

MATH 100

FACTORING POLYNOMIALS

GOALS

- ☐ FACTORING OUT GCF OF POLYNOMIAL
- ☐ FACTOR A TRINOMIAL
- ☐ RECOGNIZE AND FACTOR PERFECT SQUARES
- ☐ RECOGNIZE AND FACTOR DIFFERENCES OF SQUARES
- * ☐ FACTOR SUMS + DIFFERENCES OF CUBES
- * ☐ FACTORING WITH FRACTIONAL EXPONENTS

FACTORING (OUT) THE GCF OF A POLYNOMIAL

PROCESS

1. IDENTIFY THE GCF OF COEFFICIENTS
2. IDENTIFY THE GCF OF VARIABLE(S)
3. COMBINE THE GCFs FROM (1) + (2)
4. DIVIDE OUT FROM EACH TERM
5. WRITE FACTORED FORM.

EXAMPLES

$$\textcircled{1} \quad 6x^3y^4 + 45x^2y^2 + 21x^2y$$

$$\boxed{3x^2y (2xy^2 + 15y + 7)}$$

GCFs		
<u>COEFF.</u>	<u>x</u>	<u>y</u>
3	x^2	y

$$\textcircled{2} \quad 4x^4 + 2x^3 - 6x$$

$$\boxed{2x(2x^3 + x^2 - 3)}$$

<u>GCFs</u>	
<u>COEFF.</u>	<u>x</u>
2	x

$$\textcircled{3} \quad 36j^4k^2 - 18j^3k^3 + 54j^2k^4$$

$$\boxed{18j^2k^2(2j^2 - jk + 3k^2)}$$

<u>GCFs</u>		
<u>COEFF.</u>	<u>j</u>	<u>k</u>
18	j^2	k^2

$$\textcircled{4} \quad 49mb^2 - 35m^2ba + 77ma^2$$

$$\boxed{7m(7b^2 - 5mba + 11a^2)}$$

<u>GCFs</u>			
<u>COEFF.</u>	<u>m</u>	<u>b</u>	<u>a</u>
7	m	1	1

FACTORIZING TRINOMIALS

PROCESS: FACTORIZING $ax^2 + bx + c$

1. LIST FACTORS OF ac
2. FIND p AND q: FACTORS OF ac WITH A SUM OF b
3. REWRITE EXPRESSION AS $ax^2 + px + qx + c$
4. PULL OUT GCF OF $ax^2 + px$
5. PULL OUT GCF OF $qx + c$
6. FULLY FACTOR.

EXAMPLES:

① $x^2 + 3x + 2$

$a = 1 \quad b = 3 \quad c = 2$

$x^2 + x + 2x + 2$

$ac = 1 \cdot 2 = 2$

$x(x+1) + 2(x+1)$

FACTORS OF ac : 1, 2.

$(x+1)(x+2)$

② $x^2 - x - 6 = (x+2)(x-3)$

SINCE $a = 1$, WE KNOW IT WILL LOOK LIKE

$(x+p)(x+q)$

GOAL: FIND FACTORS OF -6 THAT ADD UP TO -1.

FACTORS OF -6: -1, 6; 6, -1; -2, 3; 3, -2

③ $6x^2 + 5x - 4$

$6x^2 - 3x + 8x - 4$

$3x(2x-1) + 4(2x-1)$

$(2x-1)(3x+4)$

FACTOR OUT $(2x-1)$

FACTORS OF $6 \cdot (-4) = -24$

1, -24

-1, 24

2, -12

-2, 12

3, -8

-3, 8 ← ADDS UP TO 5

4, -6

-6, 4

④ $2x^2 + 9x - 18$

$2x^2 + 12x - 3x - 18$

$2x(x+6) - 3(x+6)$

$(x+6)(2x-3)$

FACTORS OF $2 \cdot 18 = 36$

1, 36

2, 18

3, 12 ← $12 - 3 = 9 \checkmark$

4, 9

6, 6

$$\textcircled{5} \quad 2x^2 - 5x - 7 = \boxed{(2x-7)(x+1)}$$

CHECK: $2x^2 + 2x - 7x - 7$
 $2x^2 - 5x - 7 \checkmark$

FACTORS OF $2 \cdot (-7) = -14$

1, -14
 -1, 14
 2, -7 ← $2-7 = -5 \checkmark$
 -2, 7

$$\begin{aligned} & 2x^2 - 5x - 7 \\ \rightarrow & 2x^2 + 2x - 7x - 7 \\ & 2x(x+1) - 7(x+1) \\ & \boxed{(x+1)(2x-7)} \end{aligned}$$

$$\textcircled{6} \quad 12x^2 + x - 1 = \boxed{(3x+1)(4x-1)}$$

RECOGNISING PERFECT SQUARES AND DIFFERENCES OF SQUARES

PERFECT SQUARES

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

PROCESS

1. CONFIRM THAT FIRST AND LAST TERMS ARE SQUARES (a^2, b^2)
2. CONFIRM THAT MIDDLE TERM IS $2ab$
3. WRITE AS $(a+b)^2$

EXAMPLES:

$$\textcircled{1} \quad 36x^2 + 60x + 25 \\ (6x + 5)^2$$

$$36 = 6^2 \quad \text{AND} \quad 25 = 5^2$$

$$\underline{\underline{\text{AND}}} \quad 2 \cdot 6 \cdot 5 = 60$$

$$\textcircled{2} \quad 25x^2 - 120x + 144 = \boxed{(5x - 12)^2}$$

~~$(12x - 5)^2$~~

$$\begin{aligned} \underline{\text{CHECK:}} \quad (12x - 5)(12x - 5) &= 144x^2 - 60x - 60x + 25 \\ &= 144x^2 - 120x + 25 \quad * \end{aligned}$$

DIFFERENCES OF SQUARES

$$\boxed{(a+b)(a-b) = a^2 - b^2}$$

WHY:

$$\begin{aligned} (a+b)(a-b) \\ a^2 - ab + ab - b^2 \\ a^2 - b^2 \checkmark \end{aligned}$$

VOCAB.

"CONJUGATE PAIR"
A+B AND A-B

EXAMPLES

$$\textcircled{1} \quad x^2 - 9 = (x+3)(x-3)$$

$a = x \quad b = 3$

$$\textcircled{2} \quad 4x^2 - 25 = (2x+5)(2x-5)$$

$$\textcircled{3} \quad 36x^2 - 1 = (6x+1)(6x-1)$$

$$\textcircled{4} \quad 16x^2 - 9 = (4x-3)(4x+3)$$

FRACTION EXPONENTS:

$$\begin{aligned}
 & x^{5/3} + 5x^{2/3} + 6x^{-1/3} \\
 \text{FACTOR OUT } x^{-1/3}: & \quad x^{-1/3} \left(\frac{x^{5/3}}{x^{-1/3}} + \frac{5x^{2/3}}{x^{-1/3}} + \frac{6x^{-1/3}}{x^{-1/3}} \right) \\
 & x^{-1/3} (x^{6/3} + 5x^{3/3} + 6) \\
 & x^{-1/3} (x^2 + 5x + 6) \\
 & \boxed{x^{-1/3} (x+2)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 x^2 \cdot x^5 &= \underline{(x \cdot x)(x \cdot x \cdot x \cdot x \cdot x)} \\
 &= x^7
 \end{aligned}$$

WHEN WE FACTOR OUT A TERM, WE
DIVIDE EVERY TERM BY THAT FACTOR

$$\begin{aligned}
 \text{e.g.} \quad & 4x^3 - 2x^2 + 6x \\
 \text{USUALLY SKIP A STEP} \quad & \left(2x \left(\frac{4x^3}{2x} - \frac{2x^2}{2x} + \frac{6x}{2x} \right) \right) \\
 & \rightarrow 2x(2x^2 - x + 3)
 \end{aligned}$$