Assignment - 1 21BDS0340

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Question 1

x_1	x_2
8	21
24	9
17	6
11	18

$\bar{x}_1 = 15 , \bar{x}_2 = 13.5$

$$\operatorname{cov}(x_1, x_2) = \frac{1}{3}\Big((8-15)(21-13.5) + (24-15)(9-13.5) + (17-15)(6-13.5) + (11-15)(18-13.5)\Big)$$

$$= \frac{1}{3}(-126) = \underline{-42}$$

$$\operatorname{cov}(x_1, x_1) = \frac{1}{3}\left((8-15)^2 + (24-15)^2 + (17-15)^2 + (11-15)^2\right)$$

$$= \frac{1}{3}(150) = \underline{50}$$

$$\operatorname{cov}(x_2, x_2) = \frac{1}{3}\left((21-13.5)^2 + (9-13.5)^2 + (6-13.5)^2 + (18-13.5)^2\right)$$

$$= \frac{1}{3}(153) = \underline{51}$$

$$S = \begin{bmatrix} 50 & -42 \\ -42 & 51 \end{bmatrix}$$

$|S - \lambda I| = 0$

$$\cancel{\Rightarrow \begin{bmatrix} 50 & -42-\lambda \\ -42-\lambda & 51 \end{bmatrix} = 0 \Rightarrow}$$

$$\Rightarrow \begin{bmatrix} 50-\lambda & -42 \\ -42 & 51-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda^2 - 101\lambda + 786 = 0$$

$$\Rightarrow \lambda = \frac{101 \pm \sqrt{101^2 - 786 \times 4}}{2}$$

$$\Rightarrow \lambda = \frac{101 \pm 84}{2}$$

$$\Rightarrow \lambda = \underline{92.5} \text{ or } \underline{8.5}$$

$$v = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$(S - \lambda I)v = 0$$

$$\Rightarrow \begin{bmatrix} 50-\lambda & -42 \\ -42 & 51-\lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

$$\Rightarrow (50-\lambda)u_1 - 42u_2 = 0$$

$$-42u_1 + (51-\lambda)u_2 = 0$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{42}{50-\lambda}, \quad \frac{u_1}{u_2} = \frac{51-\lambda}{42}$$

[illegible] $v_1 = \begin{bmatrix} 42 \\ 50-\lambda \end{bmatrix}, \quad v_2 = \begin{bmatrix} 51-\lambda \\ 42 \end{bmatrix}$

$$\Rightarrow v_1 = \begin{bmatrix} 42 \\ -42.5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -41.5 \\ 42 \end{bmatrix}$$

$$\Rightarrow e_1 = \begin{bmatrix} 0.703 \\ -0.711 \end{bmatrix}, \quad e_2 = \begin{bmatrix} -0.703 \\ 0.711 \end{bmatrix}$$ (Normalised)

1^{st} Principle

$$e^T \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.703 & -0.711 \end{bmatrix} \begin{bmatrix} x_{1k} - \bar{x}_1 \\ x_{2k} - \bar{x}_2 \end{bmatrix}$$

For (8,21)

$= [0.703 \quad -0.711] \begin{bmatrix} 8-15 \\ 21-13.5 \end{bmatrix} = \underline{-10.25}$

[illegible]

For (24,9)

$= (0.703 \quad -0.711) \begin{pmatrix} 24-15 \\ 9-13.5 \end{pmatrix} = \underline{9.53}$

For (17,6)

$= (0.703 \quad -0.711) \begin{pmatrix} 17-15 \\ 6-13.5 \end{pmatrix} = \underline{6.74}$

For (11,18)

$= (0.703 \quad -0.711) \begin{pmatrix} 11-15 \\ 18-13.5 \end{pmatrix} = \underline{-6.01}$

PCA conclusion

x1	x2	1st Principle
8	21	-10.25
24	9	9.53
17	6	6.74
11	18	-6.01

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Question 2

x1 = (0, 1, 1, 0)

x2 = (1, 0, 1, 1)

x3 = (0, 0, 0, 1)

x4 = (1, 1, 1, 0)

Assuming

$\eta = 1$

$$\text{weights} = \begin{bmatrix} 0.1 & 0.9 & 0.7 & 0.7 \\ 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

For x1 = (0, 1, 1, 0)

$d_1^2 = 0.6$, $d_2^2 = 2.26$

unit 1 wins

weight change

$w_{new} = (0.1\ 0.9\ 0.7\ 0.7) + 1((0\ 1\ 1\ 0) - (0.1\ 0.9\ 0.7\ 0.7))$

$= (0\ 1\ 1\ 0)$

$$w = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

For x2 = (1, 0, 1, 1)

$d_1^2 = 3$, $d_2^2 = 2.96$

unit 2 wins

weight change

$w_{new} = (0\ 0.1\ 0.2\ 0.1) + 1((1\ 0\ 1\ 1) - (0\ 0.1\ 0.2\ 0.1))$

$= (1\ 0\ 1\ 1)$

$$w = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

For $x3 = (0,0,0,1)$

$d_1^2 = 3$, $d_2^2 = 2$

unit 2 wins

weight change

$$w_{new} = (1\ 0\ 1\ 0) + 1((1\ 0\ 0\ 0\ 1) - (1\ 0\ 1\ 0))$$

$$= (0\ 0\ 0\ 1)$$

$$w = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For $x4 = (1,1,1,0)$

$d_1^2 = 1$, $d_2^2 = 4$

unit 1 wins

weight change:

$$w_{new} = (0\ 1\ 1\ 0) + 1((1\ 1\ 1\ 0) - (0\ 1\ 1\ 0))$$

$$= (1\ 1\ 1\ 0)$$

$$w = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Organisation (After 1 Iteration)

$x1, x4 \rightarrow$ unit 1

$x2, x3 \rightarrow$ unit 2