

Homework 1 – Suggested Answers

Question 1:

Orange is currently selling at £18.04 per share. You buy 1000 shares by using £15,000 of your own money and borrowing the remainder of the purchase price from your broker. The rate on the margin loan is 6%. The account has an initial margin of 50 percent and a maintenance margin of 25 percent.

(a) What is the percentage increase in the net worth of your brokerage account if the price of ORAN immediately changes to: (i) £24; (ii) £14?

The purchase costs you £18,040. You borrow £3,040 from the broker and invest £15,000 of your own funds. The initial net value of the account is £15,000. Note that returns are always measured relative to the equity value you invest in your account.

- (i) The net account value increases to £24,000-£3,040=£20,960. Thus, the return is $20,960/15,000-1=39.73\%$
- (ii) The net account value falls to £14,000-£3,040=£10,960. Thus the return is $10,960/15,000-1=-26.93\%$

(b) How low can the price of ORAN fall before you get a margin call?

The value of the 1,000 shares is $1000 \cdot P$. Equity is $1000 \cdot P - 3,040$. You will receive a margin call when

$$\frac{1000P - 3,040}{1000P} = 0.25 \Rightarrow P = 4.05.$$

(c) How much money would you need to send your broker to satisfy the margin call if the stock price drops below the level computed in b?

You will need to add cash until your margin equals the initial margin of 50 percent. The value of the securities is £4,050 ($4.05 \cdot 1000$) and you can borrow at most 50 percent of this ($4050/2=2025$). Thus, you need to pay back £1,015 ($3040-2025$) of the original margin loan of £3,040.

(d) What is the rate of return on your margined position if ORAN is selling after one year at: (i) £24; (ii) £14? (You can ignore the dividend payments by ORAN).

The margin loan with accumulated interest will be $£3,040 \times 1.06 = £3,222.40$. The equity in your account is $1000P - £3,222.4$. Therefore your rates of return are:

$$(i) \frac{1000 \times 24 - 3222.4}{15,000} - 1 = 38.52\%$$

$$(ii) \frac{1000 \times 14 - 3222.4}{15,000} - 1 = -28.15\%$$

Question 2:

Mr. K. wants to retire from his emotionally demanding job twenty years from now. He wants to save enough so that he can have a pension of £10,000 a month for ten subsequent years (starting at the end of his job). Assume that the annual interest rate is equal 5%.

(a) How much would he have to save each month during the first twenty years to achieve his goal? Assume that all cash flows occur at the end of each month.

Note that the financial needs of Mr. K. require monthly cash flows; hence, it is natural to use monthly frequency in this problem.

Step 1: Choose $t=240$ (in months) as a reference point. The stream of cash flows Mr K. would receive is an annuity with $T=120$, $C=10,000$, and $r=5/12\%$. Using the formula from class for the annuity value:

$$PV(\text{Annuity}) = £942,813.50$$

Therefore, Mr K. has to accumulate savings of 942,813.50 during the first 20 years.

Step 2: Calculate the present value of the amount as of $t = 0$.

$$\text{Savings} = 942813.5 / (1 + 5/12\%)^{240} = £347,563.00$$

Step 3: The amount above is the value of the annuity that would be able to produce such savings. We need to calculate the cash flow that would produce such value. Again, using the formula for annuity from class, we can invert and solve for C .

$$C = £2293.76$$

Hence, Mr. K. has to save £2293.76 every month.

Note that alternatively, we could skip Step 2 from the above and calculate PMT directly going from step 1 to step 3, assuming $FV = £942,813.50$.

(b) Suppose that instead of having a 10-year stream of cash flows, Mr. K. would like to create an infinitely long stream of monthly cash flows of £8000 starting 20 years from now to provide for his descendants. How much would he have to save each month during the first twenty years?

This question is very similar, except for Mr. K. wants to receive a perpetuity instead of annuity after 20 years of saving. We assume this perpetuity has a constant cash flow of £8,000.

The value of perpetuity at the end of year 20 would be:

$$PV(\text{Perpetuity}) = C/r = 8000/5/12\% = \text{£}1,920,000$$

Therefore, Mr K. has to accumulate savings of £1,920,000.00 during the first 20 years.

We calculate the present value of the amount as of $t = 0$.

$$\text{Savings} = 1920000 / (1 + 5/12\%)^{240} = \text{£}707,796.93$$

The amount above is the value of the annuity that would be able to produce such savings. We need to calculate the cash flow that would produce such value. Using the formula from class:

$$C = \text{£}4671.15$$

Hence, Mr. K. has to save £4671.15 every month.

Again, you could have skipped one step by going directly from FV of annuity into the payment.

(c) Which of the two savings plans requires a higher monetary commitment? Why?

As you can see from the above numbers, clearly the second option requires greater savings than the first one. But the answer require to look at the following tradeoff: on one hand, perpetuity is a claim that requires higher capital investment (holding everything else constant), but in this question perpetuity comes with a smaller cash flow per month to begin with so it is not obvious that it is a more intensive option. Based on the numbers it is indeed the case that the effect of infinite compounding dominates the effect of higher cash flows demanded by the annuity.

Question 3:

You are among the OTC dealers (intermediaries) in the stock of Bio-Engineering, Inc. and quote a bid of 102 1/4 and an ask of 102 1/2. Suppose that you have a zero inventory.

(a) On Day 1, you receive buy orders for 10,000 shares and sell orders for 4,000 shares. How much do you earn on the 4,000 shares that you bought and sold? What is the value of your inventory at the end of the day? (Hints: It is possible to have negative inventory. Further, there is more than one correct way to value an inventory, but please state what assumption your valuation is based on.)

You have sold 10,000 shares at the ask price of 102 1/2. You bought 4,000 shares at a bid price 102 1/4. Thus, 6,000 shares are sold short (sold without already owning the security). Your revenue from the 4,000 round trip purchase and sale produces a profit equal to the bid minus the ask times the volume done. Hence, the profit on the round trip trades is $0.25 \times 4,000 = 1,000$.

The value of your inventory is equal to the value of your short position of 6,000 shares. Since there is both a bid and an ask price, this question can be answered in various ways depending on what you assume:

The *conservative* valuation is to value your position at the ask price of 102.50. Then, you have a position of -615,000. This conservative valuation is useful because, if you cover your short position by buying from another dealer at his ask price of 102 1/2, you would have to pay 615,000. (Also, in this example it is the price for which you sold the securities.)

The *aggressive* valuation is to value your position at the bid price of 102.25. Then, you have a position of -613,500 (i.e. less negative than above). This value is implicitly based on an expectation that some investors will come to you and sell you 6000 shares at your current bid price.

Often, real-world market makers will value their inventory at the mid-price, in this case 102 3/8. Then, you have a position of -614,250.

(b) Before trading begins on Day 2 the company announces trial testing of a cure for acne in mice. The quoted bid and ask jump to 110 1/4-1/2. During Day 2 you receive sell orders for 8,000 shares and buy orders for 2,000 shares. What is your total profit or loss over the two-day period? What is the value of your inventory at the end of Day 2?

You have bought 8,000 shares during Day 2 at 110 1/4 and sold 2,000 shares at 110 1/2. On the 2,000 you bought and sold during the day you earn $2,000 \times 0.25 = 500$. You also added 6,000 shares to your inventory at a price of 110 1/4. Since you were short 6,000 shares at 102 1/2 from yesterday's trading, your loss on these 6,000 shares is $-7.75 \times 6,000 = -46,500$. Thus your total profit/loss over the two-day period is $1,000 + 500 - 46,500 = -45,000$. Your inventory at the end of Day 2 is zero since you purchased 6,000 shares that offset the 6,000 share short position at the end of day 1.

(c) What is a dealer's objective? Is there anything you could have done during Day 1, consistent with a dealer's objective that would have improved your performance over the two-day period?

A dealer's objective is to earn the bid-ask spread, and not necessarily to speculate on equilibrium price movements. The 6,000 share short position at the end of Day 1 left him vulnerable to a jump in quoted prices. Perhaps he should have increased the prices during Day 1 as he observed more buying than selling. Also, he could have reduced his short inventory position by buying from other dealers at 102 1/2 as the market closed on Day 1. There are two factors that may have prevented him from doing this: (1) His expected profit of 0.25 on the 6,000 shares if market prices remained unchanged at 102 1/4-1/2. (2) Unwillingness of other dealers to accommodate his purchase of 6,000 shares at the 102 1/2 ask price.

Question 4:

Palm makes handheld computers and was owned by 3Com, a company selling computer network systems and services. On March 2nd, 2000, 3Com sold a fraction of its stake in Palm to the general public via an initial public offering (IPO). In this transaction, called an equity carve-out, 3Com retained ownership of 95 percent of the shares. 3Com announced that it would eventually spin off its remaining shares of Palm to 3Com's shareholders before the end of the year. 3Com shareholders would receive about 1.5 shares of Palm for every share of 3Com that they owned. After the first day of trading, Palm closed at \$95.06 a share and 3Com was trading at \$81.81.

(a) Can you generate a trading strategy using Palm and 3Com stock that will always generate positive profits? (Hint: You need to go buy a fixed number of stocks in one company and short sell a fixed number of stocks in the other company so that your payoffs are never negative.)

At the close of the first trading day you can either buy Palm directly for \$95.06 a share or you can buy Palm indirectly by purchasing 3Com stock, which trade at only \$81.81. If you purchase 3Com stock you will receive 1.5 Palm stock before the end of the year. Therefore by buying 3Com you buy indirectly 1.5 Palm stock and you will own a portion of the non-Palm activities of 3Com that have a positive value. Thus, if you want to buy Palm you should buy 3Com stock, because you eventually get more Palm stock at a lower price!

One possible trading strategy is to short-sell 1.5 Palm stocks for \$142.59 and buy one 3Com stock for \$81.81. This transaction generates a risk-free cash flow of \$60.78. By the end of the year, 3Com will spin off the remaining shares of Palm and you will receive 1.5 Palm stocks. You cover the short position in Palm by returning the stocks to the securities lender. At that time you will still own the 3Com stock, which has a non-negative value.

	March 2 nd , 2000	End of 2000
Buy one 3Com (COMS)	$-S_t^{\text{COMS}} = -\$81.81$	$S_T^{\text{COMS}} + 1.5(S_T^{\text{PALM}})$
Short-sell 1.5 Palm	$1.5(S_t^{\text{PALM}}) = \142.59	$-1.5(S_T^{\text{PALM}})$
Total Cash Flows	\$60.78	S_T^{COMS}

An alternative trading strategy involves buying one 3Com stock and short-selling one Palm stock. This strategy generates a profit of \$13.25 on March 2nd, 2000. At the end of 2000, you will receive 1.5 Palm stocks from 3Com, you give one Palm stock to the person who lent you Palm and thereby close out the position. Thus, at the end of 2000 you will own a stock of 3Com and 0.5 stocks of Palm, which both have non-negative values.

	March 2 nd , 2000	End of 2000
Buy one 3Com (COMS)	$-S_t^{\text{COMS}} = -\$81.81$	$S_T^{\text{COMS}} + 1.5(S_T^{\text{PALM}})$
Short-sell one Palm	$S_t^{\text{PALM}} = \$95.06$	$-1(S_T^{\text{PALM}})$
Total Cash Flows	\$13.25	$S_T^{\text{COMS}} + 0.5S_T^{\text{PALM}}$

Such transactions are called arbitrages, because they generate only positive cash flows. You can make money without taking any risk. This is a money machine!

(b) What happens if you cannot short-sell companies like Palm?

There are two issues. First, you must find Palm shares to borrow. Many market participants were not able to borrow Palm stocks for a couple of months after Palm's IPO. In this case you cannot do the transaction described above. Second, 3Com might change their mind and not distribute the 1.5 Palm stocks by the end of the year. If this is possible, then the transaction described above is not a pure arbitrage transaction, because it involves risk. It is then speculation. Actually, 3Com did distribute the Palm stocks to their shareholders in July 2000.

(c) How can you explain these prices?

This is clearly a mispricing. Palm is too expensive relative to 3Com. In equilibrium, this kind of situation would be difficult to sustain as the prices would most likely adjust to their true levels. Surprisingly, in reality this situation persisted for a while, mostly because this strategy was not easy to execute due to the shortage of the Palm shares to short. This is something that lots of investors used to get really excited about these days.

Question 5:

Suppose that the economy is either in a recession next period or in a boom. Financial markets are frictionless (i.e., you can buy and sell assets without any constraints and transactions costs). Two assets, stocks and gold, are affected differently by the economic environment. The following table summarizes the rates of returns in these two states:

	Boom	Recession
Return of Stocks	20%	-10%
Return of Gold	-10%	10%

a) Show how to create a portfolio of stocks and gold that has zero risk.

Let w_{Gold} be the proportion invested in gold and $w_{\text{Stocks}} = 1 - w_{\text{Gold}}$ be the weight invested in stocks.

A portfolio of stocks and gold with zero risk should have the same return in booms as in recessions (thus no surprise in general):

$$R_{\text{Boom}} = R_{\text{Recession}}$$

$$w_{\text{Gold}} * (-0.1) + (1 - w_{\text{Gold}}) * 0.2 = w_{\text{Gold}} * (0.1) + (1 - w_{\text{Gold}}) * (-0.1).$$

Solving for w_{Gold} gives $w_{\text{Gold}} = 60\%$ and $w_{\text{Stocks}} = 1 - w_{\text{Gold}} = 40\%$. Let's check that the return is identical in booms and recessions:

$$R_{\text{Boom}} = w_{\text{Gold}} * (-0.1) + (1 - w_{\text{Gold}}) * 0.2 = 0.6 * (-0.1) + 0.4 * 0.2 = 0.02 = 2\%$$

$$R_{\text{Recession}} = w_{\text{Gold}} * (0.1) + (1 - w_{\text{Gold}}) * (-0.1) = 0.6 * (0.1) + 0.4 * (-0.1) = 0.02 = 2\%$$

Note that you did not have to assume anything about the probabilities of boom and recession to get it right.

b) Suppose that risk-free T-Bills have a return of 4%. In light of results in a) is there any interesting investment opportunity here? If yes, please describe in detail the possible investment strategy and how it would make you a happy investor.

The portfolio described in (a) has a return of 2% in the two possible future states. On the other hand, T-Bills have a return of 4%. We can make money by borrowing at 2% and investing the borrowed funds at 4%. This generates risk-free profits. To borrow funds at 2%, we short-sell the portfolio of 60% gold and 40% stocks. The difference of 2% is the risk-free profit we can pocket.