An introduction

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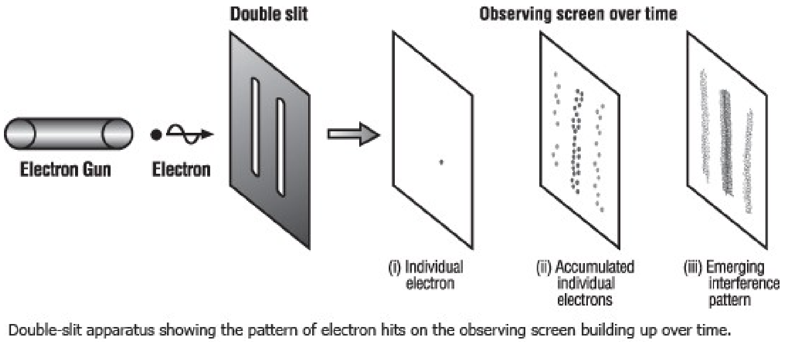

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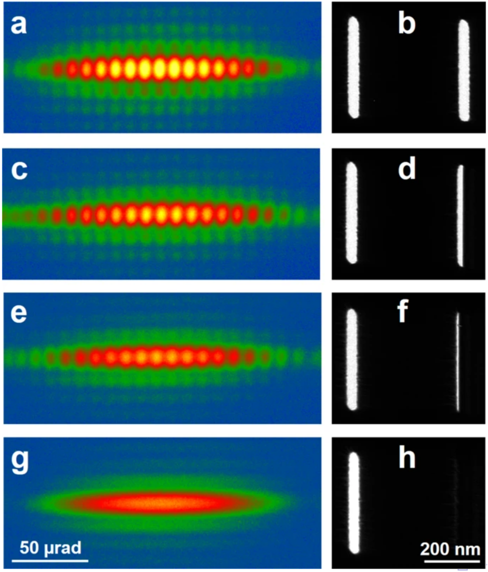
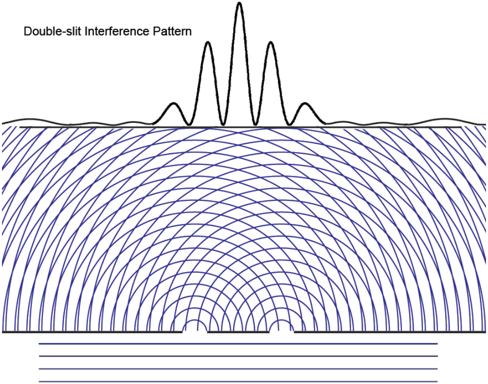
Quantum world if fascinating

Particles behave like waves

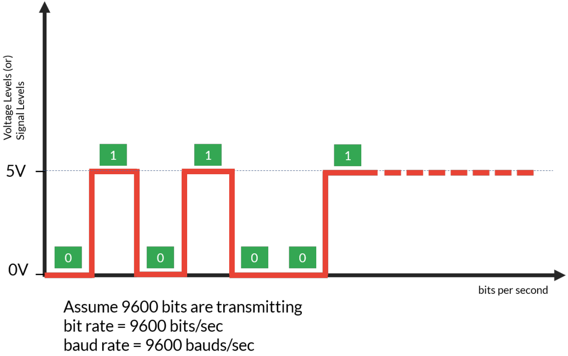


The state of a particle after passing through either one of the slits can be described as a *wave* function (probability distribution) namely $\Psi = (\alpha_0\psi_0 + \alpha_1\psi_1)$ with $\{\alpha_0, \alpha_1\} \in \mathbb{C}$



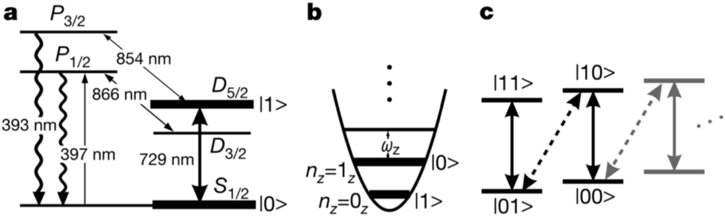
Basic Unit of information: Bits

Traditional computation works with 0 and 1 as basic units of information. A physical realization of this is voltage from 0V to 5V



Basic Unit of information: Qubits

Quantum computation works with $|0\rangle$ and $|1\rangle$ as basic units of information. A physical realization of this would be a spin $1/2$ particle.



Computational basis states

Qubits can be in different states *other* than $|0\rangle$ or $|1\rangle$. It is possible to form *linear combinations* of states, called superpositions:

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

The numbers α_0 and α_1 are complex numbers and $|\alpha_0|^2 + |\alpha_1|^2 = 1$.

Where $|0\rangle$ and $|1\rangle$ are vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ in $\mathbb{C}^2$.

A superposition state is a linear combination $\psi = \alpha_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Quantum NOT gate

Classical computer circuits consists of wires and logic gates. E.g. the NOT gate which has a truth table $0 \to 1$ and $1 \to 0$.

The analogous quantum operation would take

$$\alpha_0 \left|0\right\rangle + \alpha_1 \left|1\right\rangle$$

to

$$\alpha_0 \left|1\right\rangle + \alpha_1 \left|0\right\rangle$$

Since a quantum state can be represented as a vector, we are looking for a matrix such that:

$$\mathrm{X} \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_0 \end{bmatrix}$$

Quantum Gates

Quantum gates are represented by matrices applied on our vectors (qubits). Are all matrices valid Quantum Gates?...no.

Recall that $|\alpha_0|^2 + |\alpha_1|^2 = 1$ for a quantum state

$$\alpha_0 |0\rangle + \alpha_1 |1\rangle$$

. This must also hold for

$$\alpha_0^{'} |0\rangle + \alpha_1^{'} |1\rangle$$

after the gate has acted. It turns out that the appropriate condition on the matrix representing the gate is tha the matrix U be unitary. That is (with $U^\dagger$ is the adjoint of U)

$$U^\dagger U = I$$

Quantum Gates

In classical computers we only have one non-trivial gate for one bit (NOT gate). In the case of Quantum computers we have many!

For example the NOT gate:

$$\mathrm{X} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The *Haddamard* gate

$$\mathrm{H} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Bibliografía

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