

A numerical estimate of escape rates

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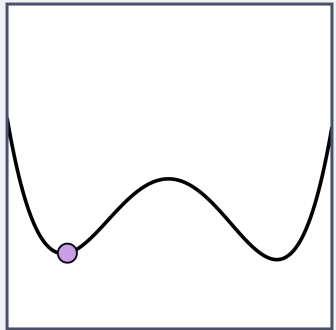


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Kramers' escape formula

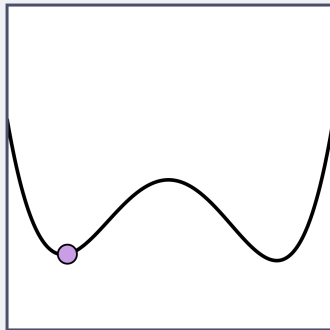
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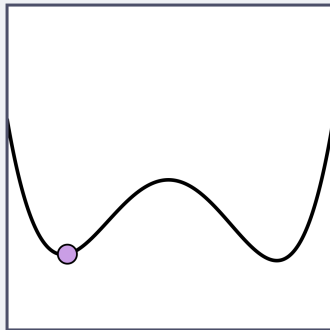
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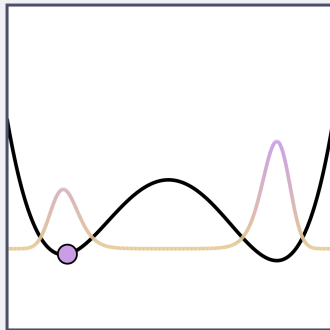
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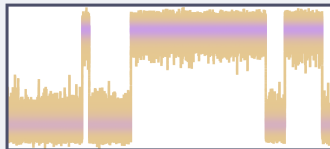
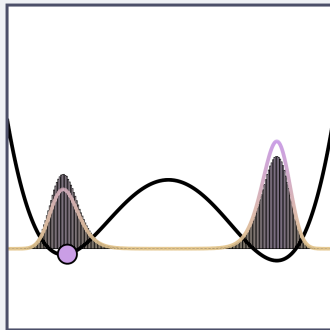
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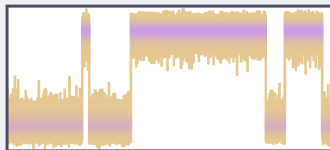
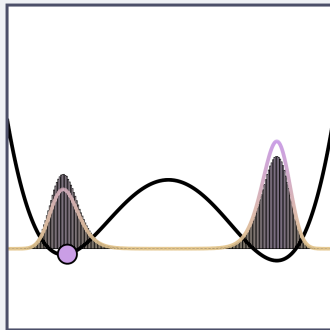
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Mean first passage time (MFPT) in the weak noise limit ($D \ll 1$):

$$\tau_{\text{esc}} \approx \frac{2\pi}{\sqrt{V''(a)|V''(b)|}} \exp\left(\frac{V(b) - V(a)}{D}\right).$$



Critical transitions and warning signs

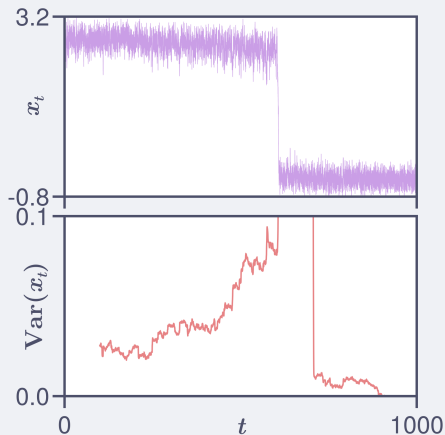
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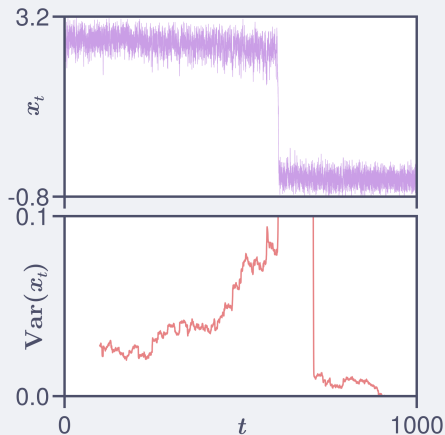
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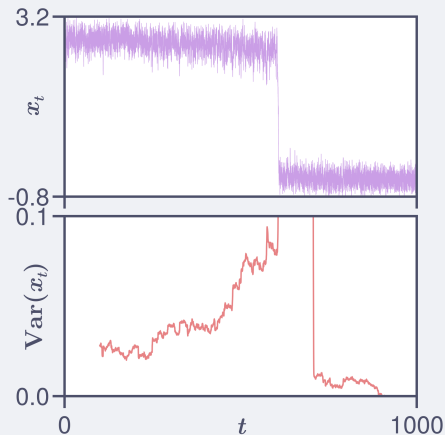
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Can we estimate it from **timeseries data alone**?

Algorithm:

- 1) detrend the timeseries;
- 2) assemble its histogram;
- 3) fit an optimal density;
- 4) reconstruct the potential;
- 5) compute τ_{esc} .

The method in action

Literature on critical transitions

-  C. Kuehn, Physica D **240** (2011).
-  M. Scheffer, et al., Science **338** (2012).
-  N. Berglund and B. Gentz, J. Diff. Eq. **191** (2003).
-  P. Ashwin, et al., Nonlinearity **30** (2017).
-  P. Ritchie and J. Sieber, Chaos **26** (2016).
-  T. Lenton, et al., Philos. T. R. Soc. A **370** (2012).

Thank you.

► Go to the beginning

► Go to the algorithm