

Assignment - 1

21BD50340

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Question 1

x_1	x_2
8	21
24	9
17	6
11	18

$\bar{x}_1 = 15, \bar{x}_2 = 13.5$

$$\begin{aligned} \text{cov}(x_1, x_2) &= \frac{1}{3} ((8-15)(21-13.5) + (24-15)(9-13.5) + \\ &\quad (17-15)(6-13.5) + (11-15)(18-13.5)) \\ &= \frac{1}{3} (-126) = \underline{-42} \end{aligned}$$

$$\begin{aligned} \text{cov}(x_1, x_1) &= \frac{1}{3} ((8-15)^2 + (24-15)^2 + (17-15)^2 + (11-15)^2) \\ &= \frac{1}{3} (150) = \underline{50} \end{aligned}$$

$$\begin{aligned} \text{cov}(x_2, x_2) &= \frac{1}{3} ((21-13.5)^2 + (9-13.5)^2 + (6-13.5)^2 + (18-13.5)^2) \\ &= \frac{1}{3} (153) = \underline{51} \end{aligned}$$

$$S = \begin{bmatrix} 50 & -42 \\ -42 & 51 \end{bmatrix}$$

$$|S - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} 50-\lambda & -42 \\ -42 & 51-\lambda \end{bmatrix} = 0$$

~~$\Rightarrow \begin{bmatrix} 50 & -42 \\ -42 & 51 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$~~

$$\Rightarrow \lambda^2 - 101\lambda + 786 = 0$$

$$\Rightarrow \lambda = \frac{101 \pm \sqrt{101^2 - 786 \times 4}}{2}$$

$$\Rightarrow \lambda = \frac{101 \pm 84}{2}$$

$$\Rightarrow \lambda = \underline{92.5} \text{ or } \underline{8.5}$$

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$(S - \lambda I)v = 0$$

$$\Rightarrow \begin{bmatrix} 50 - \lambda & -42 \\ -42 & 51 - \lambda \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\Rightarrow (50 - \lambda)v_1 - 42v_2 = 0$$

$$-42v_1 + (51 - \lambda)v_2 = 0$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{42}{50 - \lambda}, \quad \frac{v_1}{v_2} = \frac{51 - \lambda}{42}$$

$$\text{Let } v_1 = \begin{bmatrix} 42 \\ 50 - \lambda \end{bmatrix}, \quad v_2 = \begin{bmatrix} 51 - \lambda \\ 42 \end{bmatrix}$$

$$\Rightarrow v_1 = \begin{bmatrix} 42 \\ -42.5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -41.5 \\ 42 \end{bmatrix}$$

$$\Rightarrow e_1 = \begin{bmatrix} 0.703 \\ -0.711 \end{bmatrix}, \quad e_2 = \begin{bmatrix} -0.703 \\ 0.711 \end{bmatrix} \quad (\text{Normalised})$$

1st Principle

$$e^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.703 & -0.711 \end{bmatrix} \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix}$$

For (8, 21)

$$= [0.703 \quad -0.711] \begin{pmatrix} 8-15 \\ 21-13.5 \end{pmatrix} = \underline{-10.25}$$

$\neq (4/9, 1/1, 1/5, 1/3)$

For (24, 9)

$$= [0.703 \quad -0.711] \begin{pmatrix} 24-15 \\ 9-13.5 \end{pmatrix} = \underline{9.53}$$

For (17, 6)

$$= [0.703 \quad -0.711] \begin{pmatrix} 17-15 \\ 6-13.5 \end{pmatrix} = \underline{6.74}$$

For (11, 18)

$$= [0.703 \quad -0.711] \begin{pmatrix} 11-15 \\ 18-13.5 \end{pmatrix} = \underline{-6.01}$$

PLA conclusion

<u>x1</u>	<u>x2</u>	<u>1st Principle</u>
8	21	-10.25
24	9	9.53
17	6	6.74
11	18	-6.01

Question 2

$$\text{Input : } x_1 = (0, 1, 1, 0)$$

$$x_2 = (1, 0, 1, 1)$$

$$x_3 = (0, 0, 0, 1)$$

$$x_4 = (1, 1, 1, 0)$$

$$\text{weights} = \begin{bmatrix} 0.1 & 0.9 & 0.7 & 0.7 \\ 0.0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

1st Iteration

$$\text{For } x_1 : (0, 1, 1, 0)$$

$$d_1^2 = 0.6, d_2^2 = 1.46$$

Unit 1 wins

weight change:

$$\begin{aligned} w_{\text{new}} &= (0.1, 0.9, 0.7, 0.7) - 1((0, 1, 1, 0) - (0.1, 0.9, 0.7, 0.7)) \\ &= (0.1, 0.9, 0.7, 0.7) - (-0.1, 0.1, 0.3, -0.7) \\ &= (0.2, 0.8, 0.4, 1.4) \end{aligned}$$

$$\text{weights} = \begin{bmatrix} 0.2 & 0.8 & 0.4 & 1.4 \\ 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

$$\text{For } x_2 : (1, 0, 1, 1)$$

$$d_1^2 = 1.8, d_2^2 = 2.46$$

Unit 1 wins

weight change:

$$\begin{aligned} w_{\text{new}} &= (0.2, 0.8, 0.4, 1.4) - 1((1, 0, 1, 1) - (0.2, 0.8, 0.4, 1.4)) \\ &= (0.2, 0.8, 0.4, 1.4) - (0.8, -0.8, 0.6, -0.4) \\ &= (-0.6, 1.6, -0.2, 1.8) \end{aligned}$$

$$\text{weight} = \begin{bmatrix} -0.6 & 1.6 & -0.2 & 1.8 \\ 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

For $x_3 = (0, 0, 0, 1)$

$$d_1^2 = 3.6, d_2^2 = 0.86$$

Unit 2 wins

weight change:

$$w_{\text{new}} = (0, 0.1, 0.2, 0.1) - 1((0, 0, 0, 1) - (0, 0.1, 0.2, 0.1))$$

$$= (0, 0.1, 0.2, 0.1) - (0, -0.1, -0.2, 0.9)$$

$$= (0, 0.2, 0.4, -0.8)$$

$$\text{weight} = \begin{bmatrix} -0.6 & 1.6 & -0.2 & 1.8 \\ 0 & 0.2 & 0.4 & -0.8 \end{bmatrix}$$

For $x_4 = (1, 1, 1, 0)$

$$d_1^2 = 7.6, d_2^2 = 2.64$$

Unit 2 wins

weight change:

$$w_{\text{new}} = (0, 0.2, 0.4, -0.8) - 1((1, 1, 1, 0) - (0, 0.2, 0.4, -0.8))$$

$$= (0, 0.2, 0.4, -0.8) - (1, 0.8, 0.6, 0.8)$$

$$= (-1, -0.6, -0.2, -1.6)$$

After 1 iteration

$$x_1 \rightarrow \text{unit 1}, x_2 \rightarrow \text{unit 1}$$

$$x_3 \rightarrow \text{unit 2}, x_4 \rightarrow \text{unit 2}$$