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Question 1

$$A^0 = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

using formula:

$$A^k(i,j) = \min(A^{k-1}(i,j),\ A^{k-1}(i,k) + A^{k-1}(k,j))$$

$$A' = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & \infty & 4 & 0 \end{bmatrix}$$

$A'(2,3) = \min(3, 6+\infty)$
$= 3$

$A'(2,4) = \min(2, 6+1)$
$= 2$

$A'(2,5) = \min(\infty, 6+8)$
$= 14$

$A'(3,2) = \min(\infty, \infty+\infty)$
$= \infty$

$A'(3,4) = \min(4, \infty+1)$
$= 4$

$A'(3,5) = \min(\infty, \infty+8)$
$= \infty$

$A'(4,2) = \infty$
$A'(4,3) = 2$
$A'(4,5) = 3$

$A'(5,2) = 5$
$A'(5,3) = \infty$
$A'(5,4) = 4$

$$A^2 = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 19 & 4 & 0 \end{bmatrix}$$

$A^2(1,3) = 5$ $\quad A^2(3,1) = \infty$ $\quad A^2(4,1) = \infty$

$A^2(1,4) = 1$ $\quad A^2(3,4) = 4$ $\quad A^2(4,3) = 2$

$A^2(1,5) = 8$ $\quad A^2(3,5) = \infty$ $\quad A^2(4,5) = 3$

$A^2(5,1) = 3$ $\quad A^2(5,3) = 19$ $\quad A^2(5,4) = 4$

$$A^3 = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 19 & 4 & 0 \end{bmatrix}$$

$A^3(1,2) = 2$ $\quad A^3(2,1) = 6$ $\quad A^3(4,1) = \infty$

$A^3(1,4) = 1$ $\quad A^3(2,4) = 2$ $\quad A^3(4,2) = \infty$

$A^3(1,5) = 8$ $\quad A^3(2,5) = 14$ $\quad A^3(4,5) = 3$

$A^3(5,1) = 3$ $\quad A^3(5,2) = 5$ $\quad A^3(5,4) = 4$

$$A^4 = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ \infty & \infty & 0 & 4 & 7 \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix}$$

$A^4(1,2) = 2$

$A^4(1,3) = 3$

$A^4(1,5) = 4$

$A^4(2,1) = 6$ $\quad A^4(3,1) = \infty$ $\quad A^4(5,1) = 3$

$A^4(2,3) = 3$ $\quad A^4(3,2) = \infty$ $\quad A^4(5,2) = 5$

$A^4(2,5) = 5$ $\quad A^4(3,5) = 7$ $\quad A^4(5,3) = 6$

$$A^5 = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ 10 & 12 & 0 & 4 & 7 \\ 6 & 8 & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix}$$

Algorithm:

Given a matrix A^0, let $n = 0$

1. Create a matrix with A^0's size called A^{n+1}
2. Copy the n^{th} row and column of A^n to A^{n+1}
3. Apply the formula for $A^{n+1}(i,j)$

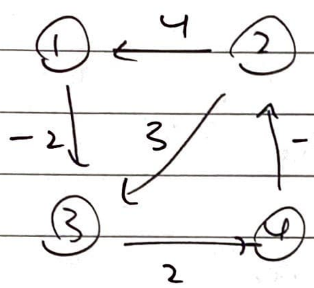
$$A^{n+1}(i,j) = \min\left[A^n(i,j),\ A^n(i,n+1) + A^n(n+1,j)\right]$$

4. Once all the entries are filled, increment n
5. Repeat the steps from 1 until n = size of matrix

Time complexity

$O(V^3)$ where V = no. of vertices

Question 2



List of edges :

(1,3), (2,1), (2, 3), (3,4), (4,2)

$n = 4$

Phase 1

$d(1) = 0$

$d(3) = d(1) - 2$

$= -2 < \infty$

$d(1) = \infty > 0$

$d(3) = \infty > -2$

$d(4) = d(3) + 2$

$= 0 < \infty$

$d(2) = d(4) - 1$

$= -1 < \infty$

$= -1$

Phase 2

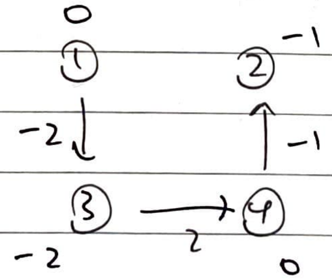
Initial distances:

$d(1) = 0$ $\quad$ $d(3) = -2$

$d(2) = -1$ $\quad$ $d(4) = 0$

No changes on relaxation

$\therefore$ The shortest paths are



Algorithm :

1. Loop from 0 to n-1
2. Go through the list of edges
3. If $d(u) + cost(u,v) < d(v)$, then update $d(v)$ as $d(u)$
4. Repeat until the looping is done or no change occurs

Time complexity

$O(v \times e)$ where v = vertices count
e = edges count

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Question 3

1. Loop from the start of the string till the end
2. check if the ith to i+lth index match the pattern of length l
3. If the pattern matches, display i
4. If no pattern matches, display as such

KMP Algorithm:

1. create a prefix table for the string pattern as pi
2 keep track of letters in the string as i
3. set j as 0
4. If i = ~~s~~ pattern[j+1], increment j
5. If the letters don't match, then set j as patter[j]'s pi and set i as the next letter in the string
6. If j = length of pattern, sequence is found

Rabin Karp Algorithm:

1. Hash the string and pattern and find the expected remainder
2. Loop through the numbers of same length as the pattern at a time
3. If the hit (mod calculation) is spurious, then check if the numbers match
4. If match, sequence is detected

Examples:

String: ABCCDDAEFG

Pattern: CDD

Naive method:

1. ABC ≠ CDD
2. BCC ≠ CDD
3. CCD ≠ CDD
4. CDD = CDD, sequence found at pos. 4

KMP method:

Pi table:

index	0	1	2
char	C	D	D
pi	0	0	1

1. ~~ABC~~ A ≠ C
2. B ≠ C
3. C = C, inc j
4. C ≠ D, go to 0
5. C = C, inc j
6. D = D, inc j
7. D = D, inc j, j = len of pattern, sequence found

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Rabin Karp method:

$m = 10$, $n = 3$

{ A, B, C, D, E, F, G, H, I, J }

1 2 3 4 5 6 7 8 9 10

choosing prime = 13

$p = CDD$

$hash(p) = \Sigma(v * m^x) \% 13$ where v = character

$m = 10$

x = position

$\therefore hash(p) = (3 \times 10^2) + (4 \times 10^1) + (4 \times 10^0) \mod 13$

$= 344 \% 13$

$= 6$

Hash value for each pair of 3 (len of pattern)

A	B	C	C	D	D	A	E	F	G
6	12	9	6	$\cancel{6}$ 12	12	0	8		

$\Rightarrow$

↑ spurious hit (under A)

↑ spurious hit ↑ exact match (under the second C)

$\therefore$ Sequence found at position 4