The CAPM

Lecture Four

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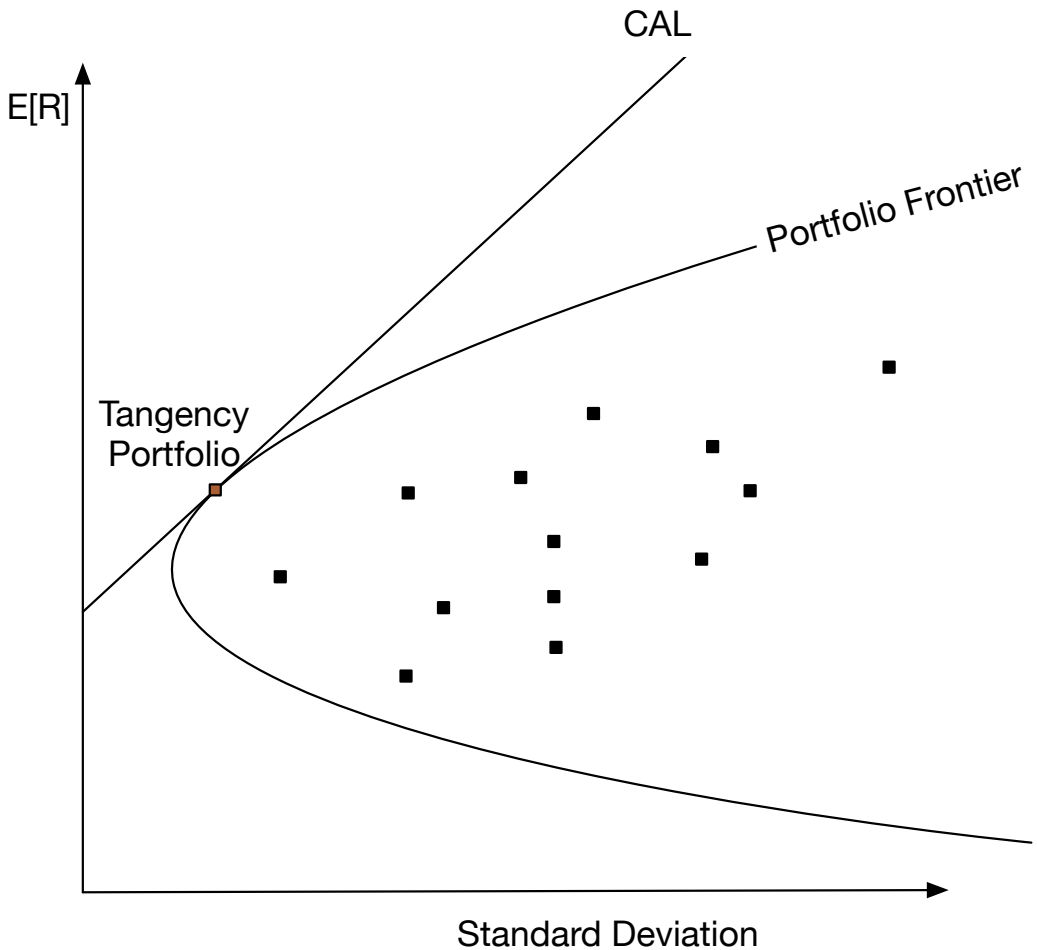
Takeaways from Last Lecture

- Portfolio risk depends mainly on covariances. Idiosyncratic risk matters very little.
- Diversification can get rid of a lot of risk.
- Hold the tangency portfolio - becomes the market portfolio!

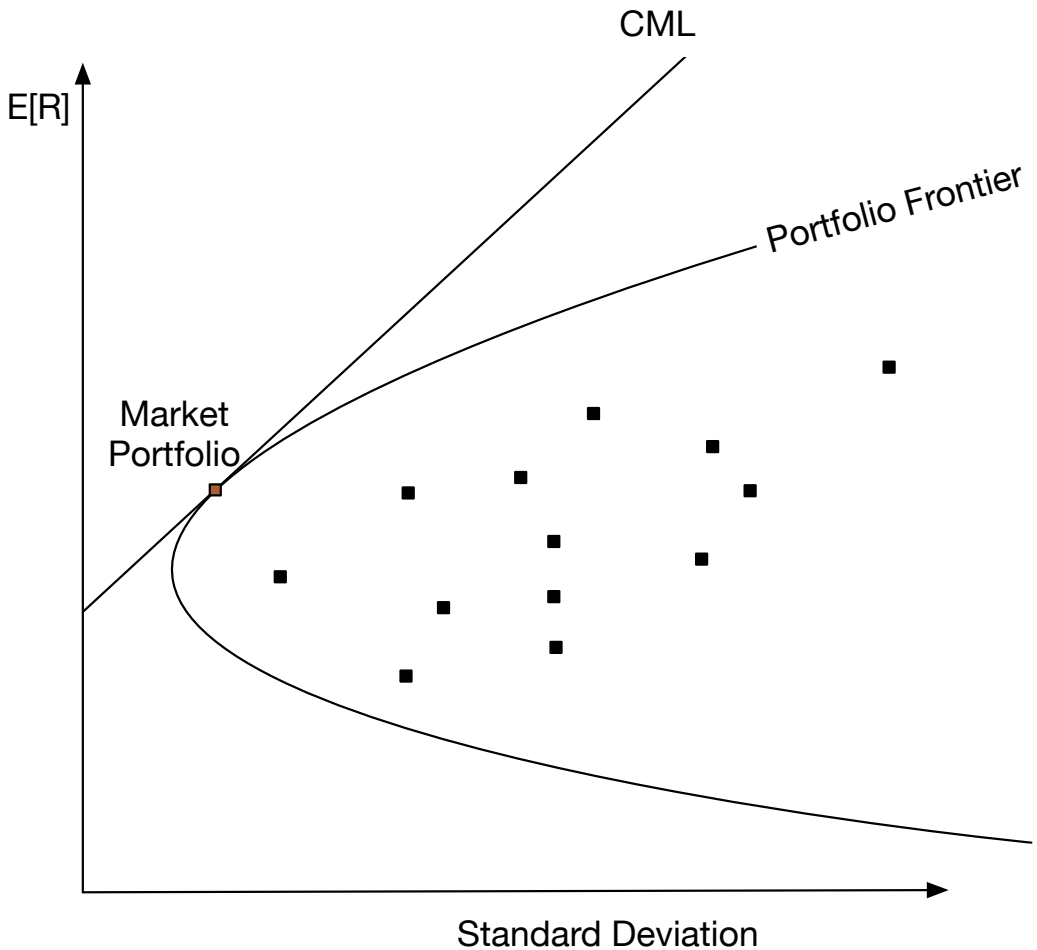
The CAPM

- The Capital Asset Pricing Model (CAPM) captures the systemic risk of a security.
  - What is the expected return of a stock?
  - Should a company undertake a risky project?
  - What are α and β?

Portfolio Theory



CAPM


CML
E[R]
Portfolio Frontier
Market
Portfolio
Standard Deviation

Market Portfolio

- All investors hold the same portfolio of risky assets.
- That portfolio is the market portfolio in equilibrium.
  - The tangency portfolio includes all assets.
  - The weight of each asset is:

$$\text{Weight}_i = \frac{\text{Market Value of Asset } i}{\text{Market Value of All Assets}}$$

Capital Market Line

- Individuals choose a portfolio on the Capital Market Line.
- The Capital Market Line connects the risk-free rate with the market portfolio.
- Risk-averse investors hold a larger portion of their assets in the risk-free asset.
- Risk-tolerant investors hold a larger portion of their assets in the market portfolio.

Riskiness of Individual Securities

- How much does the risk of the market portfolio depend on its components?

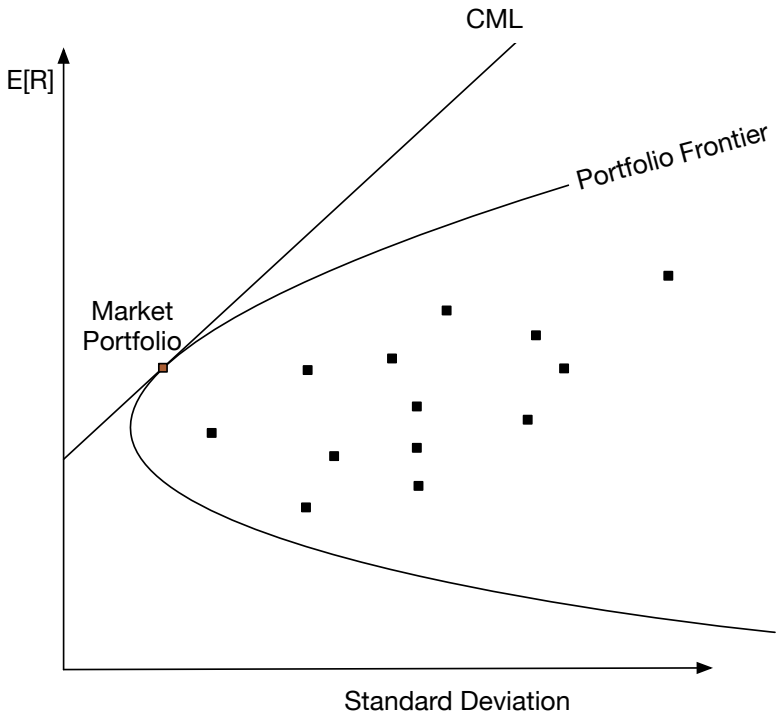
$$
\begin{aligned}
V[R_M] &= \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(R_i, R_j) \\
&= \sum_{i=1}^{N} w_i \left[\sum_{j=1}^{N} Cov(R_i, w_j R_j)\right] \\
&= \sum_{i=1}^{N} w_i Cov(R_i, R_M)
\end{aligned}
$$

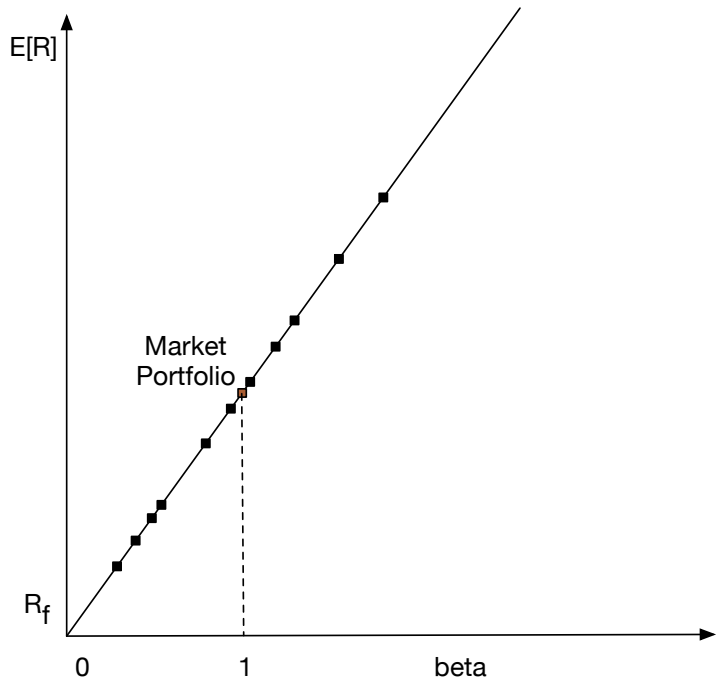
Expected Return of Individual Securities

- Equilibrium demands that risk-reward ratio is the same for all portfolios

$$\begin{aligned}
\frac{E[R_i] - R_f}{Cov(R_M, R_i)} &= \frac{E[R_M] - R_f}{V[R_M]} \\
E[R_i] &= \frac{E[R_M] - R_f}{V[R_M]}(Cov(R_M, R_i)) + R_f \\
E[R_i] &= R_f + \beta_i \left[E[R_M] - R_f\right]
\end{aligned}$$

Capital Market Line vs Securities Market Line





What is β?

- β is a measure of the systemic risk of an asset.
- CAPM says only systemic risk is priced.
- Idiosyncratic risk can be and is diversified away, and so is not priced.

CAPM

- What is the expected return of MSFT, and gold?
- βs are $\beta_{MSFT} = 1.25$ and $\beta_{Gold} = -0.05$
- The expected return of the market is $E[R_M] = 10\%$
- The risk-free interest rate is $R_F = 1\%$.

Flight to Quality

"The sub-prime mortgage crisis was the catalyst that pushed gold to 28-year highs, and now we're seeing investors make a flight to quality as fundamentals are supporting strong prices. In 2007, gold produced a return just below 30% while the S&P 500 increased less than 8%. The uncertainty in the U.S. stock market, stemming from the sub-prime crisis, has caused investors to move their assets into stable assets"
–Olivier Blanchard Gold rose from $750 an ounce on 11/09/08 to $900

an ounce on 29/09/08. $1218 as of Friday.

Performance Measures

- Consider three mutual funds.
- All three have $E[R] = 15\%$ and $V[R] = 20\%$.

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- Consider three mutual funds.
- All three have $E[R] = 15\%$ and $V[R] = 20\%$.
- βs are 0.2, 0.8, and 1.4.
- Who's the most skilled?

Application of CAPM(1)

- Analysts compare stocks' returns with their fair expected return from the CAPM
- A stock's *alpha* is the unexpected deviation from the fair return.
- Stocks with high alpha are under-valued and should be bought.

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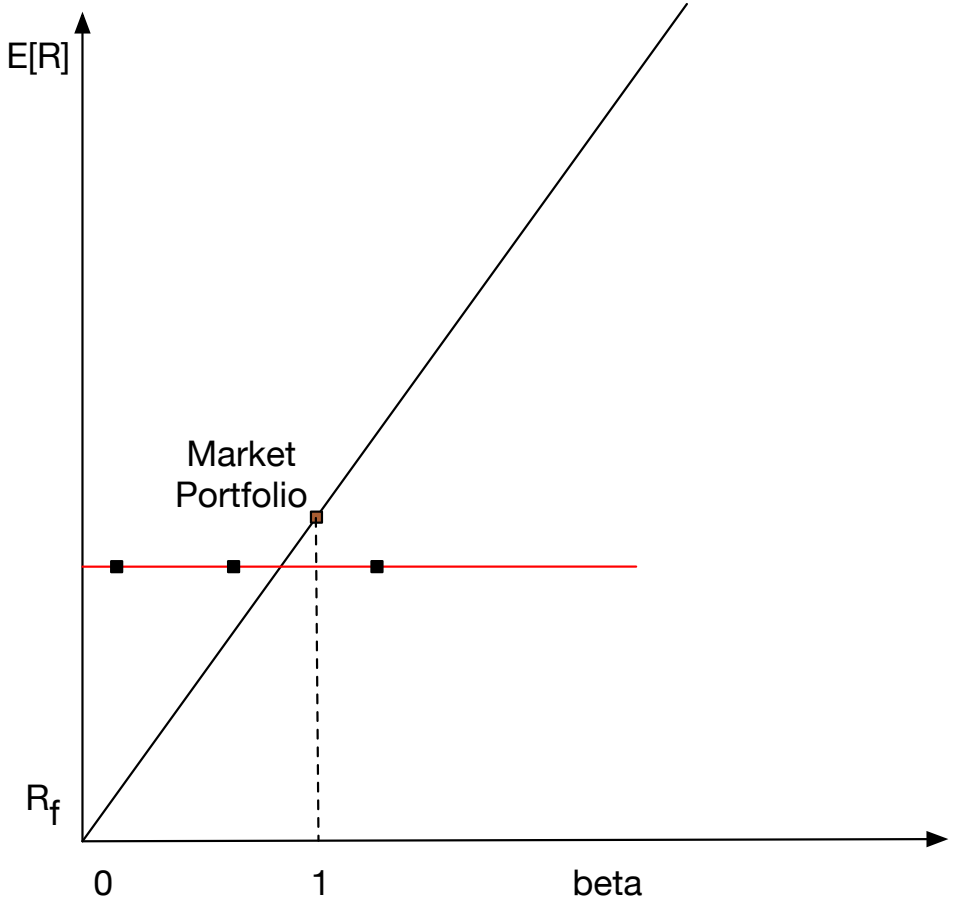
$$\alpha = E[R_s] - [R_f + \beta(E[R_M] - R_f)]$$

- α is the excess performance above the CAPM return.

Portfolio Theory



Application of CAPM(2)

- Capital Budgeting Decisions: should firms undertake risky projects?
- Calculate Net Present Value (NPV) to determine.
- Manager's Objective: Only undertake positive NPV projects.
- Compare the decision with IRR approach.
- How do you discount cash-flows?

NPV Example

- Project costs \$1000 today, returns \$300 in three years, \$600 in four years, and \$900 in five years.
- Cost of capital is given by:

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- Suppose that $\beta_i = 1.75$, $R_f = 4\%$, and $E[R_M] = 12\%$. Then $E[R_i] = 18\%$.
- Discount at $0.18 \rightarrow$ NPV$= -114.54$.
- If $\beta_i = 0.5$, then NPV$= 291.69$.
- $E[R_i] > IRR$ implies that you should reject the project, and vice-versa.

CAPM in Practice

- CAPM relies on a theoretical market of all possible assets.
- An index model uses an actual stock index to proxy this theoretical market.
- CAPM makes forward-looking predictions. Index models use historical data.

The Index Model

- The CAPM states:

$$E[R_i] - R_f = \beta_i E[R_M - R_f]$$

- The Index Model states‘:

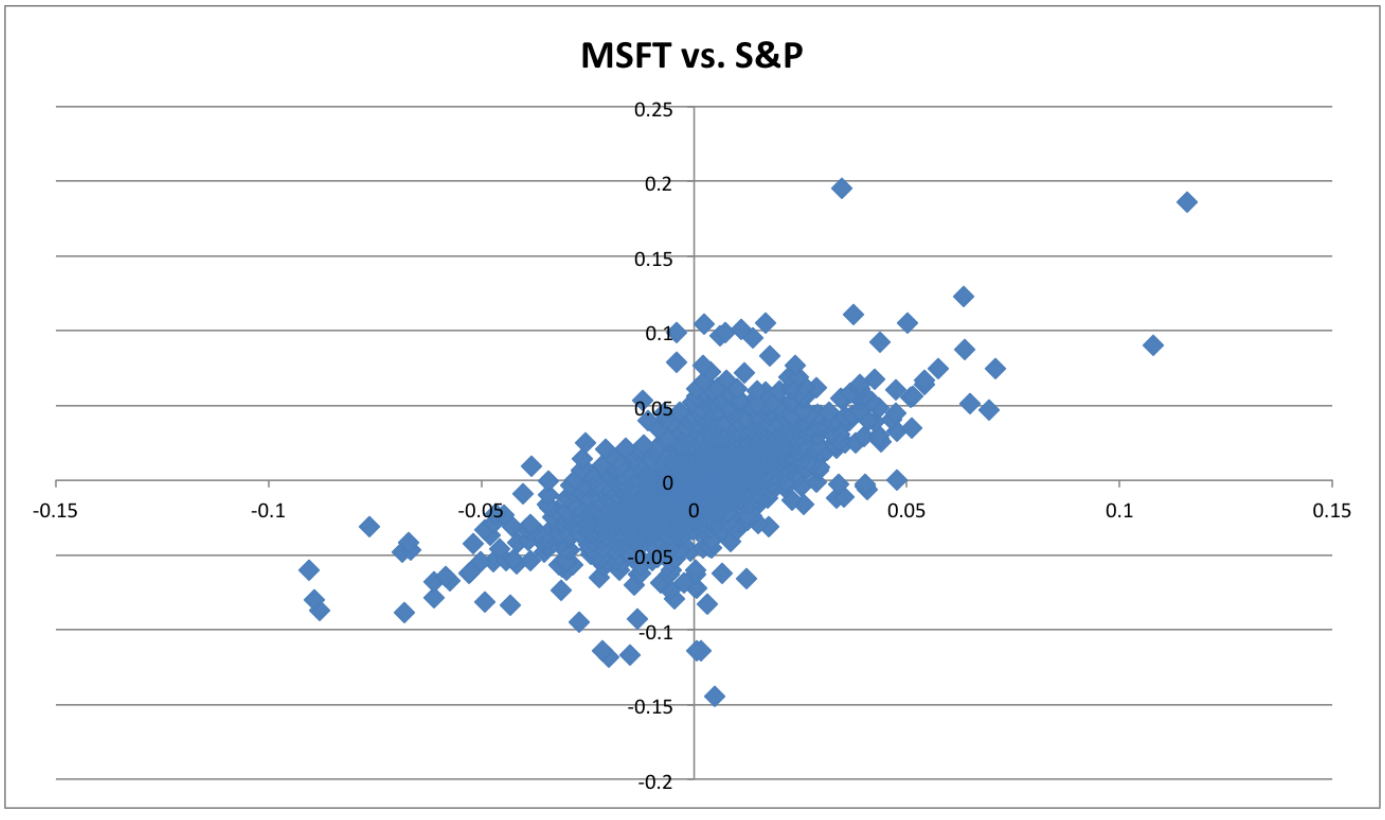
$$R_i - R_f = \alpha_i + \beta_i [R_M - R_f] + \epsilon_i$$

- α is the abnormal return, and ϵ is firm-specific risk.

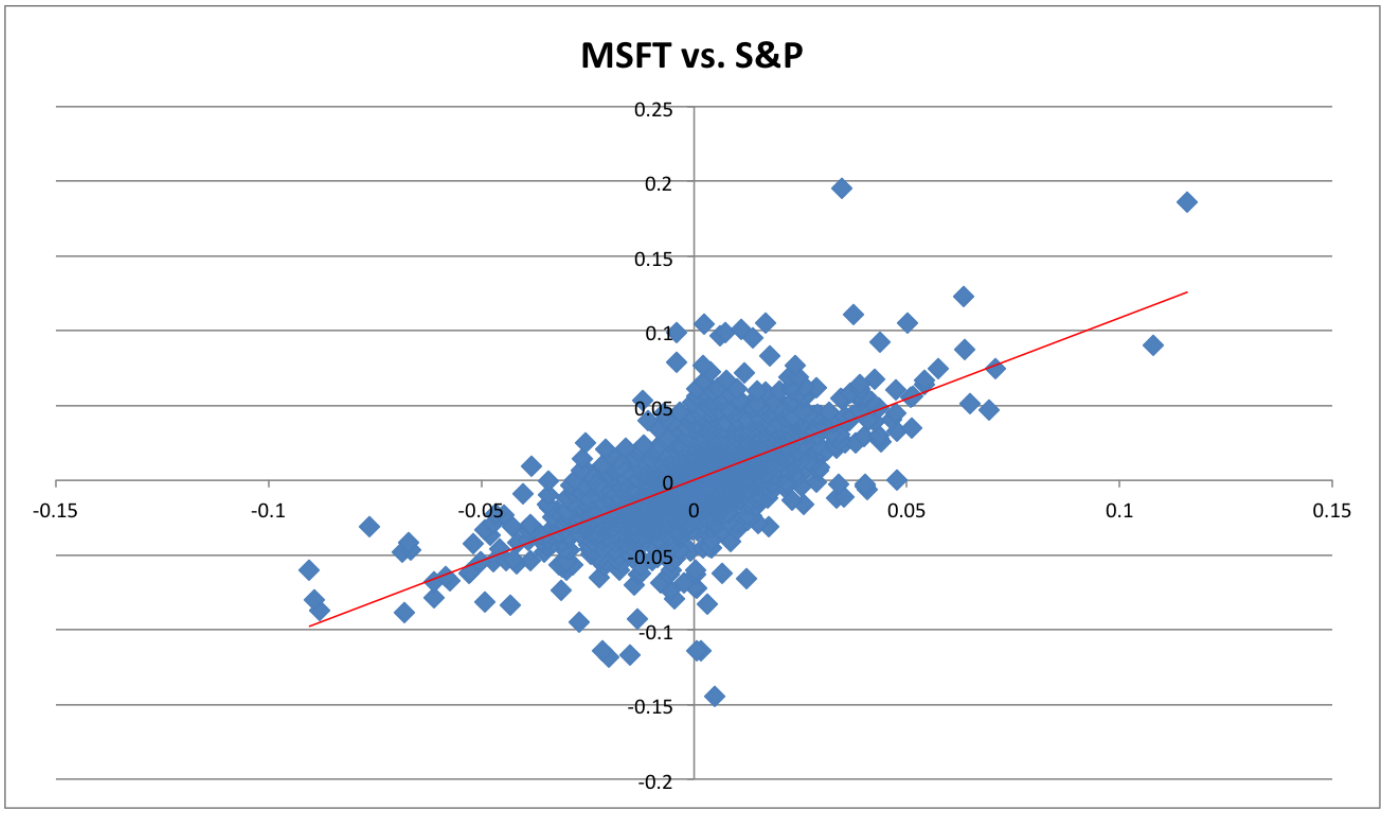
CAPM in Practice

- Estimate the index model using Microsoft data
- Use daily historical data from 1997 - 2017
- The Market Index is the S&P
- Risk Free Rate is the Treasury Bill
- Use a linear regression to estimate the index model.

CAPM



CAPM



Linear Regression

- Regression line:

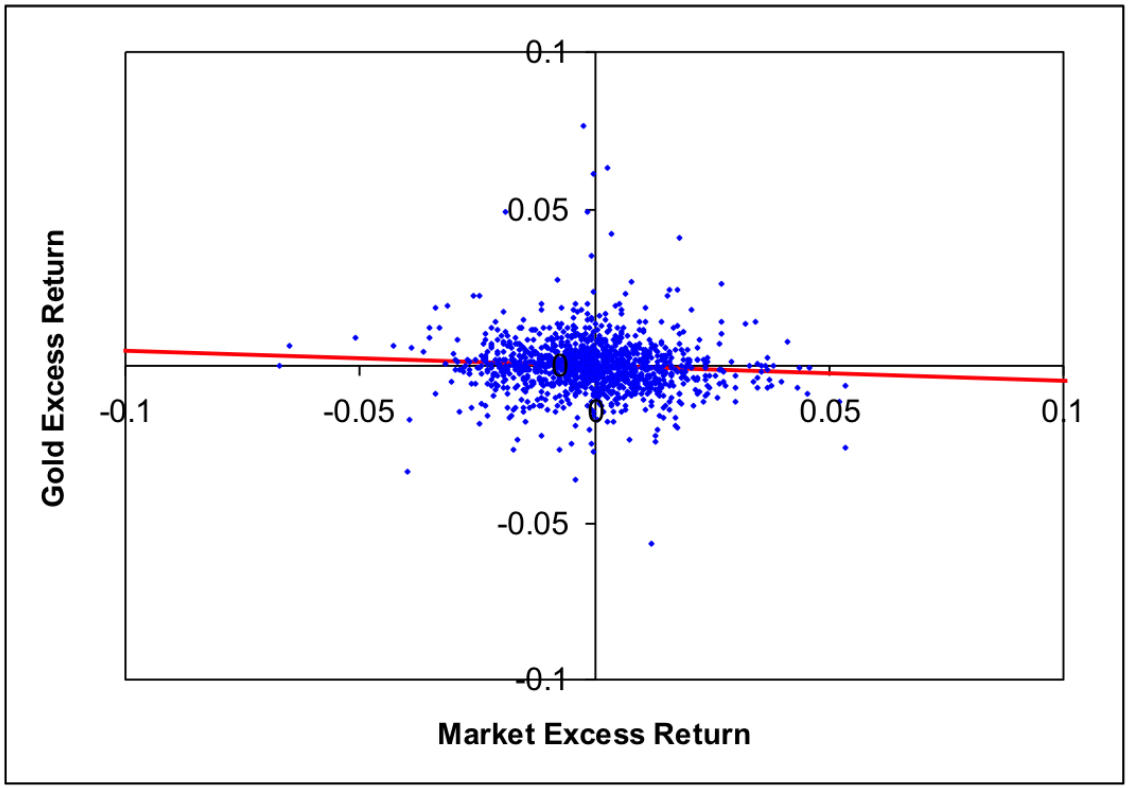
$$\begin{aligned} R_i - R_f &= \alpha_i + \beta_i[R_M - R_f] + \epsilon_i \\ R_i - R_f &= 0.04 + 1.25[R_M - R_f] + \epsilon_i \end{aligned}$$

- $R^2 = 0.42$

Interpretation

- The slope of the line is MSFT's β.
- If the Market return goes up by 1%, MSFT's goes up by 1.25%.
- The intercept is MSFT's α.
- MSFT outperformed the CAPM model by 0.04$ per day.
- R^2 is a measure of the fit: 42% of the variation in MSFT is systemic – the rest is firm-specific.

CAPM


0.1
0.05
0
-0.05
-0.1
-0.1
-0.05
0
0.05
0.1
Gold Excess Return
Market Excess Return

Betas

Battle Mountain Gold Company	.40
Boeing Corporation	.90
Bristol-Myers Squibb	.95
California Water Company	.45
Caterpillar Inc.	1.20
Coca-Cola	.95
Dow Chemical	1.15
Exxon Corporation	.65
The Gap, Inc.	1.45
General Electric	1.15
Harley-Davidson	1.65
Idaho Power Company	.65
Intel Corporation	1.35
Kaufman & Broad Home	1.65
Kellogg	1.00
Merrill Lynch & Company	1.90
Oshkosh B'Gosh (clothing mfg.)	.60
Outback Steakhouse	2.10
Procter & Gamble	1.05
Ralston Purina	.90
Telefonos de Mexico	1.35
Tootsie Roll Industries	.75
Toys 'Я' Us	1.45
Western Digital	1.85

CAPM in Practice

- The CAPM equation can be written as the index model:

$$R_i = R_f + \beta_i(R_M - R_f) + \epsilon_i$$

- Assume $E[\epsilon_i] = 0$, $Cov[\epsilon_i, R_M] = 0$.
- Implies:

$$\begin{aligned} V[R_i] &= V[R_f + \beta_i(R_M - R_f) + \epsilon_i] \\ \sigma_i^2 &= V[\beta_i(R_M - R_f) + \epsilon_i] \\ &= \beta_i^2\sigma_M^2 + \bar{\sigma}_i^2 \end{aligned}$$

Interpretation

- ABC Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%
- What is the equilibrium expected return of ABC stock?
- What is the proportion of ABC Internet's variance which is diversified away in the market portfolio?

Portfolio Beta

Suppose you have a portfolio comprised of two assets A and B with weights w_A and w_B.

$$\begin{aligned}
\beta_P &= \frac{Cov(R_M, w_A R_A + w_B R_B)}{V[R_M]} \\
&= w_A \frac{Cov(R_M, R_A)}{V[R_M]} + w_B \frac{Cov(R_M, R_B)}{V[R_M]} \\
&= w_A \beta_A + w_B \beta_B
\end{aligned}$$

Sin Stocks

Sin stocks have $\beta = 0.7$. while ethical stocks have $\beta = 1.3$. How do you allocate your money to get *no market risk*?

Multiple Systemic Risks

- What if there are multiple sources of risk?

$$E[R_i - R_f] = \beta_{i1} S_1 + ... + \beta_{iK} S_K + \epsilon_i$$

where S_k is the risk premium of factor k.