Solutions to Homework 3, FINC-UB.0002.02

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Topic 6: Equity Valuation

1. Suppose that the consensus forecast of security analysts of your favorite company is that earnings *next year* will be $E_1 = \$5.00$ per share. Suppose that the company tends to plow back 50% of its earnings and pay the rest as dividends. If the Chief Financial Officer (CFO) estimates that the company's growth rate will be 8% from now onwards, answer the following questions.

   (a) If your estimate of the company's required rate of return on its stock is 10%, what is the equilibrium price of the stock?

   The equilibrium price of a stock can be determined using Gordon's growth formula as follows:

$$P_0 = \frac{E_1(1-b)}{R-g}$$

   where $b = .5$ the plow back ratio, $R = .10$ the required rate of return, and $g = .08$ is the growth rate. Thus, the price should be:

$$P_0 = \frac{\$5(1-.5)}{.10-.08} = \$125$$

(b) Suppose you observe that the stock is selling for \$50.00 per share, and that this is the best estimate of its equilibrium price. What would you conclude about either *(i)* your estimate of the stock's required rate of return; or *(ii)* the CFO's estimate of the company's future growth rate?

Recall from the formula above, that a higher required rate of return implies a lower equilibrium price of the stock. On the other hand, a higher growth rate implies a higher stock price. Thus, a lower equilibrium stock price ($\$50 < \$125$) could indicate that either: (1) the required rate of return is higher than originally expected, and/or (2) the growth rate is less than originally estimated.

(c) Suppose your own 10% estimate of the stock's required rate of return is shared by the rest of the market. What does the market price of \$50.00 per share imply about the market's estimate of the company's growth rate?

To solve for the expected growth rate, use Gordon's growth model:

$$\$50 = \frac{\$5(1-.5)}{.10-g}$$

Solving for g, you get $g = 0.05$.

2. This question requires data collection. You can find all numbers on `finance.yahoo.com`. The questions concern Microsoft (ticker: MSFT). The answers are for the numbers as of August 5, 2013.

[Figure 1 about here.]

(a) What is the current price and the current price-earnings ratio?

The current price is \$31.74, the current earnings per share (EPS) are \$2.58, and the current price-earnings ratio is 31.74/2.58=12.30. All these statistics are in the table under “Summary".

(b) What is the current plow-back ratio?

The earnings per share are \$2.58, and the dividends per share are \$0.92, so that the plow-back ratio is (2.58-0.92)/2.58 = 0.6434 or 64.34%. Microsoft reinvests (plows back) almost 64% of its earnings and pays out the other 36% to its shareholders.

(c) What is the growth rate of earnings for the next 5 years according to the analysts? Hint: look for annual growth rates under “analyst estimates"

The consensus forecast for the next 5 years is an average annual earnings growth rate of 8.63%

(d) What is the beta of MSFT? Hint: look for “key statistics". If the risk-free rate (R_f) is 4% and the market risk premium $E[R_M - R_f]$ is 6%, what is the required rate of return on MSF according to the CAPM?

The beta of MSF is 0.90. The security market line of the CAPM tells us that the required rate of return is given by:

$$E[R_{MSF}] = R_f + \beta_{MSF}(E[R_M - R_f]) = .04 + 0.90 * .06 = 0.0940$$

(e) Assume that Microsoft will have earnings and dividends that will grow at the analysts forecasted rate forever after; i.e., the Gordon growth model (GGM) applies. What is the price-earnings ratio that the GGM predicts for Microsoft?

The price-earnings ratio in the GGM is given by:

$$\frac{P_0}{E_0} = \frac{(1-b)(1+g)}{(R-g)} = \frac{(1-.6434)(1+.0863)}{(.0940-.0863)} = 50.3084$$

(f) What growth rate does the market price-earnings ratio price in/imply? The market price-earnings ratio is 12.30.

$$
\begin{aligned}
\frac{P_0}{E_0} &= \frac{(1-b)(1+g)}{(R-g)} \\
\Leftrightarrow \frac{\frac{P_0}{E_0}}{(1-b)} &= \frac{(1+g)}{(R-g)} \\
\Leftrightarrow \frac{12.30}{(1-.6434)} &= \frac{(1+g)}{(R-g)} \\
\Leftrightarrow 34.4924 &= \frac{(1+g)}{(R-g)} \\
\Leftrightarrow 34.4924R - 34.4924 * g &= 1+g \\
\Leftrightarrow g &= \frac{34.4924R - 1}{1 + 34.4924} \\
&= \frac{34.4924 * 0.0940 - 1}{1 + 34.4924} = 0.0632
\end{aligned}
$$

The market prices in a 6.32% earnings growth rate, which is lower than the 8.63% growth rate of the analysts. This makes sense because the price-earnings ratios in the data is much lower than the price-earnings ratios in the GGM.

Topic 7: Arbitrage

3. The stock PolarBear.com trades on both the South Pole Stock Exchange and the North Pole Stock Exchange.

   (a) Suppose the price on the North Pole is $18. What does the No-Arbitrage Condition say about the price on the South Pole? (Assume no trading costs.)

   To rule out arbitrage, the price on the South Pole must be the same as on the North Pole, $18.

(b) Suppose the price on the North Pole is \$18 and the price on the the South Pole is \$17? How can you make an arbitrage profit? (Assume no trading costs.)

You can make an arbitrage by buying on the the South Pole for \$17 and selling on the North Pole for \$18. This transaction has a riskless profit of \$1 per share.

(c) Suppose that the price on the North Pole is \$18, that buying or selling on the North Pole costs \$2, and that buying or selling on the South Pole is free. What does the No-Arbitrage Condition say about the price on the South Pole?

The price on the South Pole must be between \$16 and \$20. If the price is lower than \$16 then you can make an arbitrage profit, net of trading costs, by buying on the South Pole and selling on the North Pole. Similarly, you can make an arbitrage if the price is higher than \$20.

4. Suppose that there are two securities RAIN and SUN. RAIN pays \$100 in there is any rain during the next world cup soccer final. SUN pays \$100 in there is no rain. Suppose that the world cup soccer final is 1 year from today (although this is not true), and suppose that RAIN is trading at a price of \$23 and SUN is trading at a price of \$70.

(a) If you buy 1 share of RAIN and 1 share of SUN, what is your payoff after 1 year, depending on the weather?

If you buy 1 share of RAIN and 1 share of SUN, then your payoff after 1 year is \$100, *no matter what the weather is.*

(b) What does the No-Arbitrage Condition imply about the price of a 1-year zero-coupon bond? (Assume no trading costs.)

A portfolio of 1 share of RAIN and 1 share of SUN replicates the payoff of a zero-coupon bond, that is, it has a sure payoff of \$100 just like a zero. Therefore, the zero must have the same price as the portfolio, namely \$93.

(c) Suppose that a 1-year zero-coupon bond is trading at \$90. Show how you would set up a transaction to earn a riskless arbitrage profit. (Assume no trading costs.)

If the 1-year zero-coupon bond is trading at \$90 then you can earn a riskless arbitrage profit as follows. You buy 1 zero-coupon bond, short-sell 1 RAIN, and short-sell 1 SUN. This gives you a profit of \$23+\$70-\$90=\$3 today, and your net cash flow next year is 0, no matter what the weather is.

(d) Suppose that trading zero-coupon bonds is costless, but trading RAIN and SUN each cost \$2 per \$100 face value. Can you still make an arbitrage profit?

If trading RAIN and SUN each cost \$2 per share then the above strategy has a trading cost of \$4. This is greater than the value of the mispricing \$3. Therefore, you cannot make an arbitrage profit.

Topic 8: Fixed Income Securities

5. Suppose you buy a five-year zero-coupon Treasury bond for \$800 per \$1000 face value. Answer the following questions:

(a) What is the yield to maturity (annual compounding) on the bond?

With F=face value, P=purchase price and t= years to maturity, the yield to maturity for a zero coupon bond is given by:

$$\begin{aligned} \text{YTM} &= \left(\frac{F}{P}\right)^{\frac{1}{t}} - 1 \\ &= \left(\frac{\$1000}{\$800}\right)^{\frac{1}{5}} - 1 = 0.0456 \end{aligned}$$

so

$$\text{YTM} = 4.56\%.$$

(b) Assume the yield to maturity on comparable zero coupon bonds increases to 7% immediately after purchasing the bond and remains there. Calculate your annual return (holding period yield) if you sell the bond after one year.

Annual return on a zero is the same as the holding period return and is given by:

$$\text{HPR} = \left(\frac{P_{t'}}{P}\right)^{\frac{1}{t'}} - 1$$

where $P_{t'}$ is the selling price of the bond and t' is the number of years the bond is held. This bond must also yield 7% to those you sell it to after one year. Using the formula for the price of a zero and recalling that the 5-year bond becomes a 4-year bond after one year, we have:

$$P_{t'} = \frac{\$1000}{(1+.07)^4} = \$762.90$$

Therefore, the annual return when you sell this bond after one year assuming yields have increased to 7% is:

$$\text{HPR} = \frac{\$762.90}{\$800} - 1 = -0.0464$$

Your return is less than the YTM because yields rose and you sold the bond at a lower price. Moreover, you actually lost money.

(c) Assume after the immediate increase to 7%, the yields to maturity on comparable zero coupon bonds remain at 7%, calculate your annual return if you sell the bond after two years.

The selling price after 2 years is:

$$P_{t'} = \frac{\$1000}{(1+.07)^3} = \$816.30$$

and your annual return over the two-year period is:

$$\text{HPR} = \left(\frac{\$816.30}{\$800}\right)^{\frac{1}{2}} - 1 = 0.0101$$

Note that even though rates are still 7%, your annual return over the two years is positive because the selling price of the bond is now higher than after one year (as the bond price gets pulled to par).

(d) Suppose after 3 years, the yield to maturity on similar zeros declines to 3%. Calculate the annual return if you sell the bond at that time.

Since the yield to maturity on comparable zeros is now 3% and there are two years left to maturity, your selling price is:

$$P_{t'} = \frac{\$1000}{(1+.03)^2} = \$942.60$$

and your annual return over three years is:

$$\text{HPR} = \left(\frac{\$942.60}{\$800}\right)^{\frac{1}{3}} - 1 = 0.056$$

(e) If yield remains at 3%, calculate your annual return after four years.

After four years your selling price is:

$$P_{t'} = \frac{\$1000}{1+.03} = \$970.87$$

and your annual return over four years is:

$$\text{HPR} = \left(\frac{\$970.87}{\$800}\right)^{\frac{1}{4}} - 1 = 0.0496$$

(f) After five years.

After five years the bond sells for its face value because it can be redeemed for \$1000. Therefore, no matter what yields are after five years $P_{t'} = \$1000$ and your annual return is:

$$\text{HPR} = \left(\frac{\$1000}{\$800}\right)^{\frac{1}{5}} - 1 = 0.0456$$

(g) What explains the relationship between annual returns calculated in (b) through (f) and the yield to maturity in (a)?

If you sell a bond prior to maturity, the annual return earned need not equal the yield to maturity implied by the price you paid when you purchased the bond. The calculations in (b) and (c) show that if yields rise, your annual return is lower while (d) and (e) show that if yields decline your annual return is higher. Nevertheless, because bond prices are "pulled to par," if you hold a zero coupon bond to its final maturity your annual return will equal the yield to maturity calculation on the day you purchased the bond.

6. Assume the government issues a semi-annual pay bond that matures in 5 years with a face value of \$1,000 and a coupon yield of 10 percent.

(a) What price would you be willing to pay for such a bond if the yield to maturity (semiannual compounding) on similar 5-year governments were 8%?

Since the yield to maturity on similar securities is 8%, you will pay a premium for a 10% coupon bond such that the yield to maturity for both securities are equal. Since interest payments are made semi-annually, with the face value being paid back at maturity, and since we are using the semi-annual compounding formula, the general expression for the price is:

$$P = \frac{C/2}{(1+YTM/2)} + \frac{C/2}{(1+YTM/2)^2} \cdots + \frac{C/2+F}{(1+YTM/2)^{2T}}$$

There are 10 terms in all. Substituting in the values from the problem:

$$P = \frac{\$50}{1.04} + \frac{\$50}{1.04^2} \cdots + \frac{\$1050}{(1.04)^{10}}$$

There are two ways to calculate this formula. The first, "by hand", is to use the annuity formula we learned earlier in the semester. Because the annuity pays twice a year, and because compounding is semiannual, use $YTM/2$ in the formula, and $2T$ for the number of periods. Then:

$$\begin{aligned} P &= \$50\left(\frac{1}{.04} - \frac{1}{.04(1.04)^{10}}\right) + \frac{\$1000}{1.04^{10}} \\ &= 405.54 + 675.56 = 1081.10 \end{aligned}$$

Financial calculators can do the calculation automatically. Make sure that you understand what you are calculating, that you know whether your calculator assumes annual or semi-annual coupons, and the other definitions of the calculator.

(b) What would be the price if the yield to maturity (semi-annual compounding) on similar governments were 12%?

Because the YTM is higher than the coupon rate, the bond is sold at a discount. The price can be computed exactly as above, using 12% instead of 8% as the YTM. The answer is $P = \$926.40$.

(c) If the price of the bond is 103 19/32 per \$100 of face value, what is the yield to maturity?

You have purchased the bond at $103\frac{19}{32} = 103.59$. Note that prices are quoted as a percent of par. To find the YTM, you need to use a computer or a financial calculator. The answer is 9.09%. Note that because the bond is selling at a premium, the YTM is less than the coupon yield.

(d) Suppose you held the bond in (c) for 6 months, at which time you received a coupon payment and then sold the bond for a price of 102 (per \$100 of face value). What would be the annualized holding period return?

Holding period return is defined as:

$$\text{HPR} = \left(\frac{V_t}{V_0}\right)^{\frac{1}{t}} - 1$$

We are holding the investment for six months. During this time we receive one coupon. Thus the holding period return equals:

$$\text{HPR} = \left(\frac{P_{1/2} + C}{P_0}\right)^{\frac{1}{.5}} - 1$$

where $P_{1/2}$ is the selling price, P_0 is the purchase price and C is the coupon received. Substituting in the numbers from the problem

$$\text{HPR} = \left(\frac{1020 + 50}{1035.9}\right)^{\frac{1}{.5}} - 1 = .0669$$

This is the annual HPR. The return you receive over 6 months — without annualizing — can be computed as

$$\frac{1020 + 50}{1035.9} - 1 = .0329.$$

Figure 1: MSFT

Microsoft Corporation (MSFT) - NasdaqGS

31.74 ↓0.15(0.49%) 12:21PM EDT - Nasdaq Real Time Price

Prev Close:	31.89	Day's Range:	31.69 - 32.00
Open:	31.90	52wk Range:	26.26 - 36.43
Bid:	31.70 x 5600	Volume:	10,685,249
Ask:	31.71 x 3000	Avg Vol (3m):	48,392,200
1y Target Est:	34.82	Market Cap:	264.35B
Beta:	0.9	P/E (ttm):	12.28
Next Earnings Date:	14-Oct-13	EPS (ttm):	2.58
		Div & Yield:	0.92 (2.90%)