FFM BPES
Winter Semester 2015
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Homework 2-Suggested Answers

Question 1.

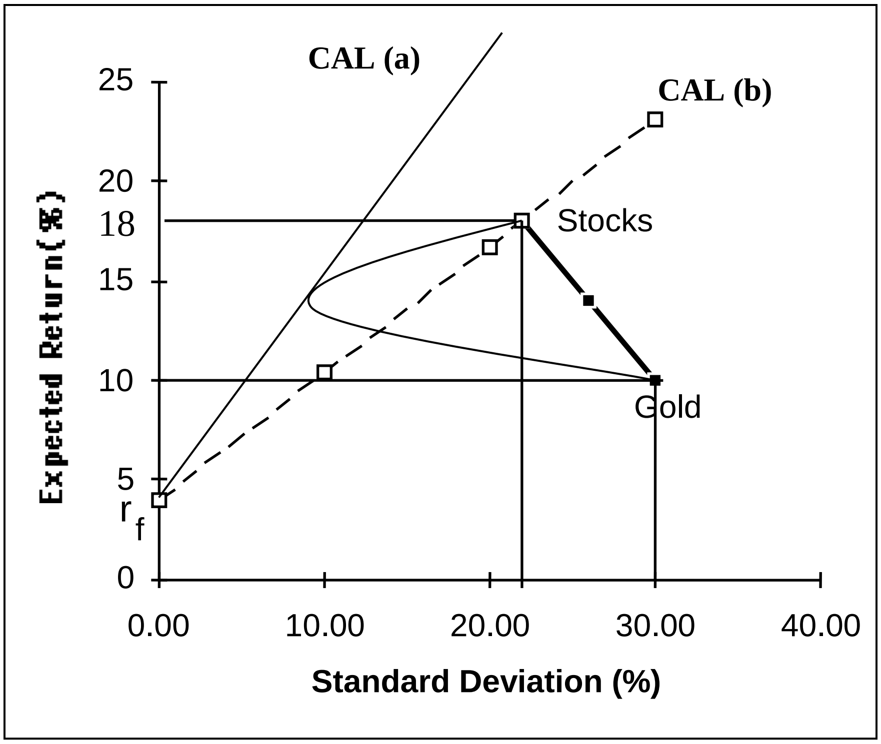
Stocks offer an expected rate of return of 18% with volatility of 22% and gold offers an expected return of 10% with volatility of 30%. Assume there is also a risk-free asset with 5% rate of return.

a. In light of the apparent inferiority of gold to stocks with respect to mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
b. How would you answer a) if the correlation coefficient between gold and stocks were +1? Draw a graph illustrating why one would or would not hold gold.
c. Could these expected returns, standard deviations, and correlation represent equilibrium (steady state) for the security market? Why or why not?

a) Even though gold seems dominated by stocks, it still might be an attractive asset to hold as a *part* of portfolio. If the correlation between gold and stocks is sufficiently low, it will be held as an element in a portfolio -- the optimal tangency portfolio. Look at the picture below and notice that the CAL for the diversified portfolio (CAL(a)) may be steeper than CAL for the portfolio which contains stock only (CAL(b)).

b) If gold had a correlation coefficient with stocks of +1, it would not be held. The set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph). The graph shows that in this case, any portfolio that contains any gold is dominated by the stock-only portfolio. Therefore, no one will hold gold. Stated differently, the CAL for the portfolio with stock only is steeper than any other CAL passing through all other possible portfolios on the portfolio frontier. (Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became an attractive enough asset to hold.)

c) The graph shows that in this case, any portfolio that contains any gold is dominated by the stock-only portfolio. Therefore, no one will hold gold. Stated differently, the CAL for the portfolio with stock only is steeper than any other CAL passing through all other possible portfolios on the portfolio frontier. (Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became an attractive enough asset to hold.)



Question 2.

Assume that you manage a risky portfolio with an expected rate of return of 17% and a standard deviation of 27%. The T-bill rate is 7%. Suppose your client initially chooses to invest 70% of a portfolio in your fund and 30% in T-bill.

a. What is the expected return and standard deviation of your client's portfolio?

b. Now, suppose that your client wonders whether to switch the 70% that is invested in your fund to the passive portfolio with the expected return of 13% and a standard deviation of 25%. Explain to your client the disadvantage of the switch.

c. Show your client the maximum fee you could charge (as percent of the investment in your fund deducted at the end of the year) that would still leave him at least as well off investing in your fund as in the passive one.

a) Expected return = .3*7% + .7 *17% = 14% per year.
Standard deviation = .7 * 27% = 18.9% per year.

b) With 70% of his money in my fund's portfolio the client gets a mean return of 14% per year and a standard deviation of 18.9% per year. If he shifts that money to the passive portfolio (which has a mean of 13% and standard deviation of 25%), his overall mean and standard deviation would become:

$$E(r_C) = r_f + .7(r_M - r_f)$$

In this case, r_f = 7% and r_M = 13%. Therefore,

$$E(r_c) = 7\% + .7 * 6\% = 11.2\%$$

The volatility of the complete portfolio using the passive portfolio would be:

$$\sigma_C = .7 * \sigma_M = .7 * 25\% = 17.5\%$$

Therefore, the shift entails a decline in the mean from 14% to 11.2% and a decline in volatility from 18.9% to 17.5%. Since both mean return *and* volatility fall, it is not yet clear whether the move is beneficial or harmful. The disadvantage of the shift is that if my client is willing to accept a mean on his total portfolio of 11.2%, he can achieve it with a lower standard deviation using my fund portfolio, rather than the passive portfolio. To achieve a target mean of 11.2%, we first write the mean of the complete portfolio as a function of the proportions invested in my fund portfolio, y:

$$E\,(r_C) = 7 + y\,(17 - 7) = 7 + 10y$$

Because our target is: $E(r_C) = 11.2\%$, the proportion that must be invested in my fund is determined as follows:

$$11.2 = 7 + 10y \qquad \Rightarrow \qquad y = \frac{11.2 - 7}{10} = .42$$

The volatility of the portfolio would be:

$$\sigma_C = y * 27 = .42 * 27 = 11.34\%.$$

Thus, by using my portfolio, the same 11.2% mean can be achieved with a standard deviation of only 11.34% as opposed to the standard deviation of 17.5% using the passive portfolio.

Note: A shorter answer to this question would be to calculate Sharpe ratios for the two strategies and compare their values. The shift would only be beneficial if the relocation increased investors' Sharpe ratio.

Sharpe Ratio (a) = (14%-7%)/18.9% = 0.37
Sharpe Ratio (b) = (13%-7%)/25% = 0.24

As is evident, the Sharpe ratio actually goes down for the alternative strategy; hence, the shift would not be advisable.

c) The fee would reduce the Sharpe ratio, i.e., the slope of the CAL. Clients will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee.

$$\text{Sharpe ratio with fee} = \frac{17 - 7 - f}{27} = \frac{10 - f}{27}$$

$$\text{Sharpe ratio (which requires no fee)} = \frac{13 - 7}{25} = .24.$$

Setting these two equal to each other we get:

$$\frac{10 - f}{27} = .24$$

$$10 - f = 27 * .24 = 6.48$$
$$f = 10 - 6.48 = 3.52\% \text{ per year}$$

Question 3.

If the CAPM is valid, which of the following situations is possible? Explain. Consider each situation separately.

a.

Portfolio	***Expected Return***	***Beta***
A	*20%*	*1.4*
B	*25%*	*1.2*

Not possible. Portfolio A has a higher beta but a lower expected return.

b.

Portfolio	***Expected Return***	***Standard Deviation***
A	*30%*	*35%*
B	*40%*	*25%*

Possible. The expected rate of return compensates only for systematic risk as measured by beta rather than the standard deviation which includes firm-specific risk.

c.

Portfolio	***Expected Return***	***Standard Deviation***
T-Bills	*10%*	*0%*
Market	*18%*	*24%*
A	*16%*	*12%*

Not possible. The reward-to-variability ratio for portfolio A is better than that of the market. Portfolio A has a reward-to-variability ratio of 0.5 (=(0.16-0.1)/0.12) and the market has a ratio of 0.33 (=(0.18-0.10)/0.24). This is impossible, because the market is the most efficient portfolio.

d.

Portfolio	***Expected Return***	***Standard Deviation***
T-Bills	*10%*	*0%*
Market	*18%*	*24%*
A	*20%*	*22%*

Not possible. Portfolio A clearly dominates the market portfolio, it has a lower standard deviation and a higher expected return.

Question 4.

A hedge fund expects that proposed elimination of the personal taxation on dividends will be enacted and that this tax reform will benefit particularly stocks paying high dividends.

The hedge fund ranks the 500 stocks in the S&P 500 index according to their dividend yield. The 250 stocks with the highest dividend yields are included in the high-dividend portfolio and the 250 stocks with the lowest dividend yields are included in the low-dividend portfolio. The high-dividend portfolio has a beta of 0.7 and the low-dividend portfolio has a beta of 1.3. The risk-free interest rate is 2% and the expected return on the S&P 500 index is 7%. The hedge fund has currently $10 million available to invest in this strategy.

a. Suppose that the hedge fund decides to go long $50 million in the high-dividend portfolio and to go short $40 million in the low-dividend portfolio. What is the beta and the expected return of the hedge fund according to the CAPM?
b. Hedge funds often want to eliminate all systematic risk by pursuing "market-neutral" strategies. Which positions in the two portfolios can the hedge fund enter into with a net investment of $10 million without incurring any systematic risk? Please give the dollar values of the long and short positions in the high- and the low-dividend portfolios. (Hint: A market-neutral portfolio has a beta of zero.)
c. Under what conditions can the hedge fund expect to make abnormal profits from the transaction in (b)?

a) First, we should determine weights applied to both the long and short strategies. The total portfolio value is 10M so the weights are 50/10 = 5 and -40/10= - 4, respectively.

Given those weights, beta of the portfolio will be the weighted average of the individual betas.

Beta(p) = 5*0.7 – 4*1.3 = -1.7

The expected return will be (using CAPM): E(R) = 0.02 – 1.7(0.07-0.02) = -6.5%.

Note that alternatively this value could be obtained by taking the weighted average of the individual returns.

Return on the long strategy (using CAPM) is: R(long)=0.02+0.7*(0.07-0.02) = 5.5%
Return on the short strategy (using CAPM) is: R(short)=0.02+1.3*(0.07-0.02) = 8.5%

Hence, the expected return is: E(R)=5*5.5%-4*8.5% = -6.5%

b) To make the portfolio beta-neutral we need to construct the portfolio whose beta is equal to 0. Using the equation for beta of a portfolio and assuming that the new weight in the long position is denoted by x, we obtain:

x*0.7 + (1-x)*1.3=0

Hence, x=2.17

Therefore, the hedge fund should invest 21.7 M in the high-dividend (HD) portfolio and short 11.7 M of the low-dividend portfolio (LD).

c) Using CAPM the expected return on this portfolio is equal to 2%, which is exactly the risk free rate. Hence, clearly this strategy would not produce any abnormal return (over the risk-free rate). For the portfolio to produce abnormal profits, we need to relax the assumption about CAPM. Clearly, if the strategy generated in (b) was underpriced we should expect to get the expected return higher than the risk free rate of 2%. Hence, alpha of the combined portfolio in (b) has to be greater than 0. For that to happen, the value weighted alpha of the portfolio has to be greater than zero. Hence,
2.17*alpha(HD) - 1.17*alpha(LD)>0,

which is equivalent to:

alpha(HD)>0.54*alpha(LD)

Question 5.

You are currently 100% invested in the S&P500. You are evaluating two trading strategies. You have historical data on the returns to these strategies and the S&P500. After analyzing the data you come up with the following estimated quantities ("1" refers to the first strategy, "2" refers to the second strategy, r_f is the risk-free rate and S&P refers to the S&P500):

$E(r_1) = 15\%$ $\sigma_1 = 0.50$ $\rho_{1,\,S\&P} = 0.35$

$E(r_2) = 6\%$ $\sigma_2 = 0.30$ $\rho_{2,\,S\&P} = 0.25$ $\rho_{1,\,2} = 0.4$

$E(r_{S\&P}) = 12\%$ $\sigma_{S\&P} = 0.15$

$r_f = 5\%$

Answer the following:

a. Let's examine how the CAPM performs when using the S&P500 as a proxy for the market portfolio. What are the alphas and betas of the two strategies?

$$\beta_1 = \frac{\mathrm{cov}(r_1, r_{S\&P500})}{\sigma^2_{S\&P}} = \frac{0.35\times 0.5\times 0.15}{0.15^2} = 1.1667$$

$$\beta_2 = \frac{0.25\times 0.3\times 0.15}{0.15^2} = 0.5$$

$$\alpha_1 = E(r_1) - r_f - \beta_1(E(r_{S\&P}) - r_f) = 0.15 - 0.05 - 1.1667\times(0.12 - 0.05) = 1.83\%$$

$$\alpha_2 = E(r_2) - r_f - \beta_2(E(r_{S\&P}) - r_f) = 0.06 - 0.05 - 0.5\times(0.12 - 0.05) = -2.5\%$$

b. Recall that you are currently holding the S&P500. Describe ***in words*** *how you would adjust your portfolio in response to the alphas calculated in part (a)?*

Since the alpha of strategy 1 is positive I would add it to my portfolio. Since the alpha of strategy 2 is negative I would short it.

c. Find a combination of the two strategies that would make you react to market risk the way S&P 500 does. What is the expected return to this portfolio? What is the alpha of this portfolio? What is the idiosyncratic risk of the portfolio?

I would like two weights w1 and w2 such that the portfolio has beta equal to one. This is exactly what it means that portfolio reacts to market risk the way S&P 500 does.

This means:

$$
\begin{aligned}
&w_1\beta_1 + w_2\beta_2 = w_1\beta_1 + (1-w_1)\beta_2 = 1 \\
&or \\
&w_1 1.1667 + (1-w_1)0.5 = 1 \\
&w_1 = 0.75 \\
&w_2 = 0.25
\end{aligned}
$$

Expected return on the portfolio equals: $E(r_p) = 0.75*15\% + 0.25*6\% = 12.75\%$

Alpha of the portfolio equals: $\alpha_P = 12.75\% - (5\% + 1*(12\% - 5\%)) = 0.75\%$

The last thing to calculate is idiosyncratic risk. We will use the formula that:

Idiosyncratic risk = $\sigma^2 - \beta^2\sigma_M^2$

We first need to calculate the variance of the portfolio returns.

$$\sigma^2 = 0.75^2 0.5^2 + 0.25^2 0.3^2 + 2*0.75*0.25*0.5*0.3*0.4 = 0.16875$$

Then idiosyncratic risk = 0.16875-1*0.0225=0.14625

You could also express this number in terms of standard deviation, which would mean taking square root of 0.14625. This is equal to 38.2%.