A numerical estimate of escape rates

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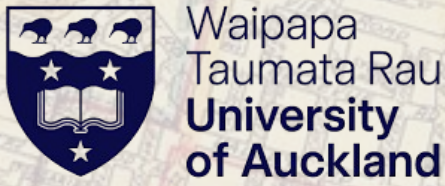

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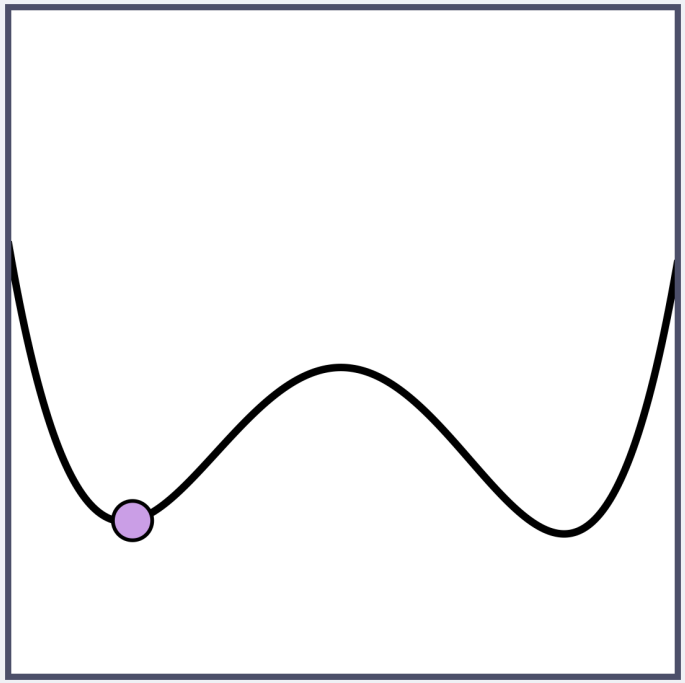

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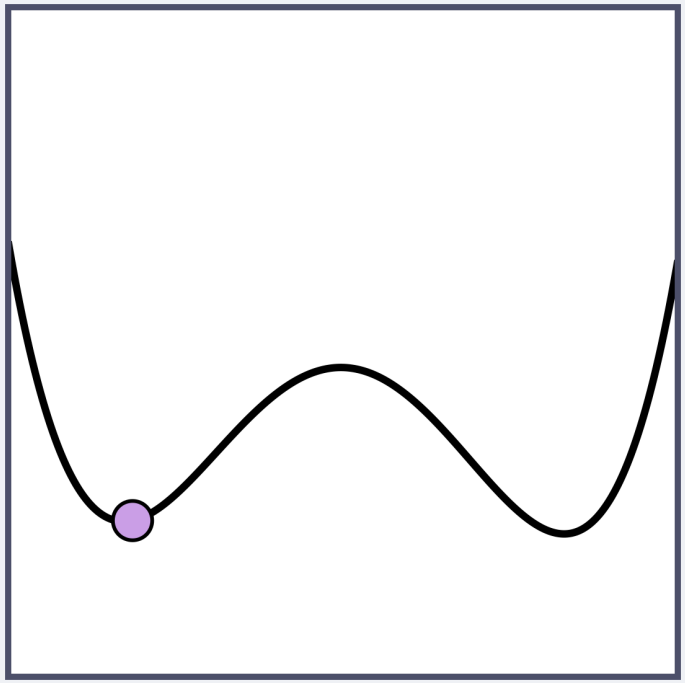

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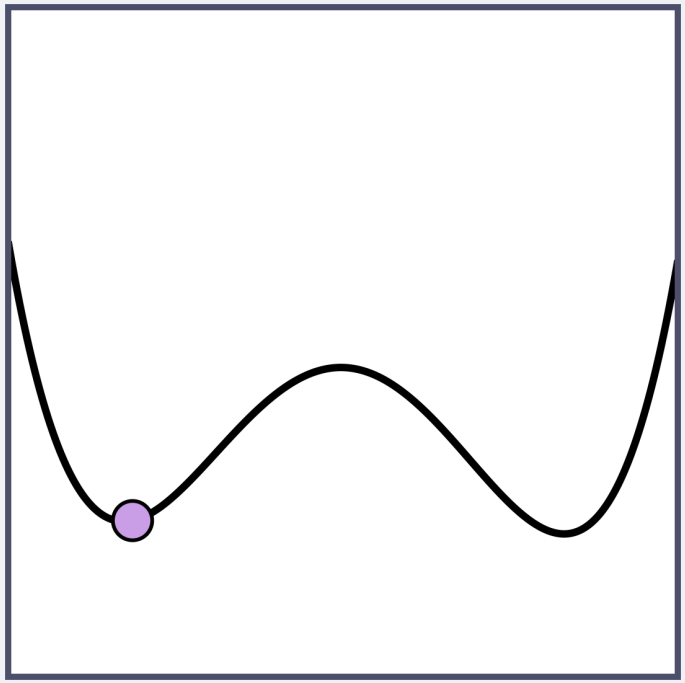

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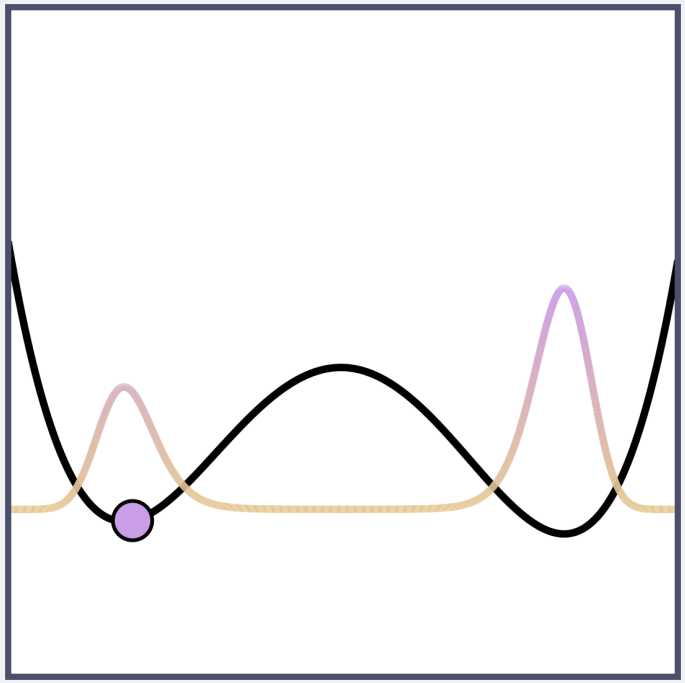

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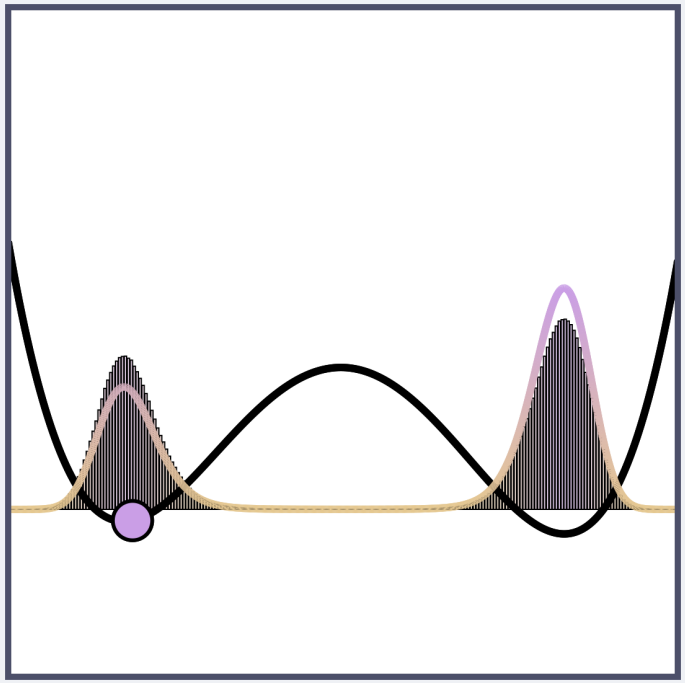

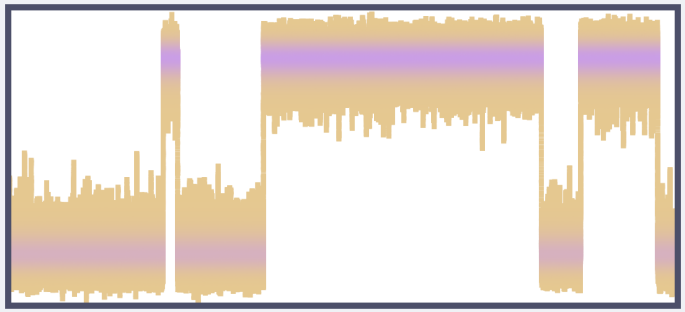

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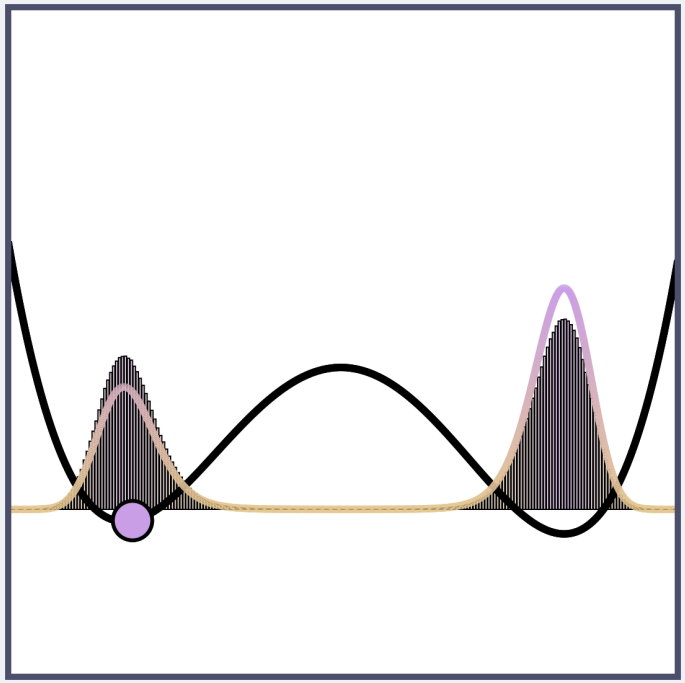
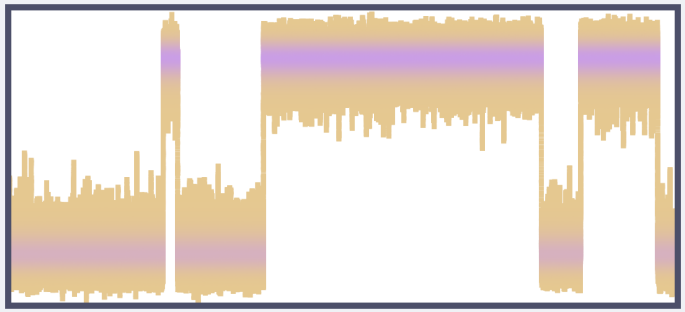
Mean first passage time (MFPT) in the **weak noise limit** ($D << 1$):

$$\tau_{\text{esc}} \approx \frac{2\pi}{\sqrt{V''(a)|V''(b)|}} \exp\left(\frac{V(b)-V(a)}{D}\right).$$





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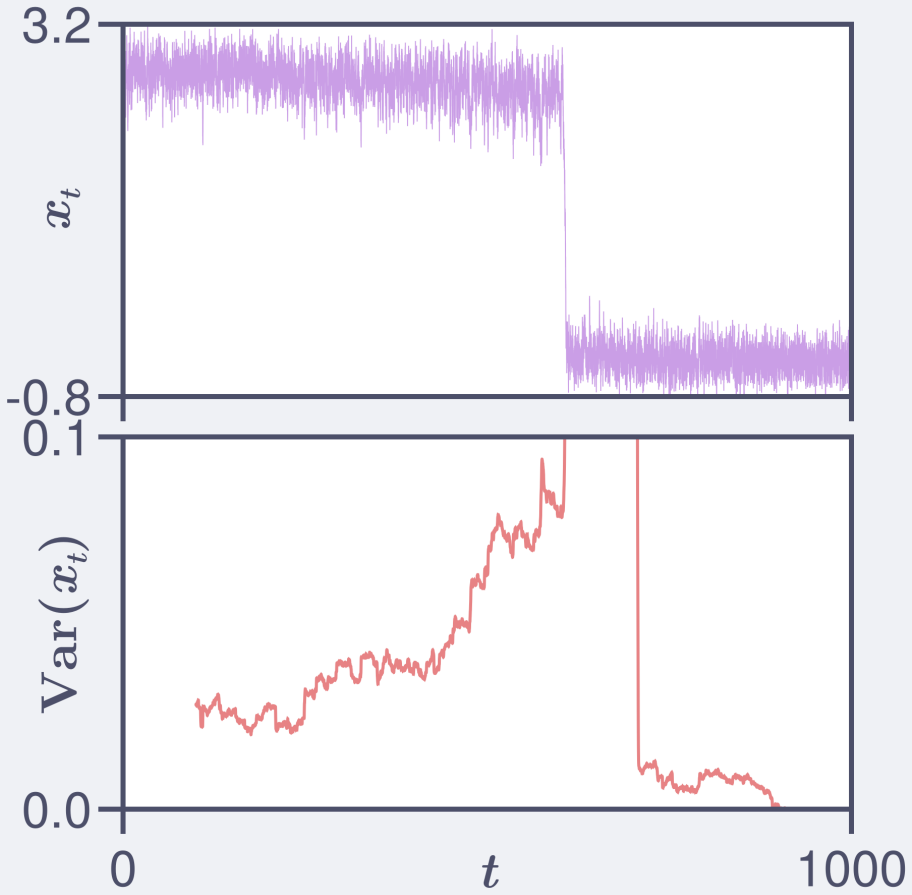

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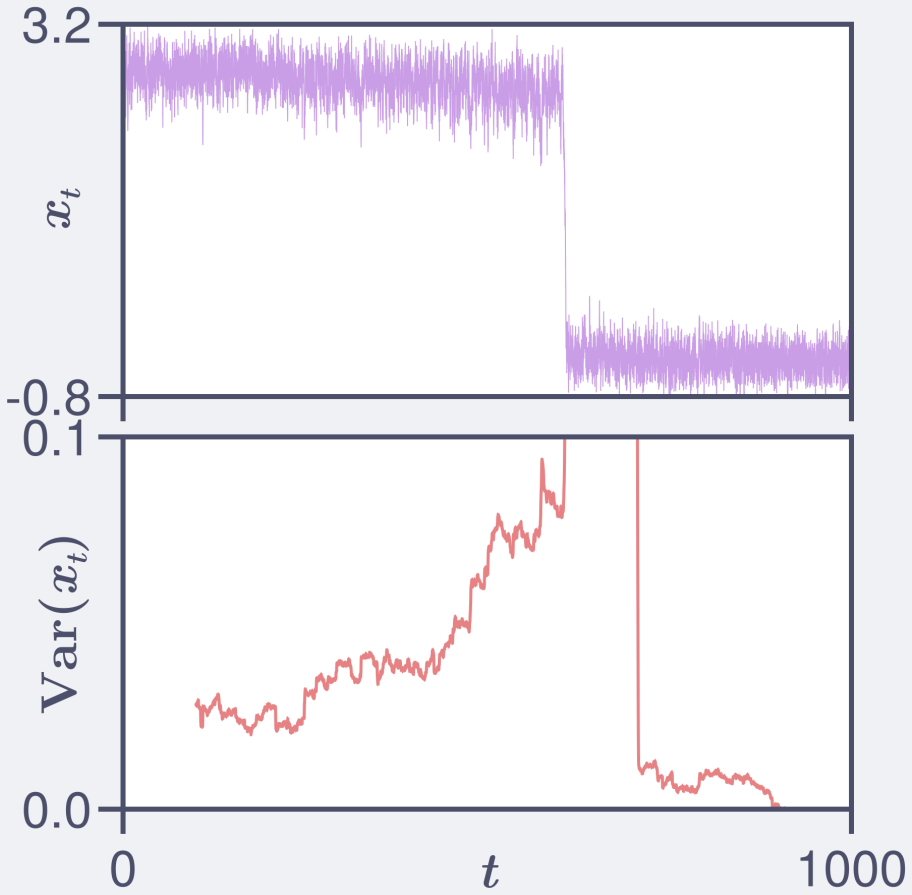

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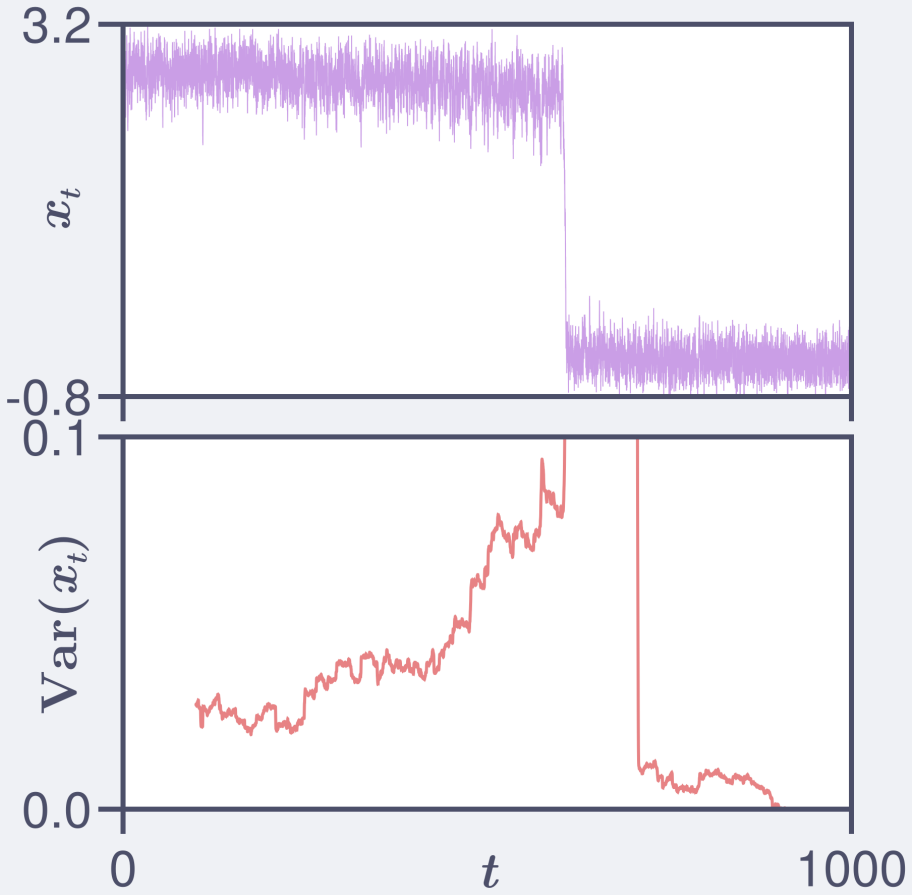

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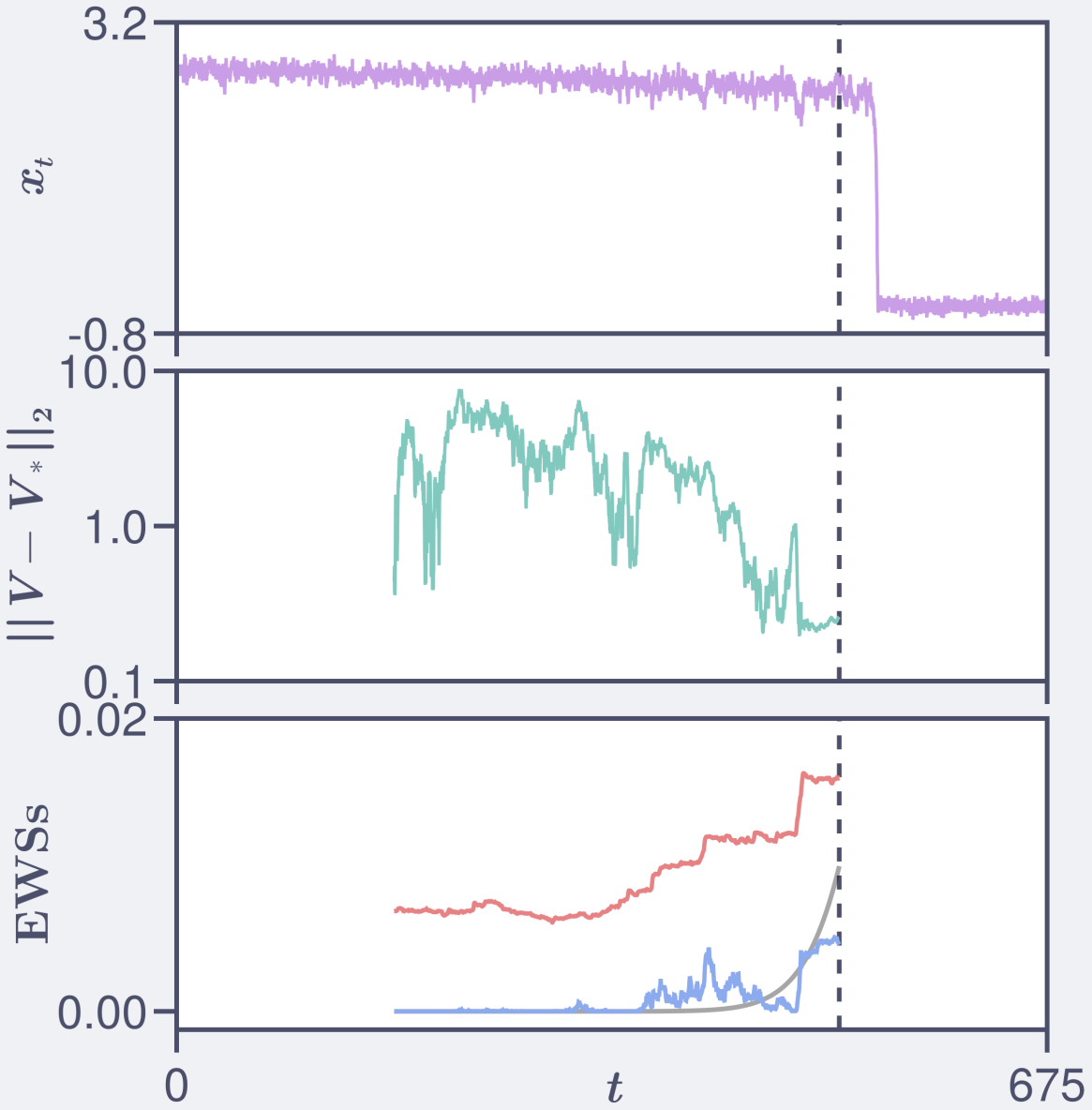
Algorithm:

1) detrend the timeseries;
2) assemble its histogram;
3) fit an optimal density;
4) reconstruct the potential;
5) compute τ_{esc}.

The method in action



Literature on critical transitions

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- N. Berglund and B. Gentz, J. Diff. Eq. **191** (2003).
- P. Ashwin, et al., Nonlinearity **30** (2017).
- P. Ritchie and J. Sieber, Chaos **26** (2016).
- T. Lenton, et al., Philos. T. R. Soc. A **370** (2012).

Thank you.

▸ Go to the beginning ▸ Go to the algorithm