Time Value of Money

Lecture Two

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What is an Asset?

- Could be property, machinery, equipment
- Patents, R&D
- Stocks, Bonds, Derivatives
- Reputation, Opportunities, Logo(!)
- Formally, it is a **sequence of cashflows**

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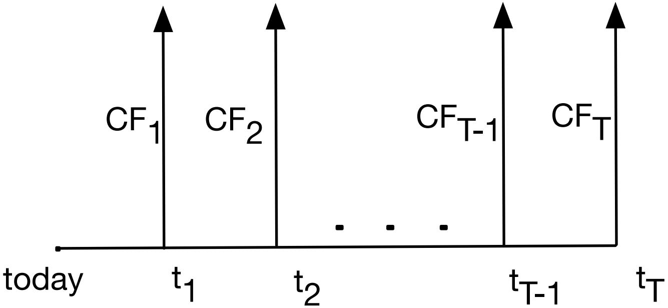
  - Dec 2, 1982 GMAC sold securities promising $10,000 on Dec 1, 2012.
  - Price? $500

Valuing the Asset



Different Currencies

- Cannot add currencies without converting into a common currency.

$$
\begin{aligned}
¥1000 + \$10 = ¥1000 + ¥1183.5 = ¥2183.5 \\
¥1000 + \$10 = \$8.45 + \$10 = \$18.45
\end{aligned}
$$

- Need to convert everything to common 'currency' or time. A few ways to convert:

Future Value and Present Value

- **One dollar today does not equal one dollar tomorrow.**
- Present Value (PV), Future Value (FV), and Yield (R) are all related.
- FV is one period away:

$$FV = PV(1 + R)$$

- FV is multiple periods away:

$$\begin{aligned} FV &= PV(1 + R)^T \\ FV &= PV(1 + RT) \end{aligned}$$

- Simple vs. Compound Interest.

Future Value: Convert to the end

- How much is \$1 invested for one year at 5% worth?
- $\text{PV} = 1$; $\text{R} = 0.05 \rightarrow \text{FV} = 1.05$
- What if it were invested for 2 years?
- $\text{PV} = 1$; $\text{R} = 0.05 \rightarrow FV_S = 1.10$, $FV_C = 1.1025$
- Compound also known as Interest on Interest

Present Value: Convert to the beginning

- How much is a cash-flow in one period worth?

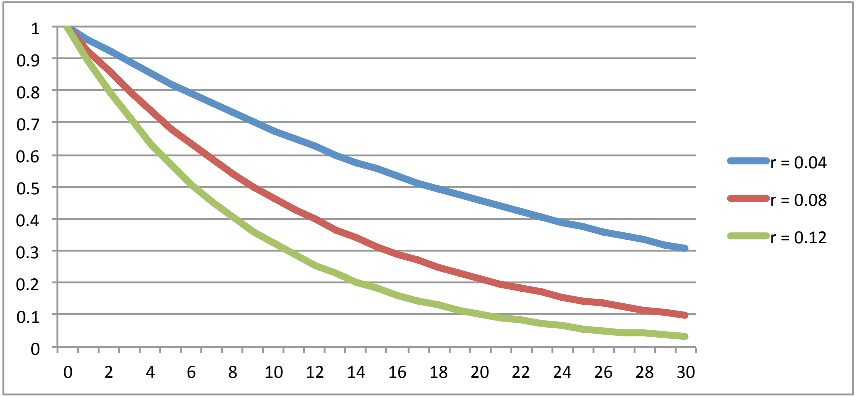
$$PV = \frac{FV}{1+R}$$

- What about in several periods?

$$PV = \frac{FV}{(1+R)^T}$$

Future Value: Convert to the end

- **Discount Factor for period t** is $\frac{1}{(1+R)^t}$.
- Who/what determines the discount factor?
- Markets and outside options. Arbitrage. PV = Price.



Simple Example: Discount Bond

- Zero Coupon Bonds, or Discount bonds
  - Issued in primary markets
  - Can be constructed by stripping coupon bonds.

  - Price increases as bond gets to maturity.
  - Changes in interest rate affect longer horizon bonds more.

Yield

- How much does an investment return in one period?

$$R = \frac{FV}{PV} - 1$$

- How much does an investment return in many periods?

$$R = \left(\frac{FV}{PV}\right)^{\frac{1}{T}} - 1$$

PV, FV, R, and T

- Any three will yield the fourth.
- If \$1000 yields \$1108 in three years, what is the yield?

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$$1108 = 1000(1 + R)^3 \qquad R = 3.5\%$$

- If \$1000 yields \$1791 at 6% how long was the investment?

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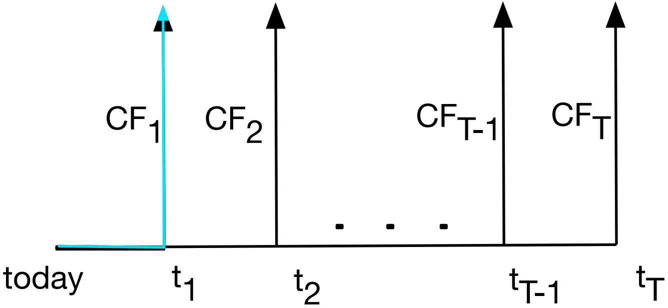
- If \$1000 yields \$1108 in three years, what is the yield?

$$1108 = 1000(1 + R)^3 \qquad R = 3.5\%$$

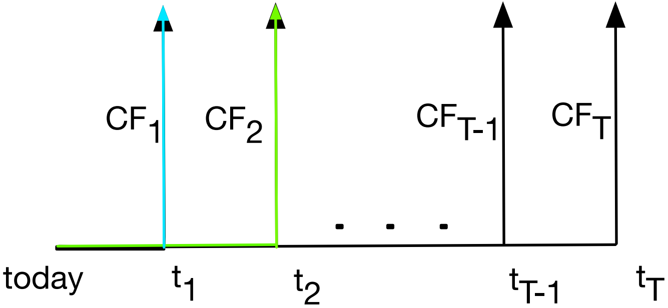
- If \$1000 yields \$1791 at 6% how long was the investment?

$$1791 = 1000(1.06)^T \qquad T = 10$$

Complicated Asset



Complicated Asset



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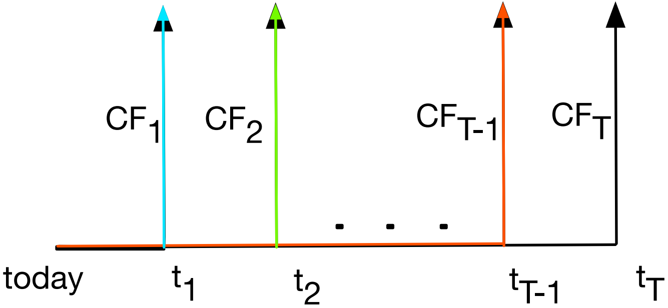

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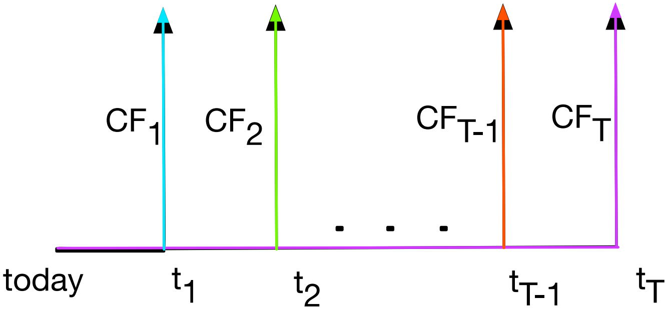


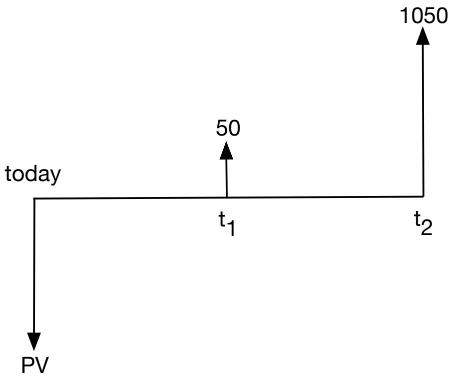
Complicated Asset

$$
\begin{aligned}
PV &= \frac{CF_1}{(1+R)} + \frac{CF_2}{(1+R)^2} + ... + \frac{CF_{T-1}}{(1+R)^{T-1}} + \frac{CF_T}{(1+R)^T} \\
FV &= CF_1(1+R)^{T-1} + CF_2(1+R)^{T-2} + ... + CF_{T-1}(1+R) + CF_T
\end{aligned}
$$

PV of sum is sum of PV.

FV of sum is sum of FV.

Multiple Payments



What is the PV when $R = 5\%$?

What about when $R = 6\%$?

PV Project Analysis

- Suppose you have to pay $\$250,000$ for a project.
- The project pays $\$100,000$ every year for three years.
- Assume that the opportunity cost is known to be 4%.
- Worth it?

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PV &= -250,000 + \frac{100,000}{1.04} + \frac{100,000}{1.04^2} + \frac{100,000}{1.04^3} \\
&= -250,000 + 96,154 + 92,456 + 88,900 \\
&= 27,510
\end{aligned}
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- What if price were $275,000?

Perpetuities

- How much is an infinite cash-flow of C each year worth?

$$
\begin{aligned}
PV &= \frac{C}{1+R} + \frac{C}{(1+R)^2} + \frac{C}{(1+R)^3} + \ldots \\
(1+R)PV &= C + \frac{C}{1+R} + \frac{C}{(1+R)^2} + \ldots \\
(1+R)PV &= C + PV \\
PV &= \frac{C}{R}
\end{aligned}
$$

Growing Perpetuities

- How much is an infinite cash-flow of C growing at g each year worth?

$$
\begin{aligned}
PV &= \frac{C}{1+R} + \frac{C(1+g)}{(1+R)^2} + \frac{C(1+g)^2}{(1+R)^3} + \ldots \\
\frac{(1+R)}{(1+g)} PV &= \frac{C}{(1+g)} + \frac{C}{1+R} + \frac{C(1+g)}{(1+R)^2} + \ldots \\
\left[\frac{(1+R)}{(1+g)} - 1\right] PV &= \frac{C}{1+g} \\
PV &= \frac{C}{R-g}
\end{aligned}
$$

What must g satisfy?

Fixed Horizon Annuities

- How much is an fixed horizon cash-flow of C each year worth?

$$
\begin{aligned}
PV &= \frac{C}{1+R} + \frac{C}{(1+R)^2} + \ldots + \frac{C}{(1+R)^T} \\
(1+R)PV &= C + \frac{C}{1+R} + \ldots + \frac{C}{(1+R)^{T-1}} \\
RPV &= C - \frac{C}{(1+R)^T} \\
PV &= \frac{C}{R}\left[1 - \frac{1}{(1+R)^T}\right]
\end{aligned}
$$

Infinite Perpetuity $-$ Date-T Perpetuity $=$ T-Period Annuity

Interest Rate vs Prices

- **Yield varies inversely with the price**.
- For perpetuities it is further inversely proportional.
- Sensitivity of price to yield depends on maturity.
- Stocks reaction to interest rate changes?

Compounding

- Interest often charged/credited more often than annually
  - Bank accounts: daily
  - Mortgages and leases: monthly
  - Bonds: semiannually
  - Effective annual rate.
- Compounding Conventions: $r_{EAR} = \left(1 + \frac{r}{n}\right)^n - 1$
- $r_{\text{yr}} = 5\% \rightarrow r_{\text{semi}} = 5.06\%, r_{\text{mth}} = 5.12\%, r_{\text{d}} = 5.13\%$

Other Issues: Assumptions

- Different discount factors between people.
    - Different values for future money.
    - One incentive for trade.
    - Risk Sharing
- Different discount factors across times
    - Sum of different discount bonds.
- Certainty vs. Uncertainty
    - What if we don't know what future cash flows are?

Real World Applications

- Who determines the yield?
- Conditional on risk, the market.
- When riskless, policy makers.