21BDS0340

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Question 1

$$y = 0.6 \sin(0.5x - 2.5t + \pi/2)$$

$\omega = 2.5$ rad/sec

$k = 0.5 \ m^{-1}$

$$f = \frac{\omega}{2\pi}$$

$\Rightarrow f = 0.398$ Hz

$$\lambda = \frac{2\pi}{k}$$

$\Rightarrow \lambda = 12.57$ m

$v = f\lambda$

$\Rightarrow v = 5$ m/s

$T = 1/f$

$\Rightarrow T = 2.51$ seconds

$\phi = \pi/2$

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$\mu = 0.2$ kg/m

$$T = \frac{\mu \omega^2}{k^2}$$

$T = 5$ N

Question 2

The role of impedance is to oppose the wave motion. This is expressed by the medium of propogation of the wave. The expression for transmission coefficient is:

$$\frac{2 z_1}{z_1 + z_2}$$

This will not affect an equation when $z_1 = z_2$. Therefore when the mediums have the same impedance, transmission coefficient does not affect.

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Question 3

$y = a e^{i(\omega t - kx)}$

wave equation: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

$\frac{\partial y}{\partial x} = -ika e^{i(\omega t - kx)}$

$\Rightarrow \frac{\partial^2 y}{\partial x^2} = -k^2 a e^{i(\omega t - kx)}$

$\frac{\partial y}{\partial t} = i\omega a e^{i(\omega t - kx)}$

$\Rightarrow \frac{\partial^2 y}{\partial t^2} = -\omega^2 a e^{i(\omega t - kx)}$

$\therefore \frac{\partial^2 y}{\partial t^2} \Big/ \frac{\partial^2 y}{\partial x^2} = \frac{\omega^2}{k^2}$

$\Rightarrow \frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \qquad \left(c = \frac{\omega}{k} \right)$

$\therefore$ This is a solution to the wave equation. The velocity is ω/k

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Question 4

Amp = 3 cm

λ = 25 cm

μ = 1g/cm

Joined to μ = 4 g/cm

$z = \sqrt{T} \cdot \sqrt{\mu} = Tk/\omega$

a. $A_2 = A_1 \cdot \frac{2z_1}{z_1 + z_2}$

$\Rightarrow A_2 = 3 \times \frac{2 \cdot \sqrt{T}\sqrt{1}}{(\sqrt{1} + \sqrt{4})\sqrt{T}}$

$\Rightarrow A_2 = \frac{6 \times 1}{3}$

$\Rightarrow A_2 = 2$ cm

$\frac{z_1}{z_2} = \frac{k_1}{k_2}$ ($\because T, \omega$ = constants)

$\Rightarrow \frac{z_1}{z_2} = \frac{\lambda_2}{\lambda_1}$

$\frac{A_1}{A_2} = \frac{1}{2}\left(1 + \frac{\lambda_1}{\lambda_2}\right)$

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$$\therefore \lambda_2 = \frac{\lambda_1}{\left(\frac{2A_1}{A_2} - 1\right)}$$

$$\Rightarrow \lambda_2 = \frac{25}{\frac{2.3}{2} - 1}$$

$$\Rightarrow \lambda_2 = \frac{25}{2}$$

$$\Rightarrow \underline{\lambda_2 = 12.5 \text{ cm}}$$

b. Reflection coefficient $= \dfrac{z_1 - z_2}{z_1 + z_2}$

$$= \frac{T - 2T}{T + 2T}$$

$$= \frac{1}{3}$$

Power is proportional to amplitude squared. Therefore, wave power reflected is $\underline{\frac{1}{9}}$ of the power

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Question 5

Gauss' Theorem uses divergence of vectors, while Stoke's theorem uses the curl. Stokes theorem calculates flux lines going through a single surface, while Gauss' Theorem calculates flux lines passing a solid, in and outgoing.