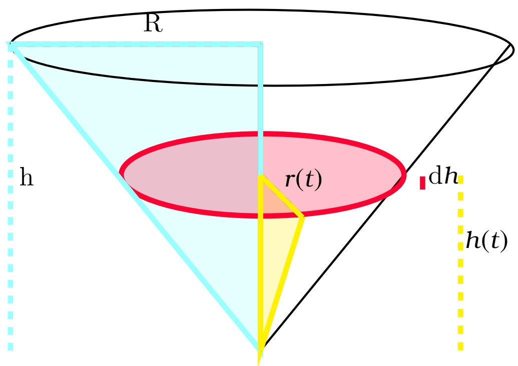
A conical glass is filled with water at a steady rate, prove that the function of the height of the surface of the water in the glass with respect to time has a cube root behavior.



From the similarity of the cyan and the yellow triangles we get

$$\frac{R}{h} = \frac{r(t)}{h(t)} \Rightarrow r(t) = c \cdot h(t)$$

the infinitesimal volume of water added is

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \pi \cdot r(t)^2 \cdot \frac{\mathrm{d}h}{\mathrm{d}t} \Rightarrow c_1 = \pi \cdot r(t)^2 \cdot \frac{\mathrm{d}h}{\mathrm{d}t}$$

with substitution we get

$$\begin{aligned}
c_1 &= \pi \cdot c^2 \cdot h(t)^2 \cdot \frac{\mathrm{d}h}{\mathrm{d}t} \\
h(t)^2 \mathrm{d}h &= c_1 \frac{\mathrm{d}t}{c^2 \pi} \\
\int h(t)^2 \, \mathrm{d}h &= \frac{c_1}{c^2 \pi} \int 1 \, \mathrm{d}t \\
\frac{h(t)^3}{3} &= \frac{c_1}{c^2 \pi} t + c_2 \\
h(t)^3 &= 3 \frac{c_1}{c^2 \pi} t + 3 c_2 \\
h(t) &= \sqrt[3]{3 \frac{c_1}{c^2 \pi} t + 3 c_2}
\end{aligned}$$

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