Phase and Group velocity of EM waves

Objective:

To understand the nature of EM waves travelling in a medium with the help of phase and group velocities.

Theory:

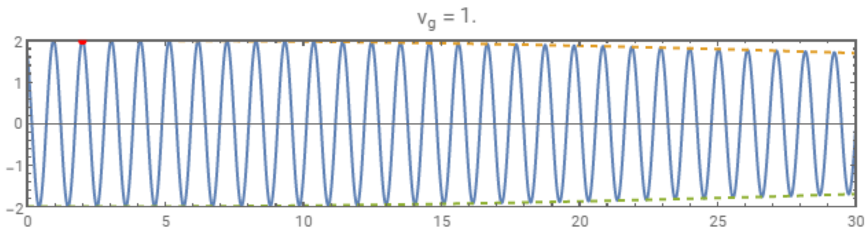
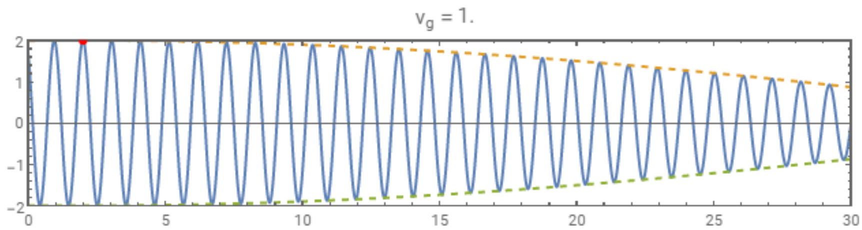
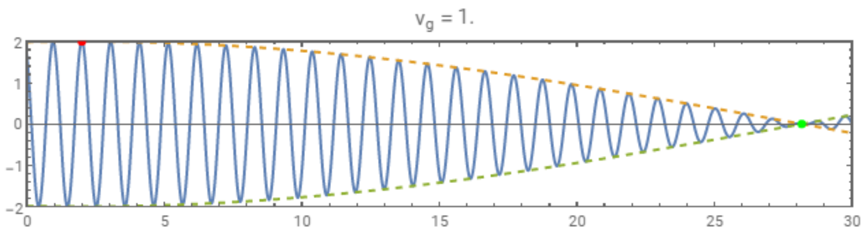
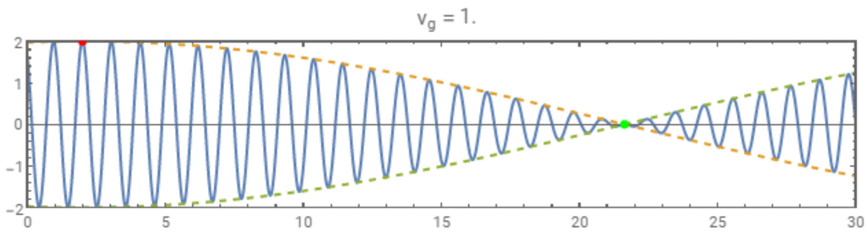
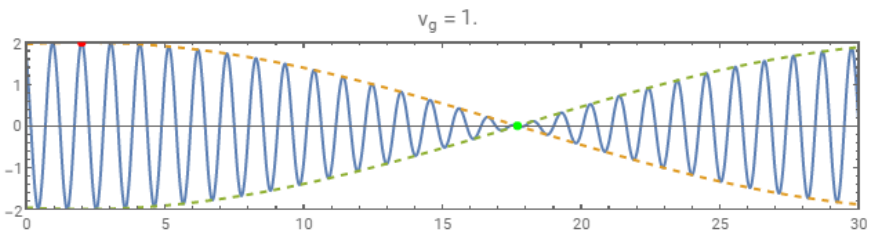
Any real signal consists of travelling waves of many different frequencies, which travel together as a group, at a speed that will always be less than or equal to the speed of light in vacuum. To gain some insight into what may happen when a real signal travels through a dispersive medium, we consider adding two waves of equal amplitude. When two travelling waves with unit amplitude $f_1(z,t) = \cos(k_1 z - \omega_1 t)$ and $f_2(z,t) = \cos(k_2 z - \omega_2 t)$ are added, we get:

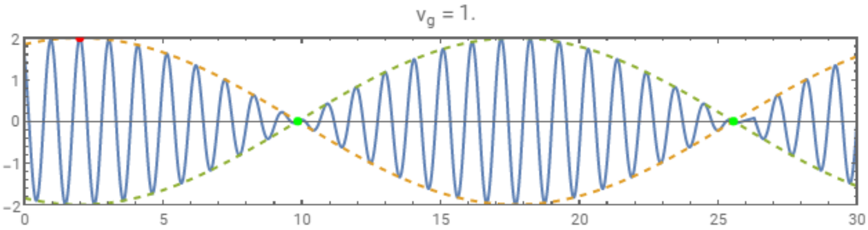
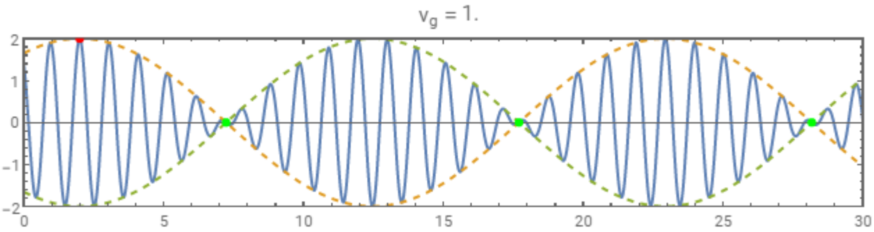
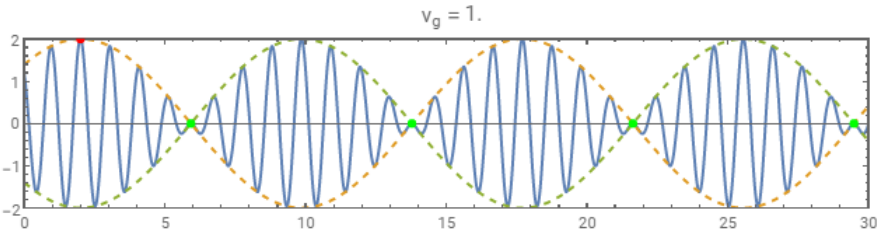
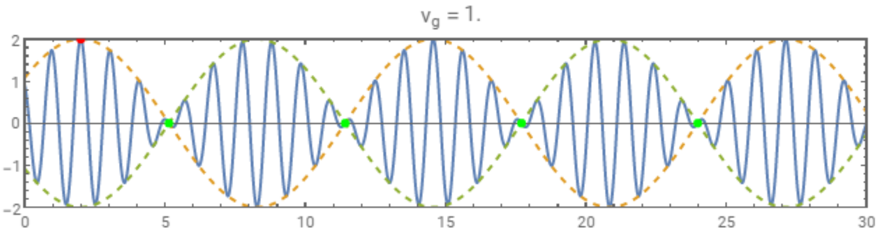
$$f_1(z,t) + f_2(z,t) = \cos(k_1 z - \omega_1 t) + \cos(k_2 z - \omega_2 t)$$
$$= 2\cos\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)\cos\left(-\bar{\omega} t + \bar{k} z\right)$$

where $\Delta k = k_2 - k_1$, $\Delta\omega = \omega_2 - \omega_1$, $\bar{k} = \frac{k_1 + k_2}{2}$, $\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$

The result is a fast oscillating wave that travels with a phase velocity $v_p = \frac{\bar{\omega}}{\bar{k}}$ and the amplitude of this wave is being modulated in space time by $2\cos\left(\frac{\Delta k}{2} z - \frac{\Delta\omega}{2} t\right)$. This modulated wave travels with group velocity $v_g = \frac{\Delta\omega}{\Delta k}$

Teacher's Signature

S. No.	Δω	Δk	Wave pattern of the resultant waves	V_g
1	0.02	0.02		1
2	0.04	0.04		1
3	0.06	0.06		1
4	0.08	0.08		1
5	0.1	0.1		1

6	0.2	0.2		1
7	0.3	0.3		1
8	0.4	0.4		1
9	0.5	0.5		1

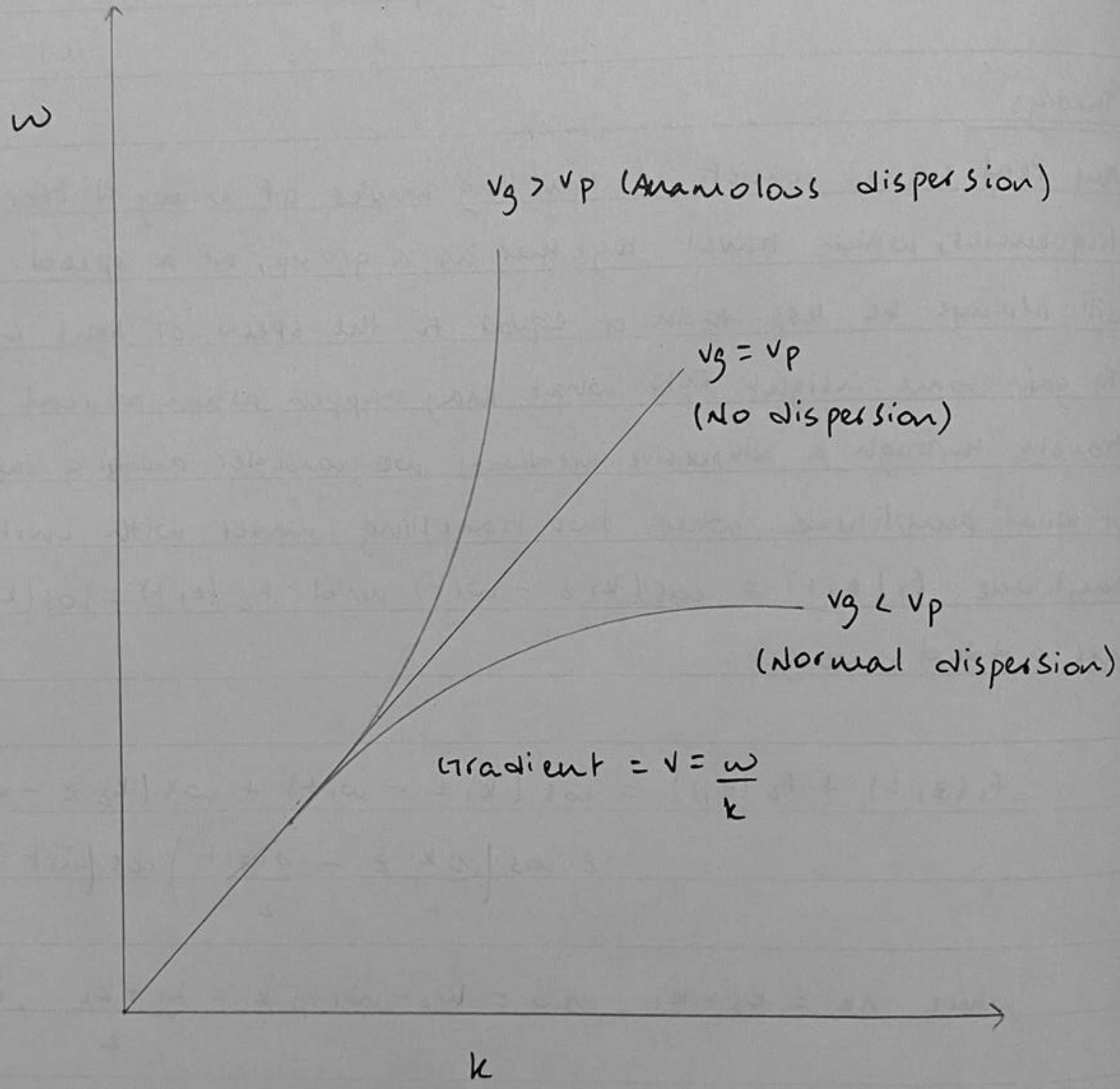


Fig 5.1. Dispersion relation curve

Inferences:

1. The wave patterns for different $\Delta\omega$ and Δk are not the same because v_p is different.
2. v_p does not depend on $\Delta\omega$ and Δk, but instead depends on $\bar{\omega}$ and $\bar{k}$. This means that the phase velocity depends on the average frequencies and wave numbers rather than the change in those parameters.
3. v_p is equal to v_g when $\frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}$
4. Image in Fig 5.1