Sample Final, FINC-UB 0002.02

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1 Multiple Choice Questions

The following are sample questions for the multiple choice section. They are organized by degree of difficulty (this ranking is a bit subjective).

1.1 Easy

1. If the (positive) yield to maturity on a zero coupon bond is constant from one year to the next, the price of the zero coupon bond over the next year will

   a. Increase
   b. Decrease
   c. Remain the same
   d. You cannot tell

2. Suppose you buy a put option with a strike price of 100 for a premium of \$10. Your maximum profit per share is

   a. \$10
   b. \$100
   c. \$90
   d. \$110

3. According to the CAPM, if a security's beta is negative, then its expected return must be

   (a) The market rate of return
   (b) Zero
   (c) A negative rate of return
   (d) The risk free rate
   (e) None of the above

4. According to the Black-Scholes-Merton model, if, for a particular call option, $N(d1)$ and $N(d2)$ are both close to 0, then which of the following is most true

   a. The call is almost worthless
   b. The call will be exercised almost certainly
   c. The call price is close to $S - X$
   d. The call is worth less than $S - X$

5. Suppose that the risk-free rate is $R_f = 3\%$ and the risk-premium is $E(R_m) - R_f = 8\%$. According to Gordon's Growth Model, if a company has a current dividend of $D_0 = \$20$ per share, a constant growth rate of $g = 6\%$, and $\beta = 1.25$, what is its stock price:

   a. the stock price is \$285.71 per share
   b. the stock price is \$302.86 per share
   c. the stock price is \$342.14 per share
   d. not enough information to tell

6. If prices reflect all publicly available information

   a. the market is semi-strong efficient
   b. stock price changes are unpredictable by public information
   c. stock price changes are unpredictable by all information, including private information
   d. one can profit from doing advanced security analysis based on public information
   e. both a and b are true
   f. All of a, b, c, and d are true

7. An upcoming event suggests that there will be significant movement in the share price, but you're not sure in which direction. Which position would you choose?

   a. Long a call
   b. Long stock and short a call
   c. A straddle
   d. Portfolio insurance

8. Assuming you hold an annual pay coupon bearing bond to maturity, its holding period return is equal to

   a. the YTM if you can and do reinvest at a fixed rate
   b. the coupon rate
   c. the YTM if you can and do reinvest at the YTM
   d. none of the above

9. According to the Expectations Hypothesis of the term structure

   a. the 1-year rate today equals the expected one-year rate next year
   b. investors are risk averse

c. when the yield curve is upward sloping, the expected one-year rate next year is higher than the one-year rate today
d. none of the above

10. The buyer of a put and seller of a call

    a. must disagree about whether the price of the underlying is expected to go up or down
    b. both have rights and not obligations
    c. both profit if the price of the underlying asset falls
    d. both b and c are correct

11. Assume a zero coupon bond has duration = 10 years and a 30 year bond has an 18% coupon and a duration =10 years. Assume further that the yields on both bonds are the same and then change by the identical small amount. Then, the % price change of the 30 year will be approximately:

    a. Equal to the % price change of the zero
    b. Less than the % price change of the zero
    c. Greater than the % price change of the zero
    d. Can't tell

12. Suppose the expected return on stock ABC is 14%. Suppose $R_f = 3\%$, $E(R_m) = 10\%$ and ABC's $\beta = 1.45$. Then the α on ABC is

    (a) Positive
    (b) Negative
    (c) Zero
    (d) Not enough information to answer

13. According to CAPM, if the expected return on asset 1, $E(R_1)$, is greater than the expected return on asset 2, $E(R_2)$, then:

    (a) R_1 must always be greater than R_2
    (b) σ_1 must be greater than σ_2
    (c) β_1 must be greater than β_2
    (d) all of the above must be true

1.2 Moderately Difficult

14. Which of the following five-year investments has the highest yield to maturity?

    a. An 8 percent coupon annual pay bond selling at 103
    b. An 8 percent coupon annual bond selling at par
    c. A zero coupon bond with $ 1000 face value selling at $665
    d. They all have the same YTM

15. A security has an equilibrium expected return less than that of the risk-free asset when:

    (a) The correlation between its return and the market return is less than 1

    (b) The security is uncorrelated with the market

    (c) A security never has an equilibrium expected return less than the risk free asset

    (d) None of the above

16. According to the Liquidity Preference Theory, an upward sloping yield implies

    a. Short-term rates are expected to rise
    b. Long-term rates are expected to rise
    c. Short-term rates are definitely not expected to decline
    d. Short-term rates may or may not be expected to rise

17. Being long a call and short a put is like:

    a. Long a call and short the stock
    b. Short selling
    c. Buying stock on margin
    d. straddle

18. If the implied volatility of a call is greater than what you think is the actual volatility, you should:

    a. Buy the call
    b. Write the call
    c. Buy the put
    d. Sell the stock

19. Suppose the yield on a one-year zero-coupon bond is 7%. The yield on a two-year zero-coupon bond is 8%. You expect the one-year yield next year to rise to 7.5%. Which of the following strategies would give you the highest expected HPR over one year?

    a. Invest in the one-year bond
    b. Invest in the two-year bond and sell after one year
    c. The expected returns on a and b are equal
    d. Impossible to tell

20. Assume you bought an 8% coupon bearing bond with 4 years to maturity at par and then sold it at a premium before maturity. If you were able to reinvest the coupons at the YTM, then:

    a. HPR = YTM
    b. HPR is less than YTM
    c. HPR is greater than YTM
    d. You cannot tell

21. Portfolio insurance

    a. combines long put with long stock
    b. profits when the underlying asset's stock price increases
    c. has downside protection
    d. all of the above

22. Which of the following statements is false:

    a. A par bond must have a coupon rate that is equal to the yield to maturity
    b. When the coupon rate is greater than the yield to maturity, the bond is selling at a premium
    c. The concept of yield to maturity suffers from the reinvestment assumption for both semi-annual and annual coupon paying bonds
    d. If I invest $100 in a par-value bond with coupon rate of 10% and maturity of two years, I will certainly have $121 at the end of the two years

23. If the stock price falls and the call price rises, then what has happened to the call option's implied volatility (assuming interest rates are unchanged)?

    a. Up
    b. Down
    c. Same
    d. Can't tell

24. The price (per $100 face value) of a 7% semi-annual pay bond with exactly 2-1/2 years to maturity and a yield to maturity of 8.75% is:

    a. 93.4381
    b. 96.9111
    c. 96.1454
    d. none of the above

25. If a company's growth rate is high then, all else the same, which of the following must be true:

   a. the P/E ratio of its stock will be higher
   b. the stock's beta will be higher
   c. the price-dividend ratio of the stock will be higher
   d. both a and c

1.3 Difficult

26. Which of the following represents an arbitrage opportunity where you would do the following: buy the call, sell the put, sell the stock, and buy a risk-free security. $S = 110$, $X = 100$, $r = 0$, $T = 1$

   a. P =2, C=12
   b. P=5, C=15
   c. P=12, C=23
   d. P=5, C=12

2 Numerical Questions

2.1 Challenging Question on Arbitrage

Assume that transaction costs are zero and there is no credit risk in any transaction. Suppose that: (1) the price at time 0 of a 2-year annual-pay coupon bond with face value \$100 and a coupon rate of 5% is \$88, and (2) the price at time 0 of a 1-year zero coupon bond with face value \$100 is \$88. What must be the time 0 price of a bond with the following (somewhat unusual) risk-free cash flows?

	Year 1	Year 2
Cash flows	\$60	\$ 210

Explain your reasoning in detail. (Hint: use no-arbitrage reasoning.)

2.2 Challenging Question on Equity Valuation

On the Yahoo Finance website you collect the following data on Intel, Inc. Intel is currently trading at \$26.52 per share. Its earnings per share are \$0.97 and its dividends per share are \$0.16. Intel's beta is 2.04. The risk-free rate is 1.2% and the expected market return is 9.5%.

(a) What is the plowback ratio of Intel?

(b) Which growth rate of dividends does the market incorporate according to the Gordon growth model?

2.3 Challenging Question on Duration

You are the CFO of a savings & loans institution (a traditional bank). The duration of your deposits is 3.5 years. Your task is to decide on an optimal portfolio of short term and long term loans that will immunize your interest rate risk. More precisely, what fraction of your loan portfolio will be 30-year mortgages and what fraction should be 1-year trade credit? Assume that there are no payments due on either type of loan until they come due.

2.4 Challenging Question on Options

You are an options trader for a major U.S. investment bank and just wrote 5,000 call options on ABC stock to a large customer. You realize this exposes you to the risk that stock prices may change. To hedge this risk, you decide to hold a position in ABC stock. What position in the underlying stock should you hold to hedge the 5,000 calls you wrote, i.e. how many shares and whether long or short?

- 1 call option = right to buy 1 stock
- The option contract is a European call option with maturity 1 year and strike price 70.
- The annual, continuously compounded, riskless interest rate is 2%
- ABC stock is currently trading at 70; ABC stock pays no dividends.
- The volatility of the annual, continuously compounded rate of return on ABC stock is 0.35.
- You believe the Black-Scholes model is the right way to value this option

2.5 Challenging Question on Futures (Attention: Futures/Forwards Not Covered This Year, so not required.)

You are an arbitrageur in the futures market for silver. Suppose 1 month prior to delivery of the futures contracts on pure silver, the futures price of pure silver is $143 per oz. and the cash price of silver is $140 per oz. Storing silver incurs a cost of $2 and the interest rate is .12 per year. Suppose the lease rate on silver is zero. Is there a cash-futures arbitrage you would like to undertake? Explain why or why not?

3 Answers Multiple Choice

1. A (The increase exactly equals the yield in this case. Suppose the maturity is first 10 years, so that $P_{10} = 100/(1+y)^{10}$, where y is the yield. One year later, if the yield remains the same, the price is $P_9 = 100/(1+y)^9$, which is $1+y$ times the price of the bond in the previous year.)

2. C (Maximum profit materializes when the price drops to \$0. In this case the payoff is \$100. Minus the \$10 cost this gives a profit of $\$100 - \$10 = \$90$.)

3. E (It should be below the risk free rate, but can still be positive).

4. A ($C_0 = S_0 e^{-\delta T} N(d_1) - X e^{-rT} N(d_2)$, so $N(d_1) = N(d_2) = 0$ implies $C = 0$.)

5. B (Discount rate is $R = R_f + \beta(E[R_M] - R_f) = 3\% + 1.25 * 8\% = 13\%$, so $P = D_0 (1+g) / (R-g) = 20 * 1.06/(0.13 - 0.06) = \$302.86$)

6. E

7. C (Straddle is established by holding both a call and a put with the same exercise price and maturity. The value of the straddle increases if the price of the underlying moves further away from the exercise price, in either direction. See also BKM, page 707.

8. C (The yield of a bond is simply the Internal Rate of Return (IRR) of the cash flows of the bond if you hold the bond until maturity. In my note on the IRR I discuss exactly when the (annualized) holding period return equals the IRR.

9. C (An upward-sloping yield curve implies that the yield on a two year bond is higher than the yield on a one-year bond. According to the Expectations Hypothesis this only happens if the expected one-year rate next year is higher than the current one-year rate.)

10. C (If the price of the underlying falls, then (i) calls become worth less, which is good for a seller of a call, and (ii) puts become worth more, which is good for the buyer of a put).

11. A (We have $\Delta P/P \approx -D\Delta y/(1+y)$, where D is the duration and y is the yield. Since D, Δy, and y are all assumed to be the same, the price change $\Delta P/P$ is approximately the same as well.

12. A (According to CAPM $E\left[R_{ABC}^{CAPM}\right] = R_f + \beta_{ABC}(E[R_M] - R_f) = 3\% + 1.45 * (10\% - 3\%) = 13.15\%$, which is lower then the actual expected return, so $\alpha = 14\% - 13.15\% = 0.85\% > 0$.

13. C (Security Market Line: higher expected return is associated with a higher beta).

14. C (The YTM of (a) is lower than 8%, the YTM of (b) is 8%, the YTM of (c) is $(1000/665)^{1/5} - 1 = 8.50\%$)

15. D (Such an asset must have a negative beta which means it must be negatively correlated with the market.)

16. D

17. C (The put-call parity says that holding a long call and short put equals holding the underlying stock and borrowing money, $C - P = S - Xe^{-rT}$ (maturity and exercise price must be the same for the two options and moreover the options must be of the European type!), so you are effectively holding the stock financed (partially) by borrowing Xe^{-rT}.)

18. B (Implied volatility higher than your expectations means the price of the call is higher than you think it should be because the price of a call increases in volatility)

19. B (We have for the two-year bond $yield = ((1 + yield_1)(1 + yield_2))^{0.5} - 1$. If we use $yield = 8\%$ and $yield_2 = 7.5\%$, then $yield_1 = (1 + yield)^2 / (1 + yield_2) - 1 = 8.5\%$)

20. C (A would have been true if you sold it at par, B would have been true if you sold it below par (at a discount)).

21. D

22. D (D would be true if the reinvestment rate for the first coupon payment is 10%, but that is not given)
23. A (Stock price fall has a negative effect on the value of a call that can only be (more than) offset by an increase in volatility).
24. C ($P = 3.5/(1.0438) + 3.5/(1.0438)^2 + 3.5/(1.0438)^3 + 3.5/(1.0438)^4 + 103.5/(1.0438)^5 = 96.1454$, where $3.5 = 7/2$ and $1.0438 = 1 + 0.0875/2$ according to the convention that coupon rate and yield are reported as an APR.)
25. D
26. D (The put-call parity states $C - P = S - Xe^{-rT}$ (maturity and exercise price must be the same for the two options and moreover the options must be of the European type!). Equivalently the put-call parity could be written as $C - P - S + Xe^{-rT} = 0$. In words: if you buy a call, sell a put, sell the stock, and invest X at the risk free rate, the final payoff at maturity will be zero and therefore todays price should be zero. If the put-call parity does not hold and instead the portfolio has a negative price, $C - P - S + Xe^{-rT} < 0$, you can buy a call, sell a put, sell the stock, invest X at the risk free rate, pocket the negative price (positive cash flow) at initiation and still the final payoff at maturity will be zero. For answer D indeed we have $C - P - S + Xe^{-rT} = 12 - 5 - 110 + 100 = -3 < 0$ ($r = 0$, so $e^{-rT} = 1$). For A,B,C we have $C - P - S + Xe^{-rT} \geq 0$.)

4 Answers to Numerical Questions

4.1 Question on Arbitrage

Draw a time line for each bond, as well as for the third asset you need to price. Lets call the latter 'asset 3'.

Assume $100 face value for each bond. The cash flows on the 2-year coupon bond are a coupon of $5 (5% of 100) at the end of year 1 and $105 at the end of year 2 (coupon plus principal). The cash flow of the 1 year bond is $100 at the end of year 1. Asset 3 has a cash flow of $60 at the end of year 1 and $210 at the end of year two.

As we did in class, our strategy will be to replicate the cash flows on asset 3 using bonds 1 and 2. The combination of bonds 1 and 2 that succeeds in exactly replicating the cash flows of asset 3 is called the replicating portfolio. This 'cash-flow matching' is the general strategy in any arbitrage question. Also, we always work backwards (from the last cash flow to the first).

The first cash-flow of asset 3 we need to match is $210 in year 2. On order to end up with $210 at the end of year two, we need to buy 2 units of the coupon bond (asset 1) at time zero. That costs 2*$88=$176 in year 0. Having matched the cash-flow in year 2, we move on to matching the cash-flow of asset 3 in year 1. We need to match $60 in year 1 using bonds 1 and/or 2. The key thing to note is that we *will already have a $10 cash flow* coming from the replicating portfolio because we just bought 2 units of bond 1 in order to match asset 3's cash flow in year 2! Hence, we only need to get an additional $50 cash flow to match the entire $60 cash flow in year 1. We do this by buying 0.5 units of bond 2 (the 1-year zero coupon bond). This costs 0.5*$88 = $44 in year zero. This matches the cash-flow of asset 3 in year 1, and we are done.

How much does the replicating portfolio cost? We spent $176 to match the time-2 cash flow and another $44 to match the time 1 cash-flow. The total cost in year zero is $220.

Finally, we recall that any two 'things' with the same riskless cash flows must carry the same price. That is the insight of the no-arbitrage framework. The two things here are asset 3 on the one hand, and the replicating portfolio of bonds 1 and 2 on the other hand. We know the price of the replicating portfolio: it is $220. Because of no arbitrage, the time 0 price of asset 3 must also be $220.

4.2 Question on Equity Valuation

(a) To find the plowback ratio, use the dividend model $D = (1 - b)E$. Solving for b, you get $b = 1 - (D/E)$. Plugging in the numbers, the plowback ratio is b=0.835.

(b) To answer this question, you need to first calculate the price-earnings ratio: $\frac{P}{E} = 26.52/.97 = 27.34$. The required rate of return according to the CAPM is: $R = .012 + 2.04 * (.095 - .012) = 0.1813$. According to the Gordon growth model:

$$\frac{P}{E} = \frac{(1+g)(1-b)}{(R-g)}.$$

Solving this equation for g, we get:

$$g = \frac{(PE * R + b - 1)}{1 - b + PE} = .1742.$$

The market expect dividends and earnings to grow at an annual rate of 17.42%.

4.3 Question on Duration

The duration of your liabilities is 3.5 years. To immunize your portfolio, you need to match the duration of assets and liabilities. We know that the duration of a portfolio is the weighted sum of the duration of its elements. The duration of the 30 year mortgages is 30. The duration of the 1 year trade credit is 1. We solve for 3.5 = w*1+(1-w)*30. The solution is w= .9138. You should allocate 91.4% of your credit to short term trade loans and 8.6% to long term mortgages.

4.4 Question on Options

The question asks for a hedge ratio. We saw in class that if you write (sell) *call* options you can hedge that position by going *long* (buying) the underlying stock. The question thus asks how many shares of the stock you must buy to hedge a position in 5,000 options (where we use the convention from class that 1 option = right to buy 1 share).

With Black-Scholes, the hedge ratio Δ has a particularly easy form:

$$\Delta = \exp(-\delta T)\mathcal{N}(d_1) > 0$$

Note that this expression is always positive, indicating that a *long* position in the stock is needed to hedge writing calls.

The first task is to calculate the d_1 from the B-S formula. Recall (and make sure your formula sheet contains):

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Plugging in the numbers that are given you should find $d_1 = 0.23214$.

The second step is to calculate $\mathcal{N}(d_1)$. It turns out that $\mathcal{N}(0.23214) = 0.5918$. How do you find this number?

Interpolation Explained Step-by-step You do this using the normal distribution tables (see page 732 in BKM). You look up d_1 in the first column. The two closest numbers to $d_1 = 0.23214$ are 0.22 and 0.24. In the second column, you find the corresponding $\mathcal{N}(0.22) = 0.5871$ and $\mathcal{N}(0.24) = 0.5948$.

Intuitively, $\mathcal{N}(0.23214)$ should be somewhere in between 0.5871 and 0.5948, about half-way in between. To do it correctly, you must *interpolate* between these two numbers. Here is how.

Step 1: Figure out where exactly 0.23214 lies in the interval 0.22 and 0.24. It turns out that it is $x = 60.7\%$ of the way between 0.22 and 0.24 (of course, 0.23 would be 50% of the way):

$$x = \frac{0.23214 - 0.22}{0.24 - 0.22} = 0.607.$$

Step 2: Interpolate between $\mathcal{N}(0.22) = 0.5871$ and $\mathcal{N}(0.24) = 0.5948$ using the $x = 60.7\%$ number you found in step 1.

$$\mathcal{N}(0.23214) = (1 - x)\mathcal{N}(0.22) + x\mathcal{N}(0.24) = 0.393 * 0.5871 + 0.607 * 0.5948 = 0.5918.$$

Back to the main question. The hedge ratio is simply $\Delta = \exp(-\delta T)\mathcal{N}(d_1) = \exp(-0 * 1) * 0.5918 = 0.5918$ because the stock pays no dividends ($\delta = 0$). The hedge ratio tells you how many shares of the stock must hold *for every 1* call option you write. To obtain the final answer, you must take into account that you wrote 5000 calls. Answer: for 5000 calls you write, you must hold long stock position of 5000*0.5918 = 2958.93 stocks.