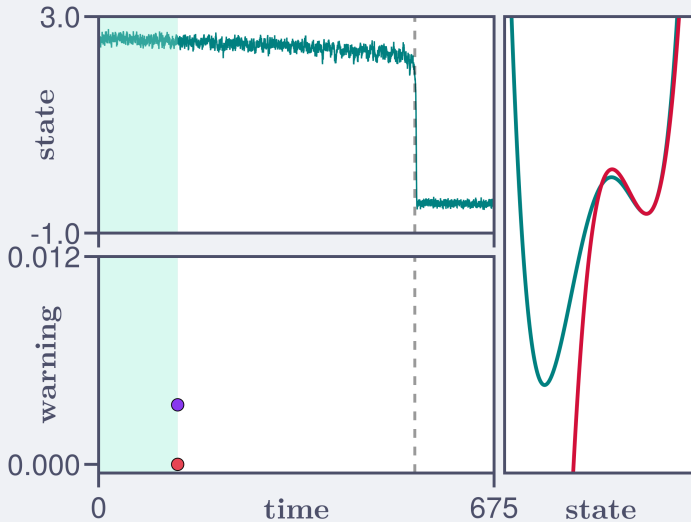


state



state

# An EWS from data





# References



H. A. Kramers, Physica **7** (1940).



C. Kuehn, Physica D **240** (2011).



M. Scheffer, et al., Science **338** (2012).

Thank you.

► Go to beginning

► Go to the variance eqns

► Go to numerical method