1. Proof Sketches for RAID+'s Properties

Below are the major properties associated with MOLS-based normal and interim data layouts. Some of them can be easily derived from the definitions and theorems about MOLS. For the others, we show the proof sketches here.

Property 1. *With normal data layout, any two blocks within a data stripe are to be placed on separate disk drives.*

Analysis. We inspect any arbitrary two blocks within a stripe, and assume that they are placed on Disks x and y respectively. According to the data distribution principle of normal data layout, x and y are the elements of two Latin squares L_a and L_b on the same location (i, j), where we have $a \neq b$ and $i > 0$.

From Theorem 2, we have $x = f_a(i, j) = a \times i + j$, and $y = f_b(i, j) = b \times i + j$. Further, we have $x - y = (a - b) \times i$. Since $a \neq b$ and $i > 0$, we have $x - y \neq 0$. In other words, we have $x \neq y$. That is, any two blocks within a stripe are not placed on the same disk drive. □

Property 2. *With normal data layout, the n disks are assigned equal shares of both data and parity blocks.*

Analysis. The ordinal numbers of disks, on which the k blocks are placed, come from the same locations of the k selected Latin squares. Since the first $k - 1$ blocks within a stripe are data blocks and the last block is parity block, the first $k - 1$ Latin squares are attached with the distribution of data blocks and the k^{th} Latin square is attached with the distribution of parity blocks.

Within any one of the k Latin squares, all elements except those in the first row are used for designing normal data layout. According to Definition 1, each number appears exactly once in each row of a Latin square. It is obvious that each number appears $n - 1$ times in the $n - 1$ rows. According to each of the first $k - 1$ selected Latin squares, RAID+ put $n - 1$ data blocks on each disk. According to the k^{th} selected Latin square, RAID+ put $n - 1$ parity blocks on each disk. Therefore, RAID+ places $(n - 1) \times (k - 1)$ data blocks on each disk, and $n - 1$ parity blocks on each disk. □

Property 3. *With the n-disk normal layout, all those blocks correlated to blocks on any given drive (i.e., blocks sharing stripes with blocks on this disk) are distributed evenly among the other disks.*

Analysis. For any one drive d ($d \in [0, n-1]$), all those blocks correlated to this drive must appear within a correlated stripe whose mapping tuple includes the element d. Let us first inspect the set S_i of all the correlated stripes, the i^{th} ($i \in [0, k - 1]$) element of their mapping tuples is d. According to the data distribution principle of normal data layout, the number of the correlated stripes is $n-1$ and the $n-1$ ds come from different locations of the i^{th} selected Lartin square.

Then, we examine what disks the $n-1$ blocks, represented by the j^{th} ($j \in [0, k - 1]$ and $j \neq i$) elements of the $n-1$ mapping tuples in the set S_i, are placed on. According to Theorem 3, the $n - 1$ blocks are placed on all the $n - 1$ drives except Disk d. Further, by traversing i and j in turn, we will easily obtain that for Disk d, all its correlated blocks are distributed evenly among all the other drives. Furthermore, the number of the correlated blocks placed on each other disk is $k \times (k - 1)$. □

Property 4. *With the $(n - 1)$-disk interim layout, any two blocks within a data stripe are still to be placed on separate disk drives.*

This property can be obtained easily similar to the analysis of *Property 1*.

Property 5. *All the $(n-1)k$ missing blocks on any single failed disk can be redistributed to all the surviving $(n-1)$ disks evenly, each receiving k additional blocks.*

Analysis. k selected Latin squares are used to construct normal data layout, and the $(k + 1)^{th}$ Latin square is selected from the left $n - k - 1$ orthogonal Latin squares for redistributing all the blocks on any one drive d ($d \in [0, n - 1]$) onto all the other drives. All those blocks on this drive must appear within a stripe whose mapping tuple includes the element d.

Let us first inspect the set S_i of all the stripes, the i^{th} ($i \in [0, k - 1]$) element of their mapping tuples is d. According to the data distribution principle of normal data layout, the number of the stripes in S_i is $n - 1$ and the $n - 1$ ds come from different rows of the i^{th} selected Latin square. The j^{th} ($j \in [1, n-1]$) one of the $n-1$ stripes can be represented by the tuple $(a_0, a_1, ..., a_i = d, ..., a_{k-1})$. We assume that θ is in the $(k+1)^{th}$ Latin square, and has the same location with a_i in the i^{th} Latin square. Then, the data block corresponding to this stripe and on Disk d can be placed on Disk θ. The new tuple for this stripe changes into $(a_0, a_1, ..., a_{i-1}, \theta, a_{i+1}, ..., a_{k-1})$. According to Definition 3, all these elements remain different. In other words, no two blocks within a stripe are placed on the same drive.

According to Theorem 3, when we traverse j from 1 to $n - 1$, the corresponding $n - 1$ blocks are placed on all the $n - 1$ drives other than Disk d. When we traverse i from 0 to $k - 1$, each one of the other disks hold k blocks from Disk d. □

Property 6. *In a RAID+ system tolerating dual-disk failures, once two disks fail simultaneously, the ratio, between the numbers of the stripes with two lost blocks and those with one lost block, is $(k-1) : 2 \times (n-k)$.*

Analysis. Let us inspect a data template in normal data layout. According to Property 3, $k \times (k-1)$ of the blocks correlated to a given disk are placed on another disk. Therefore, once two disks fail simultaneously, the number of the stripes with two lost blocks is $k \times (k-1)$. Within a data template, there is $(n-1) \times k$ blocks on each disk. So, a disk failure results in $(n-1) \times k$ degraded stripes. Except $k \times (k-1)$ stripes with two lost blocks, there is $(n-k) \times k$ stripes with one lost block. Considering that two disks fail simultaneously, there is $2 \times (n-k) \times k$ stripes with one lost block in total. As a result, the ratio between them is $(k-1) : 2 \times (n-k)$. $\square$