Papa kofi Akoto Asante

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Candidate's I.D. 39122022 Question Quest 3 i

Find the limits of

$$\lim_{n \to \infty} \frac{\ln(n) + 4}{5n^4 + 7n + 6}$$

$$\lim_{n \to \infty} \left(\frac{d/dn\,(\ln(n) + 4)}{d/dn\,(5n^4 + 7n + 6)} \right)$$

$$\lim_{n \to \infty} \frac{1/n}{20n^3 + 14n}$$

$$\lim_{n \to \infty} \left(\frac{1}{n \cdot (20n^3 + 14n)} \right)$$

$$\lim_{n \to +\infty} \left(\frac{1}{n \times (20n^3 + 14n)} \right)$$

$$\lim_{n \to +\infty} (1) \quad , \quad \lim_{n \to +\infty} (n \times (20n^3 + 14n))$$

$$1 + \infty = 0$$

$$= 0$$

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Quest 3 Part I ii

Question ii

ii

$$\lim_{n \to \infty} \frac{2^n}{\log_2(n)}$$

$$\lim_{n \to +\infty} \left(\frac{d/dn \ (2^n)}{d/dn \ (\log_2(n))} \right)$$

$$\lim_{n \to +\infty} \left(\frac{\cancel{x} \ln(2) \cdot 2^n}{\frac{1}{\ln(2) n}} \right)$$

$$\lim_{n \to +\infty} \left(\ln(2)^2 \times 2^n \cdot n \right)$$

$$\lim_{n \to +\infty} (n) = +$$

$$= +\infty + \infty = +\infty$$

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Candidate's I.D. 39122022 Question 2

Quest 3 Part 2 i) and ii)

Give ~~the~~ quickly the approximations of the following

i) $\sum_{k=0}^{30} k^2 = \frac{n \times (n+1) \times (2n+1)}{6}$

$= \frac{30(30+1) \times (2(30)+1)}{6}$

$= 9{,}455$

ii) $\sum_{k=0}^{100} k^3 = \frac{n^2 \times (n+1)^2}{4}$

$\frac{100^2 \times (100+1)^2}{4}$

$= 25{,}502{,}500 = 2.55025 \times 10^7$

Quest 3 Part 3 i

Prove by induction that

i) $\sum_{k=0}^{n} k \binom{n}{k} = n^{2-1}$

$\sum_{k=0}^{n} kn + \sum_{k=0}^{n} k \cdot k = n^{2-1}$

Base case, the statement must be true for $= n=1$

$\sum_{k=0}^{1} 0 = \frac{0(0+1)}{2} = \cancel{\frac{2}{2}} = 1 \; 0$

if $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$, is true for an arbitery

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Candidate's I.D. 39122022 Question 3 Part 3i

i) In the statement must be true for n+1,

A. $\sum_{k=0}^{n+1} k = n+1$

$= n+1 + n \cdot \sum_{k=0}^{n} k + \sum^{n} k^2$

$= (n+1) + n \times \frac{n(n+1)}{2} + \sum k^2$

$= n+1 + \frac{n^2(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n+2+n^2(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n+2+n^3+n}{2} + \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n+2+n^3}{2} + \frac{n(n+1)(2n+1)}{6}$

and $\sum$

ii.) $2 \cdot \sum_{k=0}^{n} 3^k = 3^{n+1} - 1$

if $|r| < 1$ (whether r is a real or complex ~~number~~), it is also true that

$$\sum_{k=0}^{n} r^k = \frac{1}{1-r}$$

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$$\cancel{\sum_{k=0}^{n} 3^k = \frac{1}{1-3}}$$

~~By summation~~

By induction. The formular should be coreet for $n=1$:

$$\sum_{k=0}^{n} 3^k = 3^0 = 1 \Leftrightarrow \frac{1-3^{0+1}}{1-3}$$

Assuming that the formular is correct for an ~~arbitary~~ arbitany n. In that case, we have

$$\sum_{k=0}^{n+1} 3^k = 3^{k+1} + \sum_{k=0}^{n} 3^k \times 2$$

$$= 3^{k+1} + \frac{1-3^{k+1}}{1-3} \times 2$$

$$= \frac{3^{k+1}(1-3) + 1 - 3^{k+1}}{1-3} \times 2$$

$$= \frac{3^{k+1} - 3 \cdot 3^{k+1} + 1 - 3^{k+1}}{1-3} \times 2$$

$$= \frac{-3 \cdot 3^{k+1} + 1}{1-3} \times 2$$

$$= \frac{-3 \cdot 3^{k+1} + 1}{-2} \times 2$$

Candidate's I.D. 39122022 Question 3 Part 3 II

$$\frac{+3 \cdot 3^{k+1} + 1}{+2} \times 2$$

$$= \frac{3 \cdot 3^{k+1} - 1}{\cancel{2}} \cdot \cancel{2}$$

$$= 3 \cdot 3^{k+1} - 1$$

Quest 4 Master Theorem.

i)

$T(n) = 7T(\frac{n}{2}) + n^2$

Soln

Comparing the above recurrence relation with

$T(n) = aT(n/b) + \Theta(n^k \log^p n)$

we have

$a = 7$

$b = 2$

$k = 2$

$p = 0$

Case 01

Now, $a = 7$ and $b^k = 2^2 = 4$

If $a > b^k$ then $T(n) = \Theta(\cancel{n \log}\, n^{\log_b a})$

since $7 > 4$, $= T(n) = \Theta(\cancel{n \log}\, n^{\log_b a})$

$= T(n) = \Theta(n^{\log_2 7})$

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Quest 4 ~~Part~~ ii

Question ~~4~~ ii

ii $T(n) = 5T(n/3) + O(n)$

Comparing it to the recurrence relation we ~~find that~~

~~$T(n) = aT(n/3) + O(n)$~~

$T(n) = aT(n/b) + \Theta(n^k \log^p n)$

we find that

$a = 5$

$b = 3$

$k = 0$

$p = 0$

Case 01

if $a > b^k$ then $T(n) =$ ~~$\Theta(n$ to~~ $\Theta(n^{\log_b a})$

$5 > 3^0$

since $5 > 3^0$,

~~hence~~

$T(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_3 5})$

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Candidate's I.D. 39122022

Quest

Question 4 III

III $T(n) = 3T(\frac{n}{2}) + \frac{3}{4}(n) + 1$

Comparing the above recurrence relation to the geneneral recurence relation ; ~~we~~

$T(n) = aT(\frac{n}{b}) + \cancel{3/4}\ \Theta(n^k \log^p n)$

we find that

$a = 3$

$b = 2$

$k = 1$

$\cancel{k}p = 0$

Case ~~D~~ 1

if $a > b^k$ then $T(n) = \Theta(n^{\log_b a})$

$3 > 2^{(1)}$

Since $3 > 2$

$T(n) = \Theta(n^{\log_2 3})$

~~Tn~~

$T(n) = \Theta(n^{\log_2 3})$