Commerce 374
Winter 2007

Homework 7

Problem 1: Multiple Choice

1) The price of a stock put option is ______ correlated with the stock price and ______ correlated with the exercise price.
    A) negatively, negatively
    B) negatively, positively
    C) positively, negatively
    D) positively, positively
2) The value of an American call option always______ with its maturity and the value of an American put option always______ with its maturity.
    A) decreases, decreases
    B) decreases, increases
    C) increases, decreases
    D) increases, increases
    E) none of the above
3) The value of a European call option always______ with its maturity and the value of a European put option always______ with its maturity.
    A) decreases, decreases
    B) decreases, increases
    C) increases, decreases
    D) increases, increases
    E) none of the above
4) The value of a call option always______ with its volatility and the value of a put option always______ with its volatility.
    A) decreases, decreases
    B) decreases, increases
    C) increases, decreases
    D) increases, increases
    E) none of the above
5) The delta of a call option with a strike price of $35, where the underlying stock trades at $27 is ______.
    A) Smaller than -1
    B) Between –1 and 0
    C) Between 0 and 1
    D) Larger than 1
6) The current stock price of MSFT is $25.65. The continuously compounded risk-free rate of return is 1%. The instantaneous standard deviation of MSFT stock is 40%. You wish to purchase a call option on this stock with an exercise price of $25 and an expiration date in half a year. Using the B-S pricing model, the call option should be worth __________ today.
    A) $2.50
    B) $2.94

   - **C) $3.25**
   - D) $3.50

7) __________ is the most risky transaction to undertake in the stock index option markets if the stock market is expected to fall substantially after the transaction is completed.
   - A) Writing an uncovered call option
   - **B) Writing an uncovered put option**
   - C) Buying a call option
   - D) Buying a put option
8) Which of the following strategies would be considered the least risky from the perspective of the brokerage house granting you access to trading options?
   - A) Purchasing options
   - B) Option writing
   - **C) Covered options**
   - D) Creating spreads
9) The profit function obtained for any trading strategy ________.
   - A) depending on the strategy, may be entirely below the S-axis
   - B) depending on the strategy, may be entirely above the S-axis
   - **C) will <u>always</u> cross the S-axis**
   - D) will <u>always</u> lie below the payoff function
10) Relative to the Black-Scholes model, the binomial model _______.
    - **A) is very flexible**
    - B) is easy to use without a computer or calculator when trading
    - C) makes more assumptions
    - D) all of the above
11) The divergence between an option's intrinsic value and its market value is usually greatest when __________.
    - A) the options is deep in the money
    - **B) the option is approximately at the money**
    - C) the option is far out of the money
    - D) time to expiration is very low
12) You are considering purchasing a put option on a stock of A&R Co. (AR) with a current price of $39. The exercise price is $35, and the price of the corresponding call option is $7. If the risk-free rate of interest is 4%, and there are 60 days until expiration, the value of the put should be equal to ___________.
    - **A) $2.78**
    - B) $3.13
    - C) $3.85
    - D) $4.70

Problem 2.

Derive the mathematical formula for the Black-Scholes value of a straddle position. Next, apply this formula using the following numbers:

Time to maturity = 6 months
Standard deviation = 50% per year
Strike price = $50
Moneyness of the respective call = 1.1

Interest rate = 10% per year

Notice first that the straddle position involves taking long position in one call and one put contract. Therefore, the price (value) of this strategy is obtained by adding together these two prices. Using Black-Scholes formula for call and put we obtain:

Value(Straddle) = C + P = $SN(d_1) - Xe^{-rT}N(d_2) + Xe^{-rT}(1-N(d_2))-S(1-N(d_1)) = 2SN(d_1) -$

- $2Xe^{-rT}N(d_2) + Xe^{-rT} - S =$ $\mathbf{(2N(d_1)-1)*S - (2N(d_2)-1)*Xe^{-rT}}$

The equation in bold is a ready-to-use formula for the straddle strategy under B-S model. Now, we can plug in the numbers into it and solve for the value of straddle.

Value(Straddle) = $(2*N(d_1)-1)*55 - (2*N(d_2)-1)*50e^{-0.1*0.5} =$

$d_1 = 0.588 \Rightarrow N(0.588) = 0.722$
$d_2 = 0.234 \Rightarrow N(0.234) = 0.592$
$= (2*0.722-1)*55 - (2*0.592-1)*47.561 =$ **14.282**

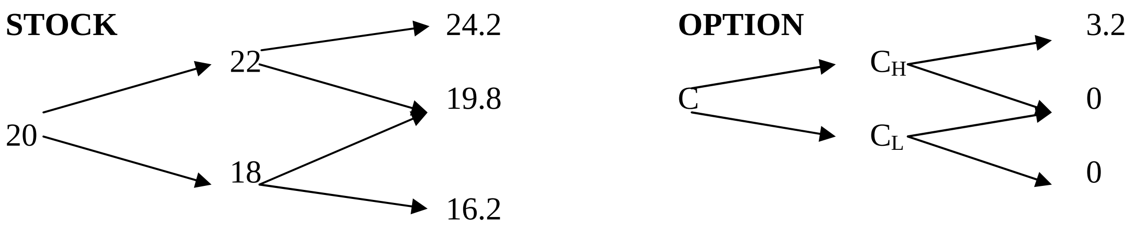
So, the straddle strategy costs approximately $14.282.

Problem 3.
If two stocks have equal betas but one stock has much higher firm-specific risk than the other firm, which firm's put option will sell for a higher price? Give an intuitive argument to support your answer.

If both stocks have the same betas this means that the two firms have equal systematic risks. Thus, the firm with higher idiosyncratic risk will have a higher total risk, and thus greater volatility. Since option prices are positively correlated with volatility, the firm with higher firm-specific risk should have a put option with a higher premium.

Problem 4.
Vinnie is considering the purchase of the European call option written on the stock with the initial price of $20. Suppose that the price of this stock can equally likely go up or down by 10% in each quarter. How much will he have to pay for the option with maturity in 6 months, strike price $21, given that the effective annual interest rate on the risk-free asset is equal to 12.55%? Please provide the value of the hedge ratio for each node of the tree. Also, calculate the price of a put option with the same strike price.

It is always the best strategy to start with two trees: one for the stock prices and one for the respective option payoffs.



As we have done it in class we have to work backward to solve for the price of the option C. This can be done by finding the delta hedge for each node of the tree. Since we work backward the first nodes to be considered are the nodes C_H and C_L. Note that the latter one should have value 0 as the future scenario predicts the value of option to be equal to 0 for sure (delta for this node will be equal to 0).

So, the only problem is to find the value of C_H. Now, we can find the value of delta for this node. It is equal to (3.2-0)/(24.2-19.8) = 3.2/4.4 = 0.73. It means that in order to find a hedging strategy we need to long 0.73 stock and short 1 call. This will provide us with the riskless payoff.

We calculate the riskless payoff having realized that for the down branch of the tree the value of option is 0. This means that the terminal payoff is equal to 0.73*19.8 = $14.45. In order to find the present value of this payoff we need to find 3-month riskless interest rate. This is equal to $(1+x)^4$ = 1+ 12.55% => x = 3%. Hence, the PV(14.45) = 14.45/1.03 = 14.03.

Now, we calculate the value of $C_{H.}$ This is equal to 0.73*22-C_H = 14.03 => C_H = $2.03.

Now, we have to do exactly the same exercise as for C_H using the new payoffs of option. Again:

1) We find the delta hedge: (2.03-0)/ (22-18) = 0.51
2) We need to long 0.51 of stock and short one call
3) The riskless payoff is equal to 0.51*18 = 9.18.
4) PV(9.18) = 9.18/1.03 = $8.91
5) Finally, the price of the call is given in equation 0.51*20 - C = 8.91 => C = 10.2-8.91 = $1.29.

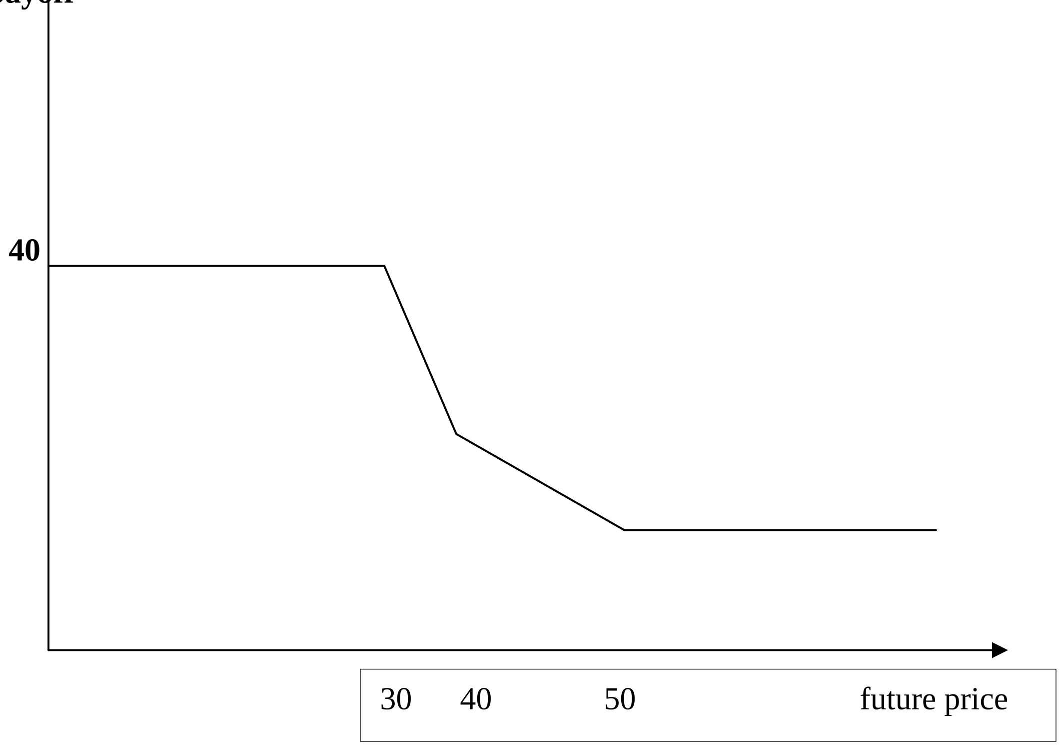
So, the price of the call is equal to **$1.29**

Problem 5.
You have a portfolio that consists of the following:
1) One share of ABC stock.
2) Long position in a European put option on shares of ABC with strike price of $40 and one year to maturity.
3) Short position in two European call options on shares of ABC with strike price of $30 and one year to maturity.

4) Long position in a European call option on shares of ABC with strike price of $50 and one year to maturity.

a) Draw a diagram of the payoff to this portfolio at the end of the year.
b) Looking at your picture, please provide a short characteristic of an investor who would like such a strategy. Can you name any other strategy with similar characteristics?
c) Using the payoff line from point a) please sketch a profit line of this strategy.

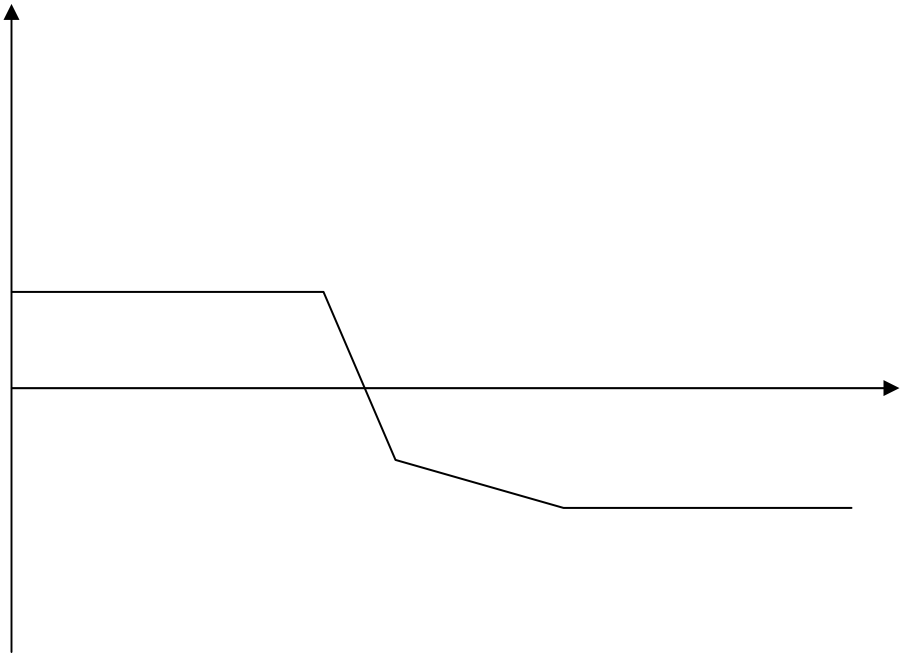
a) payoff



B and C) Notice that this strategy seems to favor investors with bearish view of the market. If we draw the graph of profits it would very much resemble the bearish spread. The profit line would look exactly the same in shape, but would be shifted down. The important thing is that the line has to cross the x-axis. It is because the strategy cannot always offer positive profits, nor can it offer only negative profits. Since we do not have the full prices of all individual components we cannot draw the precise location of the graph. Important, however, are the elements I mentioned above.

c) Profit plot

Profit



Problem 6.

Companies M&S Co. (MS) and C&P Ltd. (CP) have been offered the following rates per annum on a $20 million 5-year loan:

	Fixed Rate	*Floating Rate*
MS	*12.0%*	*LIBOR + 0.1%*
CP	*13.4%*	*LIBOR + 0.6%*

MS requires a floating rate loan; CP requires a fixed rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and appear equally attractive to both companies.

Every time we see the problem with interest rate swaps we should start with the comparative advantage exercise. Notice that if we take the difference in interests between MS and CP for both types of rates we will get 1.4% and 0.5% for fixed and floating rate, respectively. This means that MS has a comparative advantage in fixed rate market while CP in the floating market.

As a simple check point for our swap design we should notice the difference between 1.4% and 0.5% = 0.9% constitutes the total gain from swaps for all three players in the contract. Therefore, for the intermediary to make 0.1% and the other two companies to be equally happy MS and CP should make a gain of (0.9%-0.1%)/2 = 0.4% each. This is an ultimate solution we are heading for.

Following the logic we discussed in class, MS will issue bonds with the fixed rate schedule while CP with the floating rate schedule. This is where their comparative advantage drives their money.

In such a situation, MS since it borrows at a fixed rate should pay a floating rate to the intermediary and get in response the fixed rate payment, while CP should pay fixed rate to the intermediary and get the floating rate in return. Now, the rest is only to make the numbers work. Please, convince yourselves that the solution to this problem will be as drawn below:

In this case MS will pay 12% + LIBOR - 12.3% = LIBOR-0.3% floating rate which is less than its initial cost of LIBOR +0.1% by 0.4%; CP will pay LIBOR +0.6% + 12.4% - LIBOR = 13% which is less than its initial fixed rate payment of 13.4% by 0.4%; and Intermediary will get LIBOR +12.4% -12.3% - LIBOR = 0.1%. These three gains will add up to 0.4% + 0.1% + 0.4% = 0.9% which is exactly the number we were supposed to get. Moreover, both CP and MS get exactly the same gain of 0.4% and Intermediary locks in a 0.1% profit from the operation.