Brief Explanation Of The Code In ”2016/hw8/cyclotron.m”

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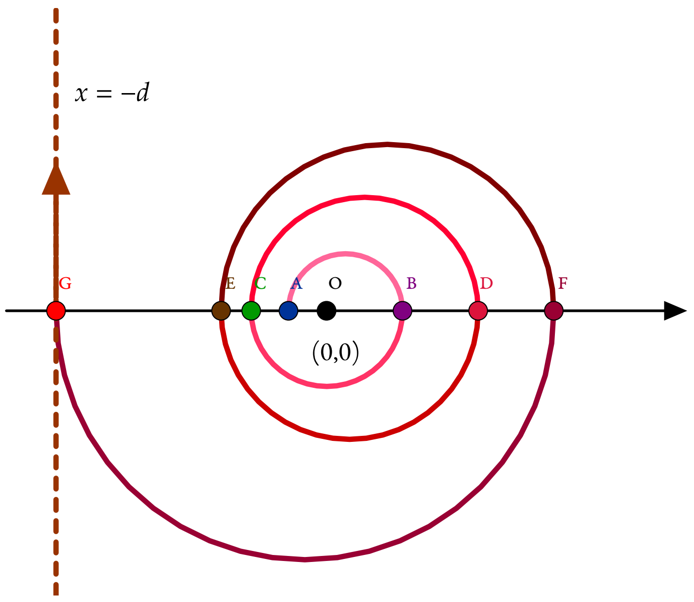
Abstract

In this brief article we explore how the cyclotron works and the implementation of the code in matlab to compute the energy in MeV the particle has acquired during the acceleration process, along with the number of semicircle trajectories it followed until it exits the device. Most students who tried to solve this problem in matlab , couldn’t understand the hint and tried to solve the problem without having understood that the centers of the semicircles the particle runs on (2-dimensional imagery), fall on different points of the line defined by the distance between the north and south D’s.

The figure we will use to show a simple case with 6 steps (only six accelerations) is shown below this paragraph. We assume that the distance between the north and south semicircles is infinitesimally small which we define as the ”x-axis” (with center O) the space in which a positively charged isotope of hydrogen, called *deuteron* is accelerated.



The function we want to define in matlab takes only one **scalar** input, which is the voltage applied to the cyclotron. The cyclotron rapidly alternates the sign of the voltage difference V in units of volts between two ”D”-shaped vacuum chambers, which are placed within a strong uniform magnetic field (perpendicular to the page). Each time the particle enters the *x-axis*, it is accelerated instantly by the voltage applied to it increasing its energy, then it enters a ”D”-shaped vacuum chamber with velocity it acquired, which remains constant until it passes again from the ”x-axis”, exits the vacuum and is accelerated again until it finally reaches the exit placed at a distance specified by the conductor of the experiment to the left of the center of the ”x-axis”, let us say $-d$, with $d > 0$. At time zero, the deuteron of mass m and charge q enters the cyclotron at point A, it is accelerated as long as it stays in the ”x-axis” space (infinitesimal fraction of second), then enters the first ”D”-vacuum with a constant speed u_1 and runs with a semi-circle trajectory due to the magnetic field of strength B, until it exits at point B. Point A lies at

$$s_0 = -\underbrace{\sqrt{\frac{m \cdot V}{q \cdot B^2}}}_{r_1}$$

and point B at $s_1 = s_0 + 2 \cdot r_1$ in the ”x-axis”, with r_1 being the radius of the first semicircle. The recursive formula given r_1, which will give us the next radii is

$$r_n = \sqrt{r_{n-1}^2 + 2 \cdot r_1^2}$$

so in oder to compute the position C, we get $r_2 = \sqrt{r_1^2 + 2 \cdot r_1^2}$ and so the position of C in the ”x-axis” is $s_2 = s_1 - 2 \cdot r_2$. Following the same computation by adding the diameters when the particle is moving to the east and subtracting when it moves to the western direction, we get $r_3 = \sqrt{r_2 + 2 \cdot r_1^2}$ and $s_3 = s_2 + 2 \cdot r_3$ (point D), respectively the computations for the other points until the exit point G are, for point

E: $r_4 = \sqrt{r_3 + 2 \cdot r_1^2}$ and $s_4 = s_3 - 2 \cdot r_4$, for point F: $r_5 = \sqrt{r_4 + 2 \cdot r_1^2}$ and $s_5 = s_4 + 2 \cdot r_5$ and lastly for point G: $r_6 = \sqrt{r_5 + 2 \cdot r_1^2}$ and $s_6 = s_5 - 2 \cdot r_6$. We started with $N = 0$ made the computations of r_N and s_N,until $s_N > -d$ and since we found the number of spins of the particle in the cyclotron which is the first thing we are asked to compute with the program we are expected to write, we are ready to answer the second question which is to compute the energy that the particle has acquired during the process, which is $E = V \cdot N \cdot 10^{-6}$ (MeV) and the program written for $d = 0.5$ in matlab follows.

```matlab
function [E,N] = cyclotron(V)
%deuteron mass in kg
m = 3.344*1e-27;
%deuteron charge 1.603 * 1e-19 in Coulomb
q = 1.603*1e-19;
%magnetic field strength in Tesla
B = 1.6;
%Initial Values for starting radius (r),initial position (p) after first
%spin and initial spin count (N)
r = sqrt((m*V)/(q*B^2));
s = r/2;
p = -s + 2 * r;
N = 1;
while p > - 0.5
    N = N + 1;
    r = sqrt(r^2 + (2*m*V)/(q*B^2));
    p = p + (-1)^(N-1) * 2 * r;
end
E = N * V * 1e-6; %MeV
end
```