

**Homework 1**  
**Due Friday 02/24**  
**One copy per group**

**Question 1:**

Consider a risky asset X with an expected rate of return of 12% and a volatility of 7%. The Treasury bill rate of return is 3%. You hold a portfolio of total market value £4000, entirely invested in the risky asset.

- (a) Consider leveraging your portfolio. You borrow an additional £2000 to invest in the risky asset. Suppose that the borrowing rate is equal to the Treasury bill rate. Calculate the Sharpe ratio of your leveraged portfolio and compare it to the Sharpe ratio of your initial portfolio.
- (b) Suppose that the borrowing rate now is higher than the Treasury bill rate and equal to 6%. Calculate the Sharpe ratio of the leveraged portfolio as constructed in part a. Comment on the result.
- (c) Using a borrowing rate of 6%, calculate the realized return of your leveraged portfolio if the asset's X price falls by 30%. If the maintenance margin is set at 25%, will you receive a margin call?
- (d) Instead of taking leverage buying on margin asset X, your broker suggests you to add another risky asset Y to your initial portfolio. The asset Y has an expected return of 8%, a volatility of 9%, and a correlation of 0.1 with asset X. Will adding asset Y improve your portfolio?

**Question 2:**

Mr. S. wants to retire from his emotionally demanding job 25 years from now. He wants to save enough so that he can have a pension of £10,000 a month for fifteen subsequent years (starting at the end of his job). Assume that the annual interest rate is equal to 4%.

- (a) How much would he have to save each month during the first 25 years to achieve his goal? Assume that all cash flows occur at the end of each month.
- (b) Suppose that instead of having a 15-year stream of cash flows, Mr. S. would like to create an infinitely long stream of monthly cash flows of £8000 starting 25 years from now to provide for his descendants. How much would he have to save each month during the first 25 years?

**Question 3:**

You are among the OTC dealers (intermediaries) in the stock of Bio-Engineering, Inc. and quote a bid of 102 1/4 and an ask of 102 1/2. Suppose that you have a zero inventory.

(a) On Day 1, you receive buy orders for 10,000 shares and sell orders for 4,000 shares. How much do you earn on the 4,000 shares that you bought and sold? What is the value of your inventory at the end of the day? (Hints: It is possible to have negative inventory. Further, there is more than one correct way to value an inventory, but please state what assumption your valuation is based on.)

(b) Before trading begins on Day 2 the company announces trial testing of a cure for acne in mice. The quoted bid and ask jump to 110 1/4-1/2. During Day 2 you receive sell orders for 8,000 shares and buy orders for 2,000 shares. What is your total profit or loss over the two-day period? What is the value of your inventory at the end of Day 2?

(c) What is a dealer's objective? Is there anything you could have done during Day 1, consistent with a dealer's objective that would have improved your performance over the two-day period?

#### Question 4:

Suppose that the economy is either in a recession next period or in a boom. Financial markets are frictionless (i.e., you can buy and sell assets without any constraints and transactions costs). Two assets, stocks and gold, are affected differently by the economic environment. The following table summarizes the rates of returns in these two states:

	Boom	Recession
Return of Stocks	20%	-10%
Return of Gold	-10%	10%

- (a) Show how to create a portfolio of stocks and gold that has zero risk.
- (b) Suppose that risk-free Treasury bills have a return of 4%. In light of results in a) is there any interesting investment opportunity here? If yes, please describe in detail the possible investment strategy and how it would make you a happy investor.

#### Question 5:

Assume that you manage a risky portfolio with an expected rate of return of 17% and a volatility of 27%. The T-bill rate is 7%. Suppose your client initially chooses to invest 70% of a portfolio in your fund and 30% in T-bill.

- (a) What is the expected return and volatility of your client's portfolio?
- (b) Now, suppose that your client wonders whether to switch the 70% that is invested in your fund to the passive portfolio with the expected return of 13% and a volatility of 25%. Explain to your client the disadvantage of the switch.
- (c) Show your client the maximum fee you could charge (as percent of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one.

**Question 6:**

Using available data sources (e.g., the Economist), collect and summarize in a table the data (as of the end of 2015 and 2016) for the following countries: United Kingdom, United States, Germany, China, India, Brazil, Japan, Switzerland, Australia, and Canada. The data to report are: short-term interest rates, long-term (10-year) interest rates, GDP growth (per year), local exchange rate (in terms of USD), and the value of local (main) stock market index.

- (a) Generate a column reporting changes in the statistics over the one-year period.
- (b) Using the data for all countries, present a scatter plot relating changes in interest rates (x-axis) to changes in currency rates (y-axis)? Using your economic intuition, discuss **briefly** the pattern you find.
- (c) In a similar vein, plot the relation between long-term interest rate changes and GDP growth. Using your economic intuition, discuss **briefly** the pattern you find.
- (d) Similarly, plot the relation between the changes in long-term interest rates and changes in local stock market value. Using your economic intuition, discuss **briefly** the pattern you find.