

Homework 2 – Suggested Answers

Part I: Multiple Choice

1. You sold short 200 shares of common stock at \$50 per share. The initial margin is 60%. Your initial investment was _____.
A) \$4,000
B) \$6,000
C) \$10,000
D) \$16,667
2. The geometric average of 10%, 20% and 25% is _____.
A) 15%
B) 18.2%
C) 25%
D) 65%
3. Suppose you pay \$9,700 for a Treasury bill maturing in six months. What is the effective annual rate of return for this investment?
A) 3.1%
B) 6%
C) 6.18%
D) 6.28%
4. Historical returns have generally been _____ for stocks of small firms as/than for stocks of large firms.
A) the same
B) lower
C) higher
D) There is no evidence of a systematic relationship between returns on small firm stocks and returns on small firm stocks
5. If you are promised a nominal return of 12%, on a one year investment, and you expect the rate of inflation to be 3%, what real rate do you expect to earn?
A) 5.48%
B) 8.74%
C) 9.00%
D) 12.00%
6. Consider a treasury bill with a rate of return of 5% and the following risky

securities:

Security A: $E(r) = .15$; $\sigma^2 = .0400$

Security B: $E(r) = .10$; $\sigma^2 = .0225$

Security C: $E(r) = .12$; $\sigma^2 = .1000$

Security D: $E(r) = .13$; $\sigma^2 = .0625$

The investor must develop a complete portfolio by combining the risk-free asset with one of the securities mentioned above. The security the investor would choose as part of his complete portfolio would be _____.

- A) **security A**
- B) security B
- C) security C
- D) security D

7. An investor invests 40% of his wealth in a risky asset with an expected rate of return of 15% and a variance of 4% and 60% in a treasury bill that pays 6%. Her portfolio's expected rate of return and standard deviation are _____ and _____ respectively.
- A) 8.0%, 12%
 - B) **9.6%, 8%**
 - C) 9.6%, 10%
 - D) 11.4%, 12%

Use the following to answer questions 8-10:

You invest \$100 in a complete portfolio. The complete portfolio is composed of a risky asset with an expected rate of return of 12% and a standard deviation of 10% and a treasury bill with a rate of return of 5%.

8. _____ of your complete portfolio should be invested in the risk-free asset if you want your complete portfolio to have a standard deviation of 9%.
- A) 100%
 - B) 90%
 - C) 50%
 - D) **10%**
9. A portfolio that has an expected outcome of \$115 could be formed if you _____.
- A) invest \$100 in the risky asset
 - B) invest \$80 in the risky asset and \$20 in the risk-free asset
 - C) **borrow \$42.86 at the risk-free rate and invest \$142.86 in the risky asset**
 - D) borrow \$33.33 at the risk-free rate and invest \$133.33 in the risky asset

10. The slope of the capital allocation line formed with the risky asset and the risk-

- free asset is _____.
- A) 1.4
 - B) .7**
 - C) .5
 - D) .3
11. You have \$500,000 available to invest. The risk-free rate as well as your borrowing rate is 8%. The return on the risky portfolio is 16%. If you wish to earn a 22% return, you should _____.
- A) invest \$125,000 in the risk-free asset
 - B) invest \$375,000 in the risk-free asset
 - C) borrow \$125,000
 - D) borrow \$375,000**
12. The return on the risky portfolio is 15%. The risk-free rate as well as the investor's borrowing rate is 10%. The standard deviation of return on the risky portfolio is 20%. If the standard deviation on the complete portfolio is 25%, the expected return on the complete portfolio is _____.
- A) 6.00%
 - B) 8.47%
 - C) 16.00%
 - D) 16.25%**
13. A portfolio is composed of two stocks, A and B. Stock A has a standard deviation of return of 25% while stock B has a standard deviation of return of 5%. Stock A comprises 20% of the portfolio while stock B comprises 80% of the portfolio. If the variance of return on the portfolio is .0050, the correlation coefficient between the returns on A and B is _____.
- A) -.225
 - B) -.474
 - C) .474
 - D) .225**
14. A portfolio is composed of two stocks, A and B. Stock A has a standard deviation of return of 5% while stock B has a standard deviation of return of 15%. The correlation coefficient between the returns on A and B is .5. Stock A comprises 40% of the portfolio while stock B comprises 60% of the portfolio. The variance of return on the portfolio is _____.
- A) .0035
 - B) .0085
 - C) .0094
 - D) .0103**
15. Consider two perfectly negatively correlated risky securities, A and B. Security A

has an expected rate of return of 16% and a standard deviation of return of 20%. B has an expected rate of return 10% and a standard deviation of return of 30%. The weight of security B in the global minimum variance is _____.

- A) 10%
- B) 20%
- C) **40%**
- D) 60%

Use the following to answer questions 16-18:

An investor can design a risky portfolio based on two stocks, A and B. Stock A has an expected return of 18% and a standard deviation of return of 20%. Stock B has an expected return of 12% and a standard deviation of return of 5%. The correlation coefficient between the returns of A and B is 50%. The risk-free rate of return is 10%.

16. The proportion of the optimal risky portfolio that should be invested in stock B is _____.

- A) .2000
- B) .2667
- C) .3636
- D) **.8000**

17. The expected return on the optimal risky portfolio is _____.

- A) **13.2%**
- B) 15.8%
- C) 16.4%
- D) 16.8%

18. The standard deviation of return on the optimal risky portfolio is approximately _____.

- A) 5%
- B) **7%**
- C) 8%
- D) 9%

19. Which of the following portfolios cannot lie on the efficient frontier?

Portfolio	Expected Return	Standard Deviation
I	8%	10%
J	16%	20%
K	15%	25%
L	25%	38%

- A) Portfolio I
- B) Portfolio J
- C) **Portfolio K**
- D) Portfolio L

20. You short-sell 200 shares of Tuckerton Trading Co., now selling for \$50 per

share. What is your maximum possible loss?

- A) \$50
- B) \$150
- C) \$10,000
- D) unlimited

Part II: Calculations

Question 1:

You have the following expectations regarding the future real performance of the stock market:

State of the Economy	Probability	Return
Boom	30%	40%
Normal	40%	10%
Recession	20%	-10%
Depression	10%	-40%

What is the mean and the standard deviation of the return on the stock market?

$$E(R) = 0.3 \cdot 0.4 + 0.4 \cdot 0.1 + 0.2 \cdot (-0.1) + 0.1 \cdot (-0.4) = 10\%$$

$$\text{Var}(R) = 0.3(0.4 - 0.1)^2 + 0.4(0.1 - 0.1)^2 + 0.2(-0.1 - 0.1)^2 + 0.1(-0.4 - 0.1)^2 = 0.06$$

$$\text{StdDev}(R) = \sigma(R) = (\text{Var}(R))^{0.5} = 24.49\%$$

Question 2:

Assume that you manage a risky portfolio with an expected rate of return of 17% and a standard deviation of 27%. The T-bill rate is 7%. Suppose your client initially chooses to invest 70% of a portfolio in your fund and 30% in T-bill.

- a) What is the expected return and standard deviation of your client's portfolio?
- b) Now, suppose that your client wonders whether to switch the 70% that is invested in your fund to the passive portfolio with the expected return of 13% and a standard deviation of 25%. Explain to your client the disadvantage of the switch.
- c) Show your client the maximum fee you could charge (as percent of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one. Hint: The fee will lower the slope of your client's CAL by reducing the expected return net of the fee.

$$\text{Expected return} = .3 \cdot 7\% + .7 \cdot 17\% = 14\% \text{ per year.}$$

$$\text{Standard deviation} = .7 \cdot 27\% = 18.9\% \text{ per year.}$$

With 70% of his money in my fund's portfolio the client gets a mean return of 14% per year and a standard deviation of 18.9% per year. If he shifts that money to the passive portfolio (which has a mean of 13% and standard deviation of 25%), his overall mean and

standard deviation would become:

$$E(r_C) = r_f + .7(r_M - r_f)$$

In this case, $r_f = 7\%$ and $r_M = 13\%$. Therefore,

$$E(r_C) = 7\% + .7 * 6\% = 11.2\%$$

The standard deviation of the complete portfolio using the passive portfolio would be:

$$\sigma_C = .7 * \sigma_M = .7 * 25\% = 17.5\%$$

Therefore, the shift entails a decline in the mean from 14% to 11.2% and a decline in the standard deviation from 18.9% to 17.5%. Since both mean return *and* standard deviation fall, it is not yet clear whether the move is beneficial or harmful. The disadvantage of the shift is that if my client is willing to accept a mean on his total portfolio of 11.2%, he can achieve it with a lower standard deviation using my fund portfolio, rather than the passive portfolio. To achieve a target mean of 11.2%, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, y :

$$E(r_C) = 7 + y(17 - 7) = 7 + 10y$$

Because our target is: $E(r_C) = 11.2\%$, the proportion that must be invested in my fund is determined as follows:

$$11.2 = 7 + 10y \quad \Rightarrow \quad y = \frac{11.2 - 7}{10} = .42$$

The standard deviation of the portfolio would be:

$$\sigma_C = y * 27 = .42 * 27 = 11.34\%.$$

Thus, by using my portfolio, the same 11.2% mean can be achieved with a standard deviation of only 11.34% as opposed to the standard deviation of 17.5% using the passive portfolio.

c) The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee.

$$\text{Slope of CAL with fee} = \frac{17 - 7 - f}{27} = \frac{10 - f}{27}$$

$$\text{Slope of CML (which requires no fee)} = \frac{13 - 7}{25} = .24.$$

Setting these slopes equal we get:

$$\frac{10 - f}{27} = .24$$

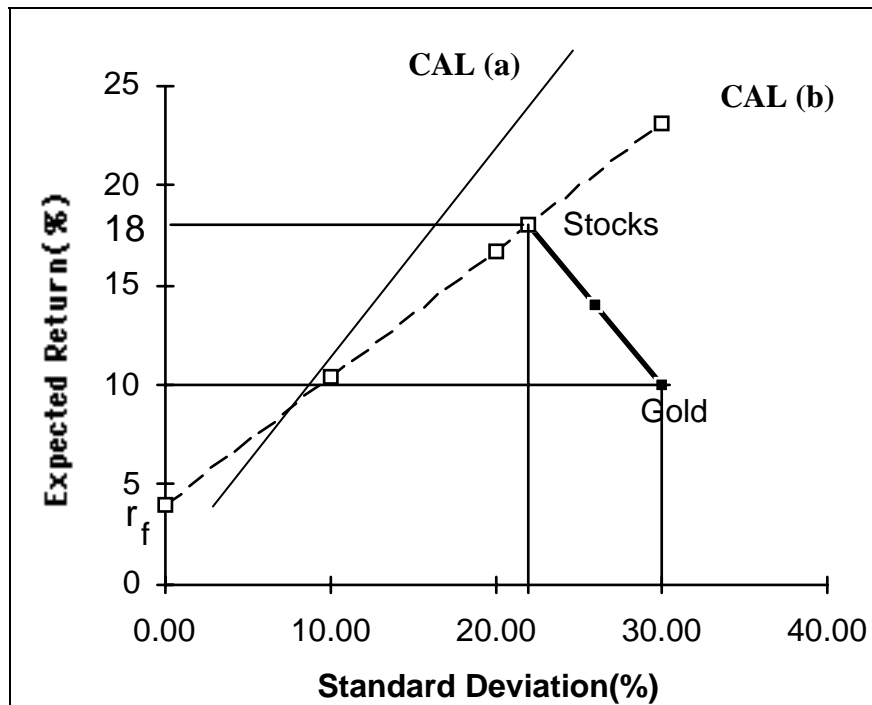
$$10 - f = 27 * .24 = 6.48$$

$$f = 10 - 6.48 = 3.52\% \text{ per year}$$

Question 3:

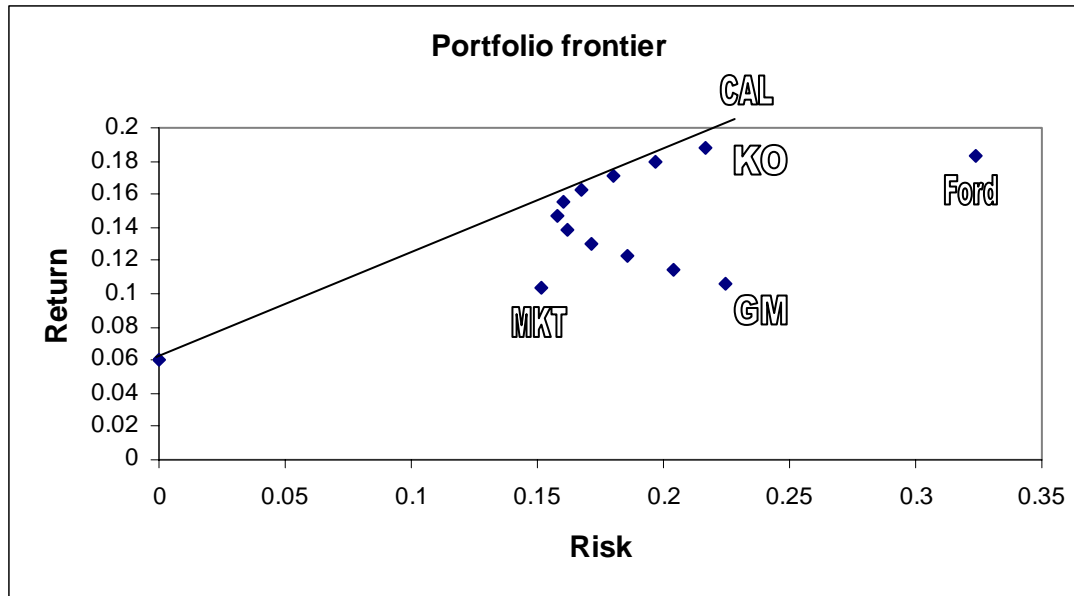
Stocks offer an expected rate of return of 18% with a standard deviation of 22% and gold offers an expected return of 10% with a standard deviation of 30%.

- a) In light of the apparent inferiority of gold to stocks with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
 - b) How would you answer a) if the correlation coefficient between gold and stocks were 1.0? Draw a graph illustrating why one would or would not hold gold. Could these expected returns, standard deviations, and correlation represent an equilibrium for the security market?
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- a) Even though gold seems dominated by stocks, it still might be an attractive asset to hold as a *part* of portfolio. If the correlation between gold and stocks is sufficiently low, it will be held as an element in a portfolio -- the optimal tangency portfolio. Look at the picture below and notice that the CAL for the diversified portfolio (CAL(a)) may be steeper than CAL for the portfolio which contains stock only (CAL(b)). I did not draw the combination line for this scenario to be tangent to CAL(a) but you can do it on your picture to convince yourselves.
 - b) If gold had a correlation coefficient with stocks of +1, it would not be held. The set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph). The graph shows that in this case, any portfolio that contains any gold is dominated by the stock-only portfolio. Therefore, no one will hold gold. Stated differently, the CAL for the portfolio with stock only is steeper than any other CAL passing through all other possible portfolios on the combination line. (Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became an attractive enough asset to hold.)



Question 4: (Computer exercise)

- Download the spreadsheet data.xls from the course site. The file contains annual holding period returns on Ford, Coca-Cola, GM, and the market (measured by S&P 500) for the period 1981-2004.
- Compute the returns of 9 portfolios consisting of combinations of GM and Coca-Cola with weights (.9, .1), (.8, .2), (.7, .3), . . . , (.1, .9). Place the returns series of these portfolios in the columns next to the calculated returns (they should take up from f2 to n24). Next calculate the standard deviations of each set of returns, including the market returns and returns on the individual stocks, on one line below the returns numbers (should fill b26 to n26). Put the means of each column of returns on the line below the standard deviations (b27 to n27).
- Plot the numbers that you generated in part b. Include a risk-free asset in your plot by inserting a standard deviation of 0 and a mean of 6% on the end of the rows containing standard deviations and means (column O). Now select the entire range of means and standard deviations (B26 to O27) and plot the data with a scatter (XY) plot. Be sure to use an (XY) plot rather than any other type of plot. Your plot should look like some of the things we have talked about in class.
- Print out your graph (I only want the graph - Please don't turn in lots of numbers). Draw in the CAL and identify the market portfolio, Ford's return, the risk-free asset, GM and Coca-Cola. Did this plot turn out how you expected it to?



I hope it looks quite nice, at least to some of you. Indeed, we can see that the portfolios with Coca Cola and GM fall on the nonlinear curve (portfolio frontier) we discussed in class. Thus, we can infer that the correlation between those two is not perfect (neither -1 or 1). It can be anything else between -1 and 1, though. Besides, the risk-free asset lies below the minimum variance portfolio on this portfolio frontier, which is also something we would expect. Finally, both market and Ford are in the feasible places to be real. If we look at market we can see that the efficient frontier could potentially include the market portfolio (note that market has the lowest risk among all assets), but would be unlikely selected by any investor using this frontier. For example, CAL drawn through say Coca Cola is steeper than CAL drawn through market portfolio.