MATH 100
WORKING WITH FRACTIONS, EXPONENTS, AND RADICALS

GOALS

- PERFORM ALL BASIC OPERATIONS WITH FRACTIONS
- APPLY PROPERTIES OF EXPONENTS TO SIMPLIFY EXPRESSIONS AND SOLVE EQUATIONS
- SIMPLIFY RADICALS

WORKING WITH FRACTIONS

ADDING / SUBTRACTING: FIND COMMON DENOMINATOR

- BRUTE FORCE: USE PRODUCT OF DENOMINATORS

e.g.
$$\frac{1}{4} - \frac{1}{6} = \frac{6}{6}\cdot\frac{1}{4} - \frac{4}{4}\cdot\frac{1}{6}$$
$$= \frac{6}{24} - \frac{4}{24}$$
$$= \frac{2}{24}$$
$$= \boxed{\frac{1}{12}}$$

$4 \cdot 6 = \underline{24}$

- EFFICIENT: USE LEAST COMMON MULTIPLE (LCM)

e.g.
$$\frac{1}{4} - \frac{1}{6} = \frac{3}{3}\cdot\frac{1}{4} - \frac{2}{2}\cdot\frac{1}{6}$$
$$= \frac{3}{12} - \frac{2}{12}$$
$$= \boxed{\frac{1}{12}}$$

MULT. OF 4: 4, 8, 12, 16, 20, 24...

MULT OF 6: 6, 12, 18, 24, ...

EXAMPLES:

① $\frac{1}{2} + \frac{1}{3} = \frac{3}{3}\cdot\frac{1}{2} + \frac{2}{2}\cdot\frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$= \boxed{\frac{5}{6}}$

② $\frac{2}{5} - \frac{1}{6} = \frac{6}{6}\cdot\frac{2}{5} - \frac{5}{5}\cdot\frac{1}{6}$

$= \frac{12}{30} - \frac{5}{30}$

$= \boxed{\frac{7}{30}}$

③ $\frac{4}{7} - \frac{1}{14} = \frac{2}{2}\cdot\frac{4}{7} - \frac{1}{14}$

$= \frac{8}{14} - \frac{1}{14}$

$= \frac{7}{14}$

$= \boxed{\frac{1}{2}}$

④ $\frac{1}{8} - \frac{1}{2} = \frac{1}{8} - \frac{4}{4}\cdot\frac{1}{2}$

$= \frac{1}{8} - \frac{4}{8}$

$= \boxed{-3/8}$

⑤ $\underbrace{\frac{1}{2} - \frac{1}{3} + \frac{1}{4}}_{\text{LCM: 12}} = \frac{6}{6}\cdot\frac{1}{2} - \frac{4}{4}\cdot\frac{1}{3} + \frac{3}{3}\cdot\frac{1}{4}$

$= \frac{6}{12} - \frac{4}{12} + \frac{3}{12}$

$= \boxed{\frac{5}{12}}$

⑥ $\frac{2}{5} + \frac{1}{6} + \frac{1}{3} = \frac{12}{30} + \frac{5}{30} + \frac{10}{30}$

$= \frac{27}{30}$

$= \boxed{\frac{9}{10}}$

MULTIPLYING/DIVIDING:

* MULTIPLY STRAIGHT ACROSS
* DIVISION = MULTIPLICATION BY RECIPROCAL

e.g. $\frac{2}{5}\cdot\frac{4}{3} = \frac{8}{15}$

$\frac{2}{5} \div \frac{4}{3} = \frac{2}{5}\cdot\frac{3}{4}$

$= \frac{6}{20}$

$= \frac{3}{10}$

SAME AS...

$\frac{\frac{2}{5}}{\frac{4}{3}} = \frac{2}{5}\cdot\frac{3}{4}$

EXAMPLES:

① $\frac{3}{7} \cdot \frac{3}{4} = \boxed{\frac{9}{28}}$

② $\frac{1}{2} \cdot \frac{7}{8} = \boxed{\frac{7}{16}}$

③ $\frac{4}{5} \div \frac{1}{10} = \frac{4}{5} \cdot \frac{10}{1}$

$= \frac{40}{5}$

$= \boxed{8}$

④ $\frac{7}{3} \div \frac{5}{9} = \frac{7}{\cancel{3}} \cdot \frac{\cancel{9}}{5}$ $\qquad \frac{63}{15}$

$= \frac{7}{1} \cdot \frac{3}{5}$ OR $=$

$= \boxed{\frac{21}{5}}$

⑤ $\frac{1}{2} \cdot \frac{3}{4} - \frac{1}{6} = \frac{3}{8} - \frac{1}{6}$

$= \frac{3}{3} \cdot \frac{3}{8} - \frac{4}{4} \cdot \frac{1}{6}$

SINCE 6 AND 8 ARE BOTH EVEN, WE KNOW THE LCM ISN'T 6·8=48.

$= \frac{9}{24} - \frac{4}{24}$

$= \boxed{\frac{5}{24}}$

⑥ $\frac{2}{5} + \frac{1}{3} \div \frac{1}{4} = \frac{2}{5} + \frac{1}{3} \cdot \frac{4}{1}$

PEMDAS

$= \frac{2}{5} + \frac{4}{3}$

$= \frac{3}{3} \cdot \frac{2}{5} + \frac{5}{5} \cdot \frac{4}{3}$

$= \frac{6}{15} + \frac{20}{15}$

$= \boxed{\frac{26}{15}}$

WORKING WITH EXPONENTS / RADICALS

PROPERTY #1: RADICALS OF PRODUCTS

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ B/c $(ab)^{1/2} = a^{1/2} b^{1/2}$

e.g. $\sqrt{200} = \sqrt{2 \cdot 100}$

$= \sqrt{2}\sqrt{100}$

$= \boxed{10\sqrt{2}}$

NOTE ABOUT $\sqrt{a+b}$: CAN'T SIMPLIFY THIS! $(a+b)^{1/2}$

PEMDAS

e.g. $\sqrt{4+9} \neq \sqrt{4} + \sqrt{9} = 2 + 3 = 5$

$\sqrt{13} \neq 5$

EXAMPLES

① $\sqrt[3]{16} = \sqrt[3]{2 \cdot 8}$

$= \sqrt[3]{2}\,\underbrace{\sqrt[3]{8}}$

$\sqrt[3]{8} = 2$

b/c $2^3 = 8$

$= \boxed{2\sqrt[3]{2}}$

LOOK FOR FACTORS OF 16

THAT ARE PERFECT CUBES...

$1^3 = 1$, $2^3 = 8$, $3^3 = 27$, ...

② $\sqrt[4]{\dfrac{16}{81}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{81}} = \boxed{\dfrac{2}{3}}$

ROOTS IN WEBWORK:

$\sqrt[4]{2}$ = 2 ^ (1/4)

③ $5\sqrt{12} - 2\sqrt{3} = 5\sqrt{4 \cdot 3} - 2\sqrt{3}$

$= 5\sqrt{4}\sqrt{3} - 2\sqrt{3}$

$= 5 \cdot 2\sqrt{3} - 2\sqrt{3}$

$= 10\sqrt{3} - 2\sqrt{3}$

$= \boxed{8\sqrt{3}}$

$3 \cdot 4$ PERF SQ.

$2 \cdot 6$ NO PERF. SQUARES

④ SIMPLIFY $\sqrt{75} = \sqrt{3 \cdot 25}$

$= \sqrt{3}\sqrt{25}$

$= \boxed{5\sqrt{3}}$

⑤ SIMPLIFY $\sqrt[3]{32} = \sqrt[3]{4 \cdot 8}$

$= \sqrt[3]{4}\,\sqrt[3]{8}$

$= \boxed{2\sqrt[3]{4}}$

⑥ SIMPIFY AND COMBINE LIKE TERMS: $14\sqrt{6} - 6\sqrt{24}$

$= 14\sqrt{6} - 6\sqrt{4\cdot 6}$

$= 14\sqrt{6} - 6\cdot 2\sqrt{6}$

$= 14\sqrt{6} - 12\sqrt{6}$

$\boxed{2\sqrt{6}}$

⑦ SIMPIFY AND COMBINE LIKE TERMS: $12\sqrt{3} - 4\sqrt{75}$

$12\sqrt{3} - 4\cdot\sqrt{3\cdot 25}$

$12\sqrt{3} - 4\cdot 5\sqrt{3}$

$12\sqrt{3} - 20\sqrt{3}$

$\boxed{-8\sqrt{3}}$

⑧ $\sqrt{64x^4} = \sqrt{64}\sqrt{x^4}$

$(x^4)^{1/2} = x^{4/2} = x^2$

$= 8x^2$

$$\boxed{\begin{aligned}(2^3)^2 &= (2\cdot 2\cdot 2)^2\\ &= (2\cdot 2\cdot 2)(2\cdot 2\cdot 2)\\ &= 2^6\end{aligned}}$$

⑨ $\sqrt[3]{\dfrac{27x^3}{y^{12}}} = \dfrac{\sqrt[3]{27}\ \sqrt[3]{x^3}}{\sqrt[3]{y^{12}}}$

$(y^{12})^{1/3} = y^{12/3} = y^4$

$= \boxed{\dfrac{3x}{y^4}}$