MATH 100
REAL NUMBERS

GOALS

- CLASSIFY NUMBERS AS NATURAL, INTEGER, RATIONAL IRRATIONAL, REAL.
- UNDERSTAND HOW ORDER OF OPERATIONS RELATES TO NUMBER TYPES
- PERFORM OPERATIONS USING PEMDAS
- EVALUATE + SIMPLIFY ALGEBRAIC OPERATIONS/EXPRESSIONS.

CONSTRUCTING NUMBER SYSTEMS

NATURAL NUMBERS: THE NUMBERS WE USE TO COUNT AND ORDER THINGS

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

WHOLE NUMBERS: NATURALS AND ZERO (NON-NEGATIVE INTEGERS)

$$\mathbb{W} = \{0, 1, 2, 3, 4, \dots\}$$

OPERATIONS ON NATURAL/WHOLE NUMBERS

- WE CAN **ADD** ANY TWO WHOLE NUMBERS AND GET ANOTHER WHOLE NUMBER

  e.g. $3 + 7 = 10$, $0 + 41 = 41$; $101 + 2002 = 2103$

- WE CAN **MULTIPLY** ANY TWO WHOLE NUMBERS AND GET ANOTHER WHOLE NUMBER

  e.g. $3 \cdot 7 = 21$

  $1 \cdot 25 = 25$

  $5 \cdot 12 = 60$

MULTIPLICATION OF WHOLE NUMBERS IS REPEATED ADDITION

$$3 \cdot 7 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 21$$

$$= 7 \cdot 7 \cdot 7 = 21$$

- WHAT IF WE SUBTRACT?
  - $7 - 3 = 4$ ✓
  - $42 - 5 = 37$ ✓
  - $6 - 8 = ?$ ← NEED TO DEFINE NEGATIVE NUMBERS

    $= -2$

INTEGERS: NATURAL NUMBERS, ZERO, AND NEGATIVE NATURALS

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

OPERATIONS ON INTEGERS

- WE CAN ADD OR SUBTRACT ANY TWO INTEGERS AND GET ANOTHER INTEGER

  e.g. $7 + 12 = 19$, $1 - 8 = -7$, $-4 + 1 = -3, \ldots$

- WE CAN MULTIPLY ANY TWO INTEGERS AND GET ANOTHER INTEGER

  e.g. $5 \cdot 4 = 20$, $3 \cdot 8 = 24$, $100 \cdot \underbrace{(-4)}_{\text{USING PARENTHESES TO BE EXTRA CLEAR}} = -400$

- WHAT ABOUT DIVIDING?
  - $10 \div 2 = 5$
  - $-32 \div 8 = 4$
  - $7 \div 3 = ?$ ← NEED TO DEFINE RATIONAL NUMBERS

    $= \frac{7}{3}$

RATIONAL NUMBERS: NUMBERS THAT CAN BE **EXPRESSED AS FRACTIONS** $\frac{a}{b}$ WHERE **a AND b ARE INTEGERS**

e.g. $\frac{1}{2}, \frac{7}{3}, \frac{4}{5}, \frac{11}{9}, 1.2, 3.12, 5.125, -0.001, \ldots$

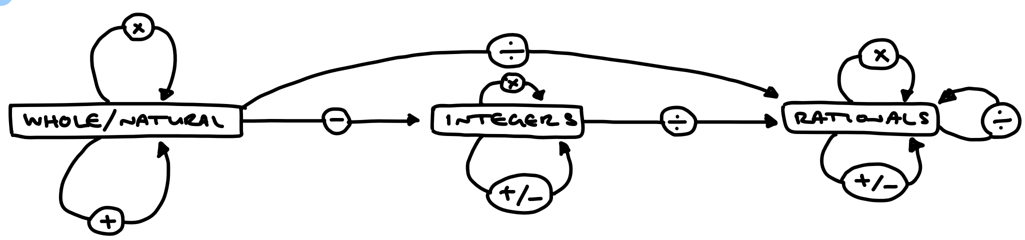
DECIMALS: RATIONALS CAN ALSO BE WRITTEN AS **DECIMALS** THAT...

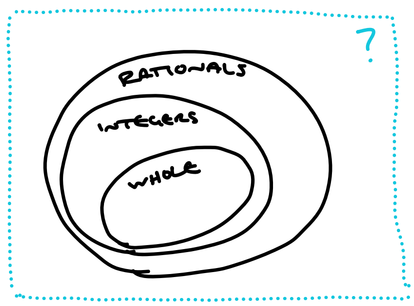
① **TERMINATE**: $1.2, 3.12, 5.125, -0.001, \ldots$

② **REPEAT**: $0.333\ldots = 0.\overline{3} = 1/3$

$$0.12121212\ldots = 0.\overline{12} = \frac{12}{99}$$

RECAP





* SO, WHY DO WE NEED MORE NUMBERS?? *

EXPONENTIATION + ROOTS

NATURAL EXPONENTS: IF n IS A NATURAL NUMBER, a^n IS **REPEATED MULTIPLICATION**.

$$\boxed{a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n}}$$

e.g. $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ $\qquad 5^1 = 5$

$(-3)^2 = (-3) \cdot (-3) = 9$

PROPERTIES:

$$a^m \cdot a^n = a^{m+n} \;:\; 2^3 \cdot 2^4 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2\cdot2) = 2^7$$

$$(a^m)^n = a^{mn} \;:\; (2^3)^4 = 2^3 \cdot 2^3 \cdot 2^3 \cdot 2^3 = 2^{3+3+3+3} = 2^{12}$$

$$\underbrace{\frac{a^n}{a^m} = a^{n-m}}_{} \;:\; \frac{2^5}{2^3} = \frac{2\cdot2\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = 2^2$$

WHAT IF n ISN'T GREATER THAN m?

ZERO AND NEGATIVE EXPONENTS

$$a^0 = 1 \;:\; \frac{a^n}{a^n} = a^{n-n} = a^0 = 1$$

$$a^{-n} = \frac{1}{a^n} \;:\; \frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$$

e.g.

① $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

② $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$

③ $\frac{x^2 y^3}{x^4 y} = \frac{y^2}{x^2}$

④ $\left(\frac{x^5}{y^4}\right)^2 = \frac{x^{10}}{y^8}$

⑤ $\left(\frac{x^5}{y^4}\right)^{-2} = \left(\frac{y^4}{x^5}\right)^2 = \frac{y^8}{x^{10}}$

⑥ $(x^2 y^3 z^5)^4 = x^8 y^{12} z^{20}$

⑦ SOLVE $x^3 = 64$

⑧ SOLVE $x^4 = 81$

$\boxed{\text{REQUIRE FRACTIONAL EXPONENTS / ROOTS}}$

FRACTIONAL EXPONENTS: $a^{m/n} = \sqrt[n]{a^m}$ or $(\sqrt[n]{a})^m$

e.g. $x^3 = 64$ $\qquad$ $x^4 = 81$

$(x^3)^{1/3} = 64^{1/3}$ $\qquad$ $(x^4)^{1/4} = 81^{1/4}$

$x = \sqrt[3]{64}$ $\qquad$ $x = \sqrt[4]{81}$

$x = 4$ $\qquad$ $x = 3$

e.g.

① $4^{3/2} = \sqrt{4^3}$ or $(\sqrt{4})^3$

$= 2^3$

$= \boxed{8}$

② $9^{3/2} = (\sqrt{9})^3$

$= 3^3$

$= \boxed{27}$

③ $8^{5/3} = (\sqrt[3]{8})^5$

$= 2^5$

$= \boxed{32}$

④ $16^{-1/2} = \frac{1}{16^{1/2}}$

$= \frac{1}{\sqrt{16}}$

$= \boxed{\frac{1}{4}}$

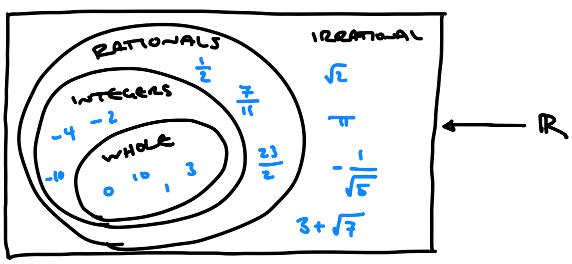
⑤ $x^2 = 2 \longrightarrow x = \sqrt{2}$??

NOT RATIONAL!

REAL NUMBERS: RATIONAL NUMBERS AND IRRATIONAL NUMBERS

$\mathbb{R}$

e.g. $\sqrt{2}$, $\sqrt[3]{5}$, π, e, $1.13215 76218\ldots$



SO WHAT...?

WE'VE FOUND THE NATURAL HIERARCHY OF NUMBERS CORRESPONDS TO A NATURAL HIERARCHY OF OPERATIONS

EXPONENTS / ROOTS

REPEATED

MULTIPLICATION / DIVISION

REPEATED

ADDITION / SUBTRACTION

* ORDER OF OPERATIONS: $\boxed{E \quad M \quad D \quad A \quad S}$ ← FOLLOWS HIERARCHY

* PARENTHESES: IMPOSE PRIORITY *

$\boxed{P \quad E \quad M \quad D \quad A \quad S}$

EXAMPLES:

① $10 + 2 \cdot (5 - 3) = 10 + 2 \cdot 2$
$= 10 + 4$
$= \boxed{14}$

② $18 + (6 - 8)^3 = 18 + (-2)^3$
$= 18 + (-8)$
$= \boxed{10}$

③ $9 - 18 \div 3^2 = 9 - 18 \div 9$
$= 9 - 2$
$= \boxed{7}$

④ $25 \div 5^2 - 7 = 25 \div 25 - 7$
$= 1 - 7$
$= \boxed{-6}$

⑤ $-2 \cdot \left[16 \div (8-4)^2\right]^2 = -2 \cdot \left[16 \div 4^2\right]^2$

$= -2 \cdot \left[16 \div 16\right]^2$

$= -2 \cdot 1^2$

$= -2 \cdot 1$

$= \boxed{-2}$

⑥ SIMPLIFY THE EXPRESSION $4x + x(13-7)$

$4x + x(13-7) = 4x + x(6)$

$= 4x + 6x$

$= \boxed{10x}$

⑦ SIMPLIFY THE EXPRESSION $9(y+8) - 27$

$9(y+8) - 27 = 9y + 72 - 27$

$= \boxed{9y + 45}$

⑧ SIMPLIFY THE EXPRESSION $9x + 4x(2+3) - 4(2x+3x)$

$9x + 4x(2+3) - 4(2x+3x) = 9x + 4x(5) - 4(5x)$

$= 9x + 20x - 20x$

$= \boxed{9x}$