Pattern classification
3 ways

↳ classification: assign classes to "known patterns

↳ clustering: assign a pattern to one of the syntactically 'labelled class'

↳ Semi-supervised classification:
Given a small collection semantically labelled patterns & a large syntactically patterns. The task is to assign a semantic label to the test pattern.

① Classification

Assign a pattern to the already known pattern (semantically labelled) class

eg. Let us consider two classes

 & rectangles are class labels.

↳ The classification problem is to

ⓐ either learn a model or directly use the training data set (collection of labelled patterns)

ⓑ assign a class label to a new pattern (test pattern) or equivalently assign the test pattern to one of the known classes

↳ This is with respect to objects ie ellipse & rectangles / given a new object we would like to classify it as either ellipse or a rectangle

Let us consider semantically labelled pattern X, where as

$X = (x_1\, c^1)\, (x_2\, c^2) \ldots\ (x_n\, c^n)$

① The no of classes k is fixed & finite.

The value of k is known a priori

Let classes be labelled as

$c_1\ c_2\ c_3 \ldots c_k$

② The ~~no~~ set X is finite & is of size (cardinality) n.

Further x_i represent the ith pattern & c^i is the corresponding semantic class label for $i = 1 \ldots n$.

so that $c^i \in C = (c_1\, c_2 \ldots c_k)$

③ ~~Let X be a test pattern~~

classification (Example)

Let us consider two classes where 'cat' & 'dog' are two objects

Then semantically labelled pattern
X can be represented as

$$X = \{(x_1, cat), (x_2\ cat)$$
$$\dots (x_{10}, dog)\}$$

Now a problem is to given a pattern
x & classify it either cat or dog

Clustering

we are given collection X,
of syntactically labelled patterns, where
$X = \{x_1, x_2 \dots x_n\}$.

↳ Patterns are syntactically labelled
using different subscripts. The problem
is partition of X into some finite
no. of blocks or clusters

↳ In other words, we partition
so that $X = C_1 \cup C_2 \cup C_3 \dots \cup C_k$

↳ where
C_i = cluster
$C_i \neq \phi$ $\quad C_i \cap C_j = \phi$ for $i \neq j$,
$\forall i, j \in \{1, 2, \dots k\}$

clustering (example)

↳ Consider a collection of patterns

$X = \{X_1, X_2 \cdots X_{10}\}$

↳ A possible partition of X may be into two classes as

$C_1 = \{X_1\ X_2\ X_4\ X_5\ X_7\ X_8$

$C_2 = \{X_3\ X_6\ X_9\ X_{10}\}$

↳ Typically the criteria used for partition is similarity or matching. A pattern in C_1 is dis-similar to pattern in C_2

(3) $\boxed{\text{Semi-supervised classification}}$

↳ In this a small set

problem is to assign new patterns (test pattern) to one of the classes or equivalently assign a semantic label to the pattern

Probability

sample space : the set of all random outcomes

set of events : subset

(3) Conditional Probility

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ B with nonzero P

P of A after occurrence of B

(1) Joint Probability :

$P(A \text{ and } B) = P(A \cap B)$

eg pack of cards Red & 4 $\frac{2}{52}$ (heart & diamond)

(2) Marginal Probablity : single event probability (marginalizing the other events)

Bayes theorem

use : to manipulate conditional probab

two events A & B

$P(B|A)$ to $P(A|B)$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|A')\,P(A')}$$

eg $P(\text{cancer}) = 0.01 = P(A)$

$P(\text{positive test} \mid \text{cancer}) = 90\% = 0.9$

$P(\text{positive test} \mid \text{no cancer}) = 8\% = 0.08$

what is the P that abc has cancer, if she is tested positive

$P(B) = P(\text{positive test})$

$P(A \mid B)$

$$P(\text{cancer} \mid \text{positive test}) = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.08 \times 0.99}$$

$$= 0.12$$

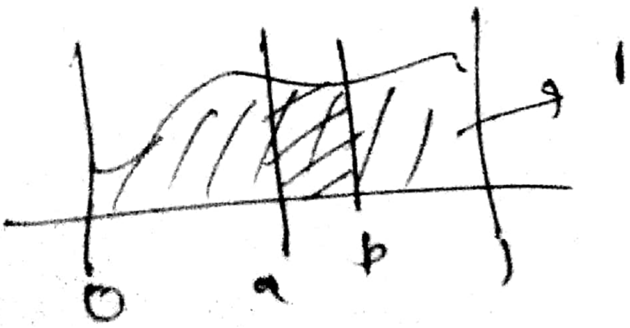
<u>Probability density</u>

It is the Prob that x will occur in an interval (a, b)

$$P(x \in (a, b)) = \int_a^b p(x)\,dx$$

$$p(x) \geq 0$$

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$



Expectations & co-variance

use: → weighted average (average value?)

avg. value of a function $f(x)$ under a prob distribution $p(x)$ is called expectation of $f(x)$

discrete distribution

$$E|f| = \Sigma p(x) f(x)$$

continuous $E|f| = \int p(x) f(x) dx$

conditional Expectations

$$E[f|y] = \Sigma (p(x|y) f(x))$$

$\boxed{\text{Variance}} (\sigma)^2 = E($

$$var(X) = \sigma^2 = E(X^2) - E(X)^2$$

$$E(X-m)^2 = E[X^2 - 2Xm + m^2]$$

$$= E(X^2) - 2m E(X) + m^2$$

$$= E(X^2) - 2m^2 + m^2$$

$$= E(X^2) - [E(X)]^2$$

Expectation → measure of central tendency

variance → measure of spread

$\boxed{\text{co-variance}}$ $E(XY) - E(X)E(Y)$

$= E(XX) - E(X)E(Y)$

Bayes classifier

① P(elephant is black) = 0.8
P(elephant is white) = 0.2

any elephant is classified is black
error = 0.2

② P(elephant is white | elephant is from region X)

If 95% of time when elephant is white, it belongs to region X

$$P(W|X) = \frac{P(X|W) \times P(W)}{P(X)} = \frac{0.95 \times 0.2}{0.2}$$

$= 0.95$

P error that elephant is not white = 0.05

classes: small, medium, large

$P(\text{small}) = \frac{1}{3}$

$P(\text{medium}) = \frac{1}{2}$

$P(\text{large}) = \frac{1}{6}$

nails, bolts, rivets

$P(\text{nail} | \text{small}) = \frac{1}{4}$ | medium $= \frac{1}{2}$ | large $\frac{1}{3}$

$P(\text{bolt} | \text{small}) = \frac{1}{2}$ $\frac{1}{6}$ | large =

$P(\text{rivet} | \text{small}) = \frac{1}{4}$ $\frac{1}{3}$ $= \frac{1}{3}$

$$P(\text{small}|\text{nail}) = \frac{P(\text{nail}|\text{small})\, P(\text{small})}{P(\text{nail}|\text{small})\, P(\text{small}) + P(\text{nail}|\text{medium}) \cdot P(\text{medium}) + P(\text{nail}|\text{large}) \cdot P(\text{large})}$$

$= 0.2143$

$P(\text{medium}|\text{nail}) = 0.6429)$

$P(\text{large}|\text{nail}) = 0.1429$

$P(\text{medium}|\text{nail}) > P(\text{small}|\text{nail})$
$P(\text{medium}|\text{nail}) > P(\text{large}|\text{nail})$

$P(\text{error}|\text{nail}) = 1 - 0.6429$
$= 0.3571$

$P(\text{small}|\text{bolt}) = 0.5455$
$P(\text{medium}|\text{bolt}) = 0.2727$
$P(\text{large}|\text{bolt}) = 0.1818$

$P(\text{error}|\text{bolt}) = 1 - 0.5455$
$= 0.4545$

small

$P(\text{small}|\text{rivet}) = 0.2727$
$P(\text{medium}|\text{rivet}) = 0.5455$
$P(\text{large}|\text{rivet}) = 0.1818$

$1 - 0.5455$
$P(\text{error}|\text{rivet}) = 0.4545$

medium

$C(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right)$

$\boxed{N(A) = N(\text{rref } A)}$ — reduced row echelon form

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \end{bmatrix} \xrightarrow[R_3 - 3R_1]{2R_1 - R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \end{bmatrix} \xrightarrow[R_3 - R_2]{R_1 - R_2} \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rref

null space

pivot

pivot

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 0x_2 + 3x_3 + 2x_4 = 0$

$0x_1 + x_2 - 2x_3 + x_4 = 0$

$x_3 \rightarrow$ free variable

$x_4 \rightarrow$ free variable

$x_1 = -3x_3 - 2x_4$

$x_2 = 2x_3 + x_4$

$$N(A) = N(\text{rref}) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

x_3, x_4 any real no.s

$A\vec{x} = 0$

$$N(A) = \text{span}\left(\begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right)$$

linear independent means, only one solution

$A\vec{x} = 0$

$\vec{x} = \vec{0}$

If, $N(A) = \{\vec{0}\}$

$N(A)$ more than just $\vec{0}$

linearly dependent set

$\boxed{\text{Basis}}$ span a subspace

also, linearly independent

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = 0$$

x_3 & x_4 ∃ free variable, let $x_3 = 0$

$x_4 = -1$

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$x_1 = -3x_1 - 2x$

$x_2 = 2x_3 + x$

$x_3 = 0$ & $x_4 = -1$

$x_1 = 2$

$x_2 = -1$

Let $x_4 = 0$

$x_3 = 1$

then $x_1 = 3$

$x_2 = -2$

$$C(A) = \text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}\right)$$