

21BDS0340

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Digital Systems Design Lab

## Task 1

S. No.	Components	Page No.	Student Check Mark	RA Check Mark
1	Aim	1	✓	
2	Components Required and Tools Required	1	✓	
3	Procedure and Theory	1, 2	✓	
4	Truth Table	2	✓	
5	SOP Boolean expression in canonical form	2, 3	✓	
6	POS Boolean expression in canonical form	2, 3	✓	
7	Boolean simplification using Boolean laws	3, 4	✓	
8	Boolean simplification using KMAP simplification	5, 6	✓	
9	Simplified SOP expression in Standard form	6	✓	
10	Simplified POS expression in Standard form	6	✓	
11	Circuit diagram using AOI logic	6	✓	
12	Circuit diagram using OAI logic	7	✓	
13	Circuit diagram using NAND logic	7	✓	
14	Circuit diagram using NOR logic	7	✓	
15	Multisim live / Circuitverse.org Simulation link for SOP F and F' circuit	8	✓	
16	Multisim live / Circuitverse.org Simulation link for POS F and F' circuit	8	✓	
17	Multisim live / Circuitverse.org Simulation link for NAND F and F' circuit	8	✓	
18	Multisim live / Circuitverse.org Simulation link for NOR F and F' circuit	8	✓	
19	Justification for optimized circuit NAND /NOR /XOR Logic	8	✓	
20	Verilog code: Gate Level	8	✓	
21	Verilog code: Data Flow	9	✓	
22	Test Bench	9	✓	
23	Snip of Output waveform with respective code	10	✓	
24	Online Verilog code Simulation links	10	✓	
25	Result	10	✓	
26	Inference	11	✓	

**Aim**

Using Reg.no. formulate expressions in SOP and POS for F and F'. Use K-Map and Boolean laws to simplify the expressions. Write a Verilog code to implement F and F' with a neat circuit diagram for all circuits designed using the following forms.

- a. SOP – AND-OR-INV logic circuit
- b. POS – OR-AND-INV logic circuit
- c. SOP – NAND-NAND logic circuit
- d. POS – NOR-NOR logic circuit

Use only two input logic elements for AND, OR, NAND and NOR logic gates.

**Components Required**

- a. AND, OR, NOT, NAND and NOR gates
- b. 5V voltage source
- c. Led indicator

**Tools Required**

- a. Multisim simulator
- b. Charlie-Coleman KMAP solver
  - <https://charlie-coleman.com/experiments/kmap/>
- c. Truth table display
  - <http://www.32x8.com/>
- d. Boolean algebra simplifier
  - <https://www.boolean-algebra.com/>

**Theory**

- a. K-maps
  - Karnaugh maps (K-maps) are graphical representations of Boolean functions.
- b. Don't care conditions
  - There may be a combination of input values which (i) will never occur, (ii) if they do occur, the output is of no concern.
- c. Canonical Forms
  - Any Boolean function F can be expressed as a unique sum of minterms and a unique product of maxterms (under a fixed variable ordering).
  - In other words, every function F() has two canonical forms:
    - Canonical Sum-Of-Products (sum of minterms)
    - Canonical Product-Of-Sums (product of maxterms)

**Procedure**

1. Write down the SOP and POS expression using registration numbers.
2. Implement the expression in K-map using Logic minimiser or Logic Friday.
3. Then design circuits using these expressions in Multisim using AND OR INV and 2 input NAND logic for  $\Sigma m$  expressions.
4. Similarly design  $\Pi M$  expressions in OR AND INV Logic and 2- input NOR Logic.
5. Compare and contrast the circuits with the number of gates used.
6. Draw suitable conclusions and inference.
7. Implement one of these 8 circuits in hardware and cross check the results with the truth table.

**Truth Table****F:**

Submit					Y		
	A	B	C	D	0	1	x
0	0	0	0	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
1	0	0	0	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
2	0	0	1	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
3	0	0	1	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
4	0	1	0	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
5	0	1	0	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
6	0	1	1	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
7	0	1	1	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
8	1	0	0	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	1	0	0	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
10	1	0	1	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
11	1	0	1	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
12	1	1	0	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
13	1	1	0	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
14	1	1	1	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
15	1	1	1	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
Submit							

**F':**

Submit					Y		
	A	B	C	D	0	1	x
0	0	0	0	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
1	0	0	0	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	0	0	1	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	0	0	1	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
4	0	1	0	0	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	0	1	0	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
6	0	1	1	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
7	0	1	1	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
8	1	0	0	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
9	1	0	0	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
10	1	0	1	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
11	1	0	1	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
12	1	1	0	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
13	1	1	0	1	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>
14	1	1	1	0	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
15	1	1	1	1	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
Submit							

**Functional Expression**

$$F(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, B, D) = \Sigma m(0, 1, 2, 3, 4, 11, 13)$$

$$= \Pi M(5, 6, 7, 8, 9, 10, 12, 14, 15)$$

$$F = A'B'C'D' + A'B'C'D + A'B'CD' + A'B'CD + A'BC'D' + AB'CD + ABC'D \text{ (SOP form)}$$

$$F = (A+B'+C+D')(A+B'+C'+D)(A+B'+C'+D')(A'+B+C+D)(A'+B+C+D')(A'+B+C'+D)(A'+B'+C+D)(A'+B'+C'+D) \text{ (POS form)}$$

$$F'(A, B, C, D) = \Sigma m(5, 6, 7, 8, 9, 10, 12, 14, 15)$$

$$= \Pi M(0, 1, 2, 3, 4, 11, 13)$$

$$F' = A'BC'D + A'BCD' + A'BCD + AB'C'D' + AB'C'D + AB'CD' + ABC'D' + ABCD' + ABCD \text{ (SOP form)}$$

$$F' = (A+B+C+D)(A+B+C+D')(A+B+C'+D)(A+B+C'+D')(A+B'+C+D)(A'+B+C'+D')(A'+B'+C+D') \text{ (POS form)}$$

### Simplified SOP Form

Start

$$\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Distributive Law:  $AB+AC = A(B+C)$

$$\overline{A}\overline{B}\overline{C}(\overline{D}+D) + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Complement Law:  $A+\overline{A} = 1$

$$\overline{A}\overline{B}\overline{C}1 + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Identity Law:  $A1 = A$

$$\overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Distributive Law:  $AB+AC = A(B+C)$

$$\overline{A}\overline{B}(\overline{C}\overline{D} + \overline{C}D) + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Absorption Law:  $AB+\overline{A} = B+\overline{A}$

$$\overline{A}\overline{B}(\overline{D} + \overline{C}) + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD$$

Apply the Distributive Law:  $AB+AC = A(B+C)$

$$\overline{A}\overline{B}(\overline{D} + \overline{C}) + \overline{B}CD(\overline{A} + A) + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D$$

Apply the Complement Law:  $A+\overline{A} = 1$

$$\overline{A}\overline{B}(\overline{D} + \overline{C}) + \overline{B}CD1 + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D$$

Apply the Identity Law:  $A1 = A$

$$\overline{A}\overline{B}(\overline{D} + \overline{C}) + \overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D$$

Apply: Distribution

$$\overline{A}\overline{B}\overline{D} + \overline{A}\overline{B}\overline{C} + \overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D$$

Apply the Distributive Law:  $AB+AC = A(B+C)$

$$\overline{A}\overline{B}\overline{C} + \overline{B}CD + \overline{A}\overline{D}(\overline{B}\overline{C} + \overline{B}) + \overline{A}B\overline{C}D$$

Apply the Absorption Law:  $AB+\overline{A} = B+\overline{A}$

$$\overline{A}\overline{B}\overline{C} + \overline{B}CD + \overline{A}\overline{D}(\overline{C} + \overline{B}) + \overline{A}B\overline{C}D$$

Apply: Distribution

$$\overline{A}\overline{B}\overline{C} + \overline{B}CD + \overline{A}\overline{D}\overline{C} + \overline{A}\overline{D}\overline{B} + \overline{A}B\overline{C}D$$

**Simplified POS Form**

$$\bar{y} = B.C + A.\bar{D} + \bar{A}.B.D + A.\bar{B}.\bar{C}$$

$$\bar{\bar{y}} = \overline{B.C + A.\bar{D} + \bar{A}.B.D + A.\bar{B}.\bar{C}}$$

$$y = (B' + C') (A' + D) (A + B' + D') (A' + B + C)$$

**K-Map Simplification****F:****Terms****Minterms:**

Comma separated list of numbers

0,1,2,3,4,11,13

**Don't Cares:**

Comma separated list of numbers

**Solutions:****Generic:**

$$f(a, b, c, d) = a'b' + a'c'd' + b'cd + abc'd$$

<i>f</i>		<i>c,d</i>			
		00	01	11	10
<i>a,b</i>	00	1	1	1	1
	01	1	0	0	0
	11	0	1	0	0
	10	0	0	1	0

**F':****Terms****Minterms:**

Comma separated list of numbers

5,6,7,8,9,10,12,14,15

**Don't Cares:**

Comma separated list of numbers

**Solutions:****Generic:**

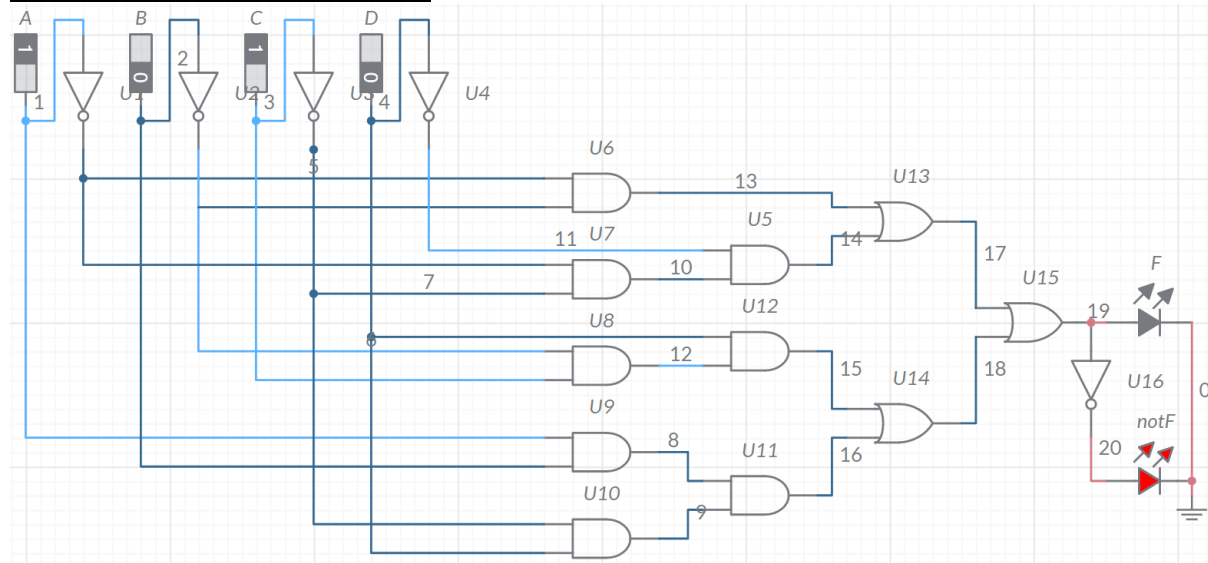
$$f(a, b, c, d) = a'bd + bc + ab'c' + ad'$$

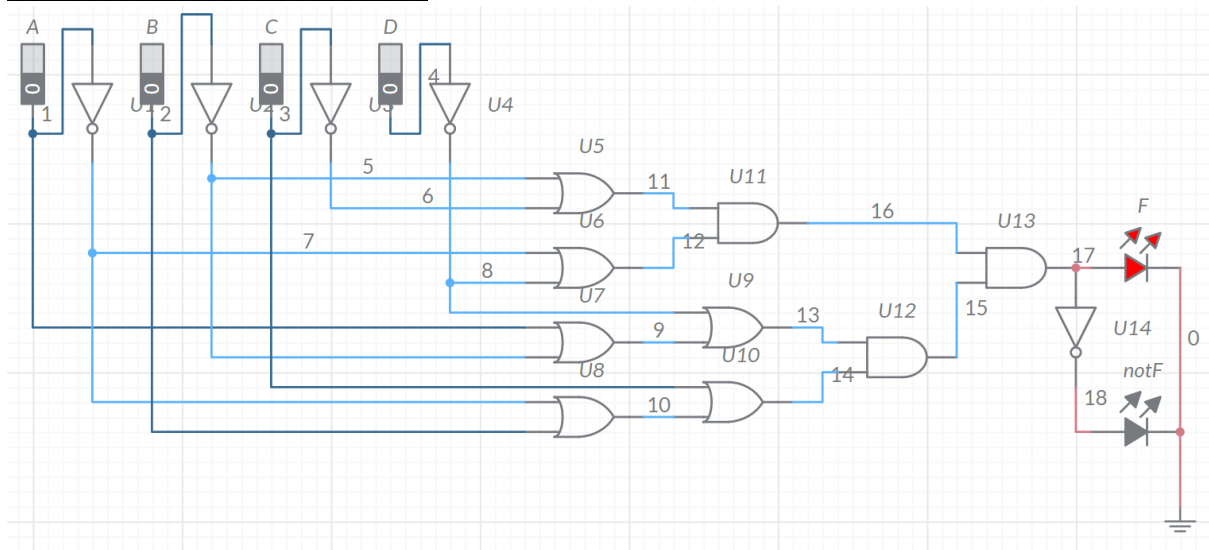
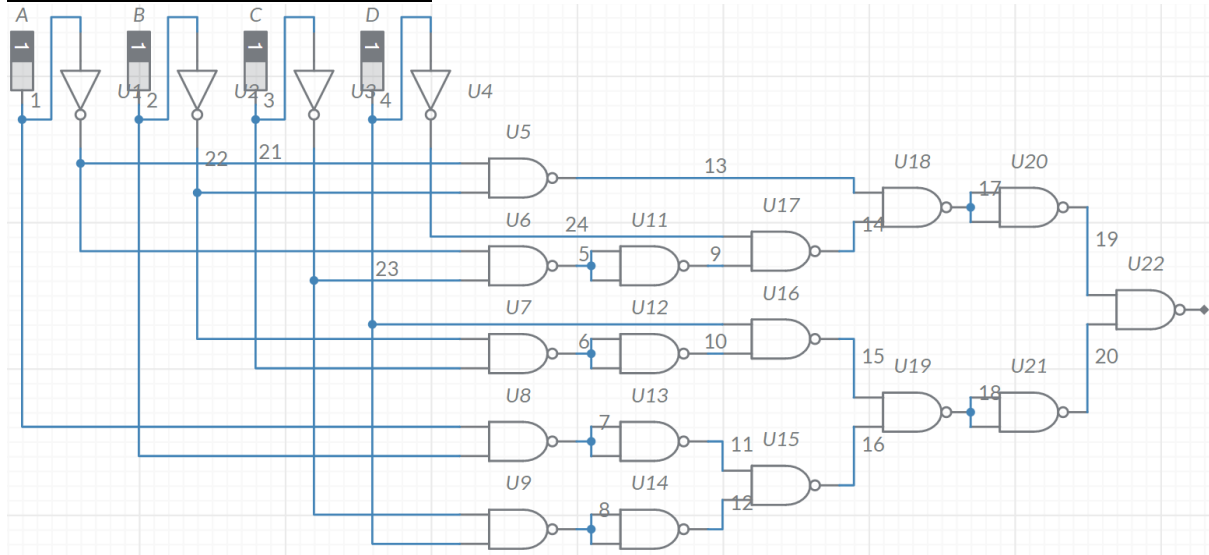
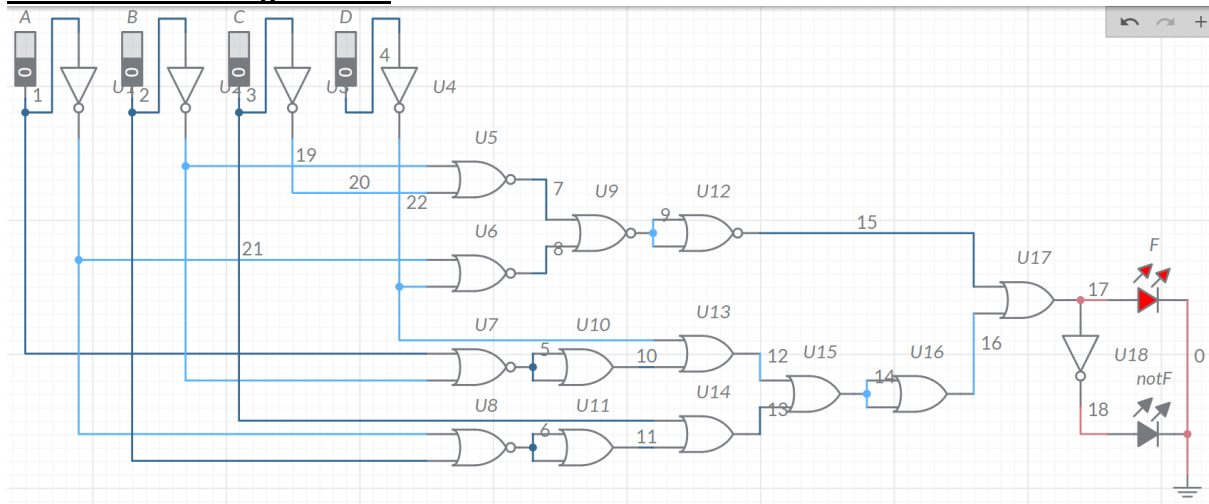
$f$		$c,d$			
		00	01	11	10
$a,b$	00	0	0	0	0
	01	0	1	1	1
	11	1	0	1	1
	10	1	1	0	1

**Simplified Forms**

$$F = A'B' + A'C'D' + B'CD + ABC'D = (B' + C') (A' + D) (A + B' + D') (A' + B + C)$$

$$F' = BC + AD' + A'BD + AB'C' = (A + B) (A + C + D) (B + C' + D') (A' + B' + C + D')$$

**SOP – AND-OR-INV Logic Circuit**

**POS – AND-OR-INV Logic Circuit****SOP – NAND-NAND Logic Circuit****POS – NOR-NOR Logic Circuit**

### **Multisim Links to Circuit Diagrams**

- a. SOP – AND-OR-INV logic circuit
  - <https://www.multisim.com/content/Dh2E5WYcPVSahgB6owGejF/21bds0340-sop-aoi-f-and-f/open/>
- b. POS – OR-AND-INV logic circuit
  - <https://www.multisim.com/content/jr8yKr5ADsHAasgvYdj5Fo/21bds0340-pos-oai-fand-f/open/>
- c. SOP – NAND-NAND logic circuit
  - <https://www.multisim.com/content/qTEa9pUv3tVrEoDRh5juhG/21bds0340-sop-nand-f/open/>
- d. POS – NOR-NOR logic circuit
  - <https://www.multisim.com/content/DtaT8SneEbofyLkTobGae7/21bds0340-pos-nor-f/open/>

### **Justification for Optimized Circuit**

The circuits constructed by NAND and NOR gates are optimised as those gates are cheaper and universal, meaning that they can replace any other gates.

### **Verilog Code: Gate Level**

```
// Gate Level Model
module regno(a, b, c, d, f);
    input a, b, c, d;
    output f;

    not(nota, a);
    not(notb, b);
    not(notc, c);
    not(notd, d);

    and(prod1, nota, notb);

    and(prod2_1, nota, notc);
    and(prod2, prod2_1, notd);

    and(prod3_1, notb, c);
    and(prod3, prod3_1, d);

    and(prod4_1, a, b);
    and(prod4_2, notc, d);
    and(prod4, prod4_1, prod4_2);

    or(sum1, prod1, prod2);
    or(sum2, prod3, prod4);
    or(f, sum1, sum2);
endmodule
```



**Verilog Code: Data Flow**

```
// Data Flow Model
module design(a, b, c, d, f);
    input a, b, c, d;
    output f;

    assign nota = ~a;
    assign notb = ~b;
    assign notc = ~c;
    assign notd = ~d;

    assign prod1 = nota & notb;

    assign prod2_1 = nota & notc;
    assign prod2 = prod2_1 & notd;

    assign prod3_1 = notb & c;
    assign prod3 = prod3_1 & d;

    assign prod4_1 = a & b;
    assign prod4_2 = notc & d;
    assign prod4 = prod4_1 & prod4_2;

    assign sum1 = prod1 | prod2;
    assign sum2 = prod3 | prod4;
    assign f = sum1 | sum2;
endmodule
```

**Test Bench**

```
module testbench;
    reg a, b, c, d;
    wire f;
    regno r(a, b, c, d, f);
    initial begin
        a = 0; b = 0; c = 0; d = 0;
        #50
        a = 0; b = 0; c = 0; d = 1;
        #50
        a = 0; b = 0; c = 1; d = 0;
        #50
        a = 0; b = 0; c = 1; d = 1;
        #50

        a = 0; b = 1; c = 0; d = 0;
        #50
        a = 0; b = 1; c = 0; d = 1;
        #50
        a = 0; b = 1; c = 1; d = 0;
    end
```

```

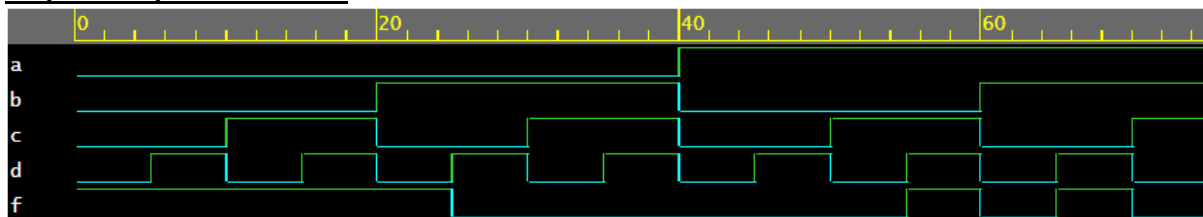
#50
a = 0; b = 1; c = 1; d = 1;
#50

a = 1; b = 0; c = 0; d = 0;
#50
a = 1; b = 0; c = 0; d = 1;
#50
a = 1; b = 0; c = 1; d = 0;
#50
a = 1; b = 0; c = 1; d = 1;
#50

a = 1; b = 1; c = 0; d = 0;
#50
a = 1; b = 1; c = 0; d = 1;
#50
a = 1; b = 1; c = 1; d = 0;
#50
a = 1; b = 1; c = 1; d = 1;
end
endmodule

```

### Snip of Output Waveform



Note: To revert to EPWave opening in a new browser window, set that option on your user page.

### Online Verilog Code Simulation Link

<https://www.edaplayground.com/x/9BLy>

### Result

I have successfully completed and simulated the combinational circuit. The result of the circuit was as expected from the truth table. I have derived functional expressions from the truth table using Boolean laws and K-map and got the same results in both techniques. I have simulated the circuit using a Multisim online simulator.

17 two input NAND gates are used in SOP F circuit simulation and 13 two input NOR gates are used in POS F circuit simulation. Both SOP and POS are using the same number of logic gates so, we can use any one of them for circuit simulation

### **Inference**

I have derived functional expressions using my registration number (21BDS0340) both in SOP and POS form. I have learnt to derive a truth table from functional expressions. I learnt Boolean expressions simplification using Boolean laws and K-map technique.