21BDS0340

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Question 1

a. $m = 4$

$n = 7$

Min distance $= 4$

Errors correctable:

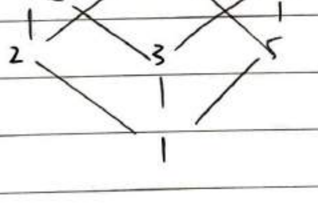
$2k+1 \le 4$

$\Rightarrow k \le 1.5$

$\therefore$ Errors correctable at most is 1

b. $\langle S_{30}, D \rangle$

Hasse Diagram: $\{1, 2, 3, 5, 6, 10, 15, 30\}$

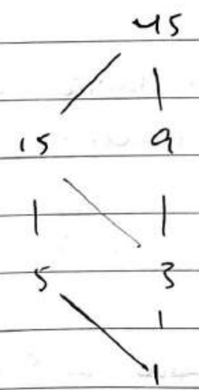


All ~~each~~ elements have a complement

$\therefore$ Lattice is distributive

$\langle S_{45}, D \rangle$

Hasse Diagram: $\{1, 3, 5, 9, 15, 45\}$



Both diagrams are complemented

(15, 9) have a GLB = 3,

$\therefore$ Not all elements have only 1 complement

$\therefore$ Lattice is not distributive

Question 2

a. Let chain = $\langle P, \leq \rangle$,

For any $a, b \in P$

If $a \leq b$, from reflexivity $a \leq a$

$\therefore$ a is lower bound of (a, b)

Similarly for a, c, where $a \leq c$

a is the lower bound of (a, c)

If $b \leq c$

Then the upper bound of $(b, c) = c$

By checking for distributivity:

$a \wedge (b \vee c)$

$= a \wedge (c)$

$= a \wedge c$

$= a$

$= a \vee a$

$= (b \wedge a) \vee (c \wedge a)$

$\therefore$ A chain lattices is always distributive

b. In a complemented, distributive lattice each element has exactly 1 complemente

If $a \leq b \Rightarrow b' \leq a'$

This true because for any ~~com~~ $a \leq b$, the complement of a, a', is $a * a' = 0$, $a \oplus a' = 1$

So, a' is on the opposite side of the lattice

Doing the same for b and b'

$\therefore$ If $a \leq b$, b' has to be $\leq a'$

For $a * b' = 0$ and $a' \oplus b = 1$

This can easily be shown by duality:

$\overline{a * b'} = \bar{0}$

$= a' \oplus b = 1$

c. $a \leq b \Rightarrow a + bc = b(a+c)$

If $a \leq b$, then $a * b = a$

Representing $ab = a * b$

$\therefore$ $a + bc$

$= ab + bc$

$= b(a+c)$

Question 3

a. $(L, \leq)$

$*$, $\oplus$ - meet, join

$a, b \in L$

Prove if $a \leq b \Rightarrow a * b = a$

$\Rightarrow a \oplus b = b$

$\therefore$ a is less than b

The GLB of $(a, b) = a$

$\therefore$ $a * b = a$

$\because$ b is greater than a

$\therefore$ b is the upper bound (least) of (a, b)

$\therefore$ $a \leq b \Rightarrow a \oplus b = b$

6. $e: B^3 \rightarrow B^6$

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \underline{A}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Code:

$e(000) = 000000$

$e(001) = 001011$

$e(010) = 010101$

$e(011) = 011110$

$e(100) = 100100$

$e(101) = 101101$

$e(110) = 110011$

$e(111) = 111000$

Question 4

a. $x_1 . x_2$

$= x_1 x_2 x_3 + x_1 x_2 \bar{x}_3$ = SOP

$(x_1)(x_2)$

$= (x_1 + \bar{x}_2 + \bar{x}_3)(x_1 + \bar{x}_2 + x_3)(x_1 + x_2 + \bar{x}_3)(x_1 + x_2 + x_3)$
$(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + x_2 + x_3)(x_1 + x_2 + \bar{x}_3)(x_1 + x_2 + x_3)$

$= \Pi M(0,1,2,3,4,5)$ = POS

b. $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$

LHS:

$= (a + b')(bc + ba' + c'a')$

$= abc + b'c'a' =$ ①

RHS

$= (a' + b)(b'c' + b'a + ac)$

$= a'b'c' + abc$ — ②

① = ②

∴ The LHS = RHS

1. $\Sigma m(0, 2, 5, 8, 9, 13, 14, 15)$

Table 1:

Group	Minterm	A	B	C	D
0	0	0	0	0	0
1	2	0	0	1	0
	8	1	0	0	0
2.	5	0	1	0	1
	9	1	0	0	1
3	11	1	0	1	1
	13	1	1	0	1
4	15	1	1	1	1

Table 2

Group	Pair	A	B	C	D
0	(0, 2)	0	0	-	0
	(0, 8)	-	0	0	0
1	(8, 9)	1	0	0	-
2	(5, 13)	-	1	0	1
	(9, 11)	1	0	-	1
3	(11, 15)	1	-	1	1
	(13, 15)	1	1	-	1

Table 3

Group	Quad	A	B	C	D
0					
1	(9,11,13,15)	1	–	–	1
2	(0,2,9,11)	–	0	–	$\bar{0}$
	(~~0,8,9,11~~)				
	(0,8,5,13)	–	–	0	–
	(0,2,13,15)	–	–	–	–

$$\therefore \Sigma m(0,2,5,8,9,13,14,15) = \bar{w}\bar{x}\bar{z} + x\bar{y}z + wxy + w\bar{x}\bar{y}$$