

Sample Midterm, FINC-UB.0002.02

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1 Multiple Choice Questions

The following are sample questions for the multiple choice section. They are organized by degree of difficulty (this ranking is a bit subjective).

1.1 Easy

1. Security A has a higher equilibrium price volatility than security B. Assuming all else is equal, the equilibrium bid-ask spread of A would be expected to be:
 - (a) Greater than B
 - (b) Less than B
 - (c) Equal to B
 - (d) It is impossible to tell
2. Security A has a higher trading volume than security B. Assuming all else is equal, the equilibrium bid-ask spread of A would be expected to be:
 - (a) Greater than B
 - (b) Less than B
 - (c) Equal to B
 - (d) It is impossible to tell
3. If you can get 7.75% return on in 10 years from your local bank, would it be wise to invest in a 10 year, \$1000 par value zero coupon bond that costs \$475? (Assume both are equally risky)
 - (a) Yes, the YTM is greater
 - (b) No, the YTM is less
 - (c) Can't tell from information given

4. What is the expected return on a two asset portfolio, where you invest 150% of your net worth in A, with a mean return of 10%, and borrow 50% of your net worth by selling short B, which has a mean return of 6%?
- (a) 8%
 - (b) 18%
 - (c) 120%
 - (d) 12%
 - (e) None of the above
5. A riskfree security pays a dividend of 200\$ after one year, 400\$ after two years, 800\$ after three years, and thereafter it never pays dividends again. The riskfree interest rate is 3%. What is the current price of the security:
- (a) 1203.3
 - (b) 1303.3
 - (c) 1345.2
 - (d) 1400
6. According to portfolio theory, which of the following is *not* true?
- (a) All systematic risk can be diversified away
 - (b) All non-systematic risk can be diversified away
 - (c) Diversification lowers the potential risk of the portfolio
7. Which of the following statements about short selling a risk-free security is true:
- (a) It is impossible to short sell risk-free securities
 - (b) Even combined with other securities, the short sale makes no sense
 - (c) This transaction is in principle equivalent to borrowing money
 - (d) This transaction is in principle equivalent to lending money
8. Which of the following is not possible when two securities are positively correlated:
- (a) Asset A's mean return is negative while asset B's is positive
 - (b) Asset A's return is sometimes below its mean when asset B's is above its mean
 - (c) Asset A's mean return is negative while asset B's mean return is also negative
 - (d) All are possible

9. A security can be in one of four states next year:
- i) a good state with a return of 35% (this happens with probability = 0.30);
 - ii) a normal state with a return of 15% (this happens with probability = 0.50); and
 - iii) a bad state with a return of 0% (this happens with probability = 0.15).
 - iv) a disaster state with a return of -50% (this happens with probability = 0.05).
- What are, respectively, the mean rate of return and the standard deviation of the rate of return?
- (a) $E(R) = 0.175$; $\sigma = 0.16$
 - (b) $E(R) = 0.155$; $\sigma = 0.19$
 - (c) $E(R) = 0.155$; $\sigma = 0.16$
 - (d) $E(R) = 0.175$; $\sigma = 0.15$
 - (e) $E(R) = 0.155$; $\sigma = 0.034$
10. What is the effective annual rate corresponding to an APR of 40% with weekly compounding?
- (a) 34.23%
 - (b) 52.12%
 - (c) 42.88%
 - (d) 48.95%

1.2 Moderately Difficult

11. Suppose you have a two asset portfolio with $\sigma_1 = .05$ and $\sigma_2 = .08$. Assume the correlation coefficient of returns on the two assets is -1.0. Assuming you must hold positive amounts of both securities, what fraction of the portfolio should you hold in asset 2 to reduce the risk of the portfolio to zero.
- (a) .62
 - (b) .5
 - (c) .42
 - (d) .38
12. If a Treasury bill pays 5%, which of the following would definitely not be chosen by a risk averse investor: (assume that investors cannot form portfolios.)
- (a) An asset paying 10%, with probability .6 or 2% with probability .4
 - (b) An asset paying 10% with probability .4 or 2% with probability .6

- (c) An asset paying 10% with probability .2 or 3.75% with probability .8
 - (d) An asset paying 10% with probability .3 or 3.75% with probability .7
13. A portfolio consisting of two risky securities that have a correlation of zero has a minimum variance portfolio that has a standard deviation equal to
- (a) The weighted average of the standard deviations of the two securities
 - (b) -1
 - (c) 0
 - (d) Greater than 0
14. Assume $\sigma_1 = 10\%$ and $\sigma_2 = 30\%$. Under what circumstances will a portfolio allocation of 25% in asset 1 and 75% in asset 2 produce a σ for the combined portfolio equal to 25%
- (a) $\rho = 0$
 - (b) $\rho = 1$
 - (c) $\rho = -1$
 - (d) None of the above
15. Suppose Kim and Susan care only about the mean and standard deviation of their portfolio return. Kim is less risk averse than Susan. Suppose that Susan holds the tangency portfolio. Which portfolio might Kim hold?
- (a) The riskfree asset
 - (b) The tangency portfolio
 - (c) The tangency portfolio leveraged by the risk-free asset
 - (d) None of the above
16. Assume the variance of IBM is .16 and the variance of Microsoft is .25. If the variance of an equally weighted portfolio of these stocks is .0525, then the covariance between these stock is:
- (a) .10
 - (b) .20
 - (c) .25
 - (d) -.10
17. John and Jim are both risk averse and only care about the mean and standard deviation of their portfolio return. They agree on the opportunity set available. There are N risky assets and a riskless asset. Which of the following statements is correct?

- (a) John and Jim must hold the same portfolio of all assets.
 - (b) John and Jim may hold completely different portfolios of risky assets.
 - (c) When choosing between 2 portfolios, John and Jim always prefer the one with the lowest standard deviation.
 - (d) John holds any two risky assets in the same ratio as Jim does in his portfolio.
18. Which of the following best explains a decline in a dealer's inventory:
- (a) bid price and offer price are too high
 - (b) bid price is too high and offer price is too low
 - (c) bid price is too low and offer price is too high
 - (d) bid price and offer price are too low

1.3 Difficult

19. Which of the following is true about risk averse investor?
- (a) They care only about risk
 - (b) They care only about returns
 - (c) They might hold a risky security as part of a portfolio even if its expected return is less than the risk-free rate
 - (d) They prefer a risk-free security to a risky security
20. Suppose that among the many stocks in the market there are two securities, A and B, with the following characteristics: A has mean .08 and $\sigma = .4$ and B has mean .13 and $\sigma = .6$. If the correlation between these two is $\rho = -1$, and if it is possible to borrow and lend at the risk-free rate, R_f , then the equilibrium risk-free rate must be:
- (a) 8%
 - (b) 10%
 - (c) 13%
 - (d) any R_f is possible

2 Numerical Questions

2.1 Challenging Question on Performance of Securities

Your bank offers you a \$60,000 mortgage loan at an annual 5.45% interest rate. If you make monthly payments of \$1,250 how many months will it take you to pay off the loan?

2.2 Challenging Question on Portfolio Theory

Suppose that the expected return and standard deviation of portfolio A is .10 and the expected return and standard deviation of portfolio B is 0.15. The correlation between A and B is 0. Suppose you put 80% of your wealth in portfolio A and the rest in portfolio B. Is this portfolio efficient (on the efficient frontier)? Explain why or why not.

2.3 Challenging Question on Portfolio Theory 2

Assume the standard deviation of Dell stock is .24 and the standard deviation of General Motors is .17. If you put 130% of your wealth in Dell and take a 30% short position in General Motors and the standard deviation of the portfolio is .298, what is the correlation coefficient between the returns of Dell and General Motors?

2.4 Challenging Question on Portfolio Theory 3

Super and Hyper are two risky securities. The expected return of Super is 6%, and its standard deviation is 8%. The expected return of Hyper is 3%, and its standard deviation is 4%. The returns on Super and Hyper are perfectly negatively correlated (their correlation is -1).

- (a) What fractions of an investor's wealth should be held in Super and Hyper in order to produce a zero-risk portfolio?
- (b) What is the expected return on the zero-risk portfolio?
- (c) Suppose that the risk-free T-bill rate is equal to 3%. How would you set up an arbitrage trade?

2.5 Challenging Question on CAPM

A regression of the return on Ebay on the return of S&P 500 gives you the following result:

$$R_{ebay} = 0.03 + 1.45R_{S\&P500} + error_{ebay}.$$

Suppose that the standard deviation of the S&P 500 return is 0.20 and the standard deviation of $error_{ebay}$ is 0.10, what fraction of the total risk of Ebay is systematic risk?

2.6 Challenging Question on NPV

A firm considers undertaking an investment project with the following cash flows: initial machine purchase: \$20,000; a purchase of raw materials of \$1000 in year one; revenue from sales of production of \$9,000 in year two and \$12,000 in year three. At the end of year three it sells the machine for \$3,000. If the cost of external capital is 7%, should the firm undertake this project?

3 Answers

3.1 Multiple Choice

1. A (Market Maker needs higher bid-ask spread to be compensated for larger risk on inventory positions)
2. B (One reason is that a Market Maker needs to be compensated with a lower bid-ask spread because inventory positions can be more easily be traded away which implies less inventory risk)
3. B ($YTM = (1000/475)^{1/10} - 1 = 7.73\%$)
4. D ($1.5 * 10\% + -0.5 * 6\% = 12\%$)
5. B ($200/1.03 + 400/(1.03)^2 + 800/(1.03)^3 = 1303.3$)
6. A
7. C
8. D
9. B ($E[R] = 0.30 * 35\% + 0.50 * 15\% + 0.15 * 0\% + 0.05 * -50\% = 15.5\%$, $\sigma = \sqrt{0.30(35\% - E[R])^2 + 0.50(15\% - E[R])^2 + 0.15(0\% - E[R])^2 + 0.05(-50\% - E[R])^2} = 0.191$)
10. D ($EAR = (1 + 0.40/52)^{52} - 1 = 48.95\%$)
11. D ($\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$; for $\rho_{1,2} = -1$ this simplifies to $\sigma_P^2 = (w_1\sigma_1 - w_2\sigma_2)^2$, using $w_1 = 1 - w_2$ this is zero if $w_2 = \sigma_1/(\sigma_1 + \sigma_2)$, plugging in numbers gives $w_2 = 0.3846$)
12. C ($E[R_C] = 0.2 * 10\% + 0.8 * 3.75\% = 5\%$, which equals the risk free rate, and therefore only adds risk without raising the expected return. $E[R_A], E[R_B], E[R_D] > 5\%$.)
13. D (A would be true for perfectly correlated assets (assuming the weights in both are positive, B is obviously wrong because standard deviation cannot be negative, C would only be true for a perfectly negative correlation (+1 or -1).
14. B ($\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$; for $\rho_{1,2} = 1$ this simplifies to $\sigma_P^2 = (w_1\sigma_1 + w_2\sigma_2)^2$ or $\sigma_P = |w_1\sigma_1 + w_2\sigma_2|$, which is the absolute value of the weighted average. Clearly $25\% = 0.25 * 10\% + 0.75 * 30\%$, so indeed we are in the special case of perfect positive correlation $\rho = 1$.)
15. C (Less risk averse investors select into riskier portfolios with higher expected returns)
16. D ($\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2COV_{1,2}$ (where $COV_{1,2} = \rho_{1,2}\sigma_1\sigma_2$); we have $w_1 = w_2 = 0.5$, $\sigma_P^2 = 0.0525$, $\sigma_1^2 = 0.16$, $\sigma_2^2 = 0.25$, and this implies $COV_{1,2} = -0.10$)
17. D

18. D (lower bid \implies lower supply from investors \implies lower inventory AND lower ask \implies higher demand from investors \implies lower inventory)

19. C (Investors care about the risk of the overall portfolio. Adding an asset that by itself is risky might still lower the risk of the overall portfolio if it is negatively correlated with the other assets (negative systematic risk, negative beta). Investors in this case might be willing to hold the asset even when its expected return is lower than the risk free rate (which has zero systematic risk, zero beta).

20. B (Similar to question 11, with perfect negative correlation we have zero risk if $w_A = \sigma_B / (\sigma_A + \sigma_B) = 0.6 / (0.4 + 0.6) = 0.6$. The expected return with $w_A = 0.6$ and $w_B = (1 - w_A) = 0.4$ is $0.6 * 8\% + 0.4 * 13\% = 10\%$. No arbitrage implies that the return on the risk free asset must equal 10%.)

4 Answers to Numerical Questions

4.1 Question on performance of Securities

This problem is a standard annuity problem. The periodic payment is \$1250, the present value is

$$PV = C[1/r - 1/r(1+r)t].$$

This question asks you to solve for t. $PV = 60,000$ the periodic rate is $r = 5.45\% / 12 = 0.45417\%$, the periodic payment is 1250. The answer is 55 full periods (you will have paid back a little bit too much 60,714). Also correct is 54 periods (you will have paid back a little bit too little 59,740) or 54.266 periods which is the exact answer.

4.2 Question on Portfolio Theory

No. Efficient portfolios lie on the increasing part of the investment opportunity set. That is the part of the investment opportunity set that lies beyond (above) the minimum variance portfolio. If $w_A = 0$ then the (20%, 80%) portfolio has an expected return of 11% and a standard deviation of 0.085. This is not on the efficient part of the investment opportunity set because if you put a little bit less in A, for example 0.75 and a little bit more in B, for example 0.25, then you get a higher expected return of 11.25% and a lower standard deviation of 0.084. This means that the original portfolio (20%, 80%) lies on the inefficient part of the investment opportunity set.

4.3 Question on Portfolio Theory 2

Correlation $\rho = .35$. The variance of the portfolio is:

$$Var[R_p] = (0.298)^2 = (1.3)^2(.24)^2 + (-.3)^2(.17)^2 + 2(1.3)(-.3)(.24)(.17)\rho.$$

It follows that the correlation coefficient is equal to:

$$\rho = \frac{[(0.298)^2 - (1.3)^2(.24)^2 - (-.3)^2(.17)^2]}{[2(1.3)(-.3)(.24)(.17)]} = .35$$

4.4 Question on Portfolio Theory 3

Part a Whenever two risky assets are perfectly negatively correlated, there exists one portfolio (weighted average of Super and Hyper) that has exactly no risk (zero standard deviation). We need to solve for weight of the assets in that portfolio. In class, we derived that the weight of Super in the zero-risk portfolio equals

$$w_S = \frac{\sigma_H}{\sigma_S + \sigma_H}.$$

Plugging in for the standard deviations, we find that $w_S = \frac{1}{3}$ and therefore we invest 1/3 in Super and 2/3 in Hyper.

Digression: Where does this formula come from? You may recall that this equation follows from the general formula for the variance of the return on a portfolio of two risky assets:

$$Var[R_p] = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}$$

In the case of $\rho_{12} = -1$, the general variance formula simplifies to:

$$Var[R_p] = (w_1\sigma_1 - w_2\sigma_2)^2$$

We need to find the weight w_1 which makes this variance equal to zero; that weight is the one from the zero-risk portfolio.

$$0 = w_1\sigma_1 - w_2\sigma_2$$

I also took the square root on both sides (the square root of zero is zero). Finally, note that $w_2 = 1 - w_1$. We can now solve this equation for w_1 :

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}.$$

Part b The expected return on this zero-risk portfolio is straightforward to calculate:

$$E[R_p] = w_SE[R_S] + w_HE[R_H],$$

which gives $E[R] = .333 * 6\% + .666 * 3\% = 4\%$.

Part c The key insight is that this portfolio (1/3 Super, 2/3 Hyper) is risk-free. By no arbitrage, there cannot be two different risk-free investments with a different rate of return.

Here, we are given that there is a risk-free bond with a return of 3%. This is a violation of no arbitrage. To exploit this violation, you would borrow at the risk-free rate of 3% (which is equivalent to shorting the risk-free bond) and you would invest the proceeds in the zero risk portfolio (1/3 Super, 2/3 Hyper) and earn 4% (long in the zero-risk portfolio). You would earn the difference of 1% per year. This is a risk-free rate of return. This investment strategy requires no initial cash outlay and it carries no risk. Hence, it is an arbitrage.

4.5 Question on CAPM

The total risk is $Var[R_{ebay}] = (1.45)^2(.2)^2 + (.1)^2 = .0941$. The first term measures systematic risk (market risk), the second term is idiosyncratic risk. The fraction of systematic risk is $.0841/.0941 = 0.894$ or 89.4%.

4.6 Question on NPV

5.33%. The cash flows are: $CF_0 = -20,000$, $CF_1 = -1,000$, $CF_2 = 9,000$, $CF_3 = 15,000$. You can verify (using your financial calculator) that the IRR is 5.328%. If the cost of capital is $7\% > 5.33\%$, then the firm should not undertake this investment project. This project will reduce the firm value rather than increase it. There is a second way of answering this question that does not require to calculate the IRR. It is to compute the NPV using the cost of capital of 7%. $NPV = -20,000 - 934.58 + 7,860.95 + 12,244.47 = -829.14$. Since the NPV is negative, do not undertake the project. Both methods give the same answer.