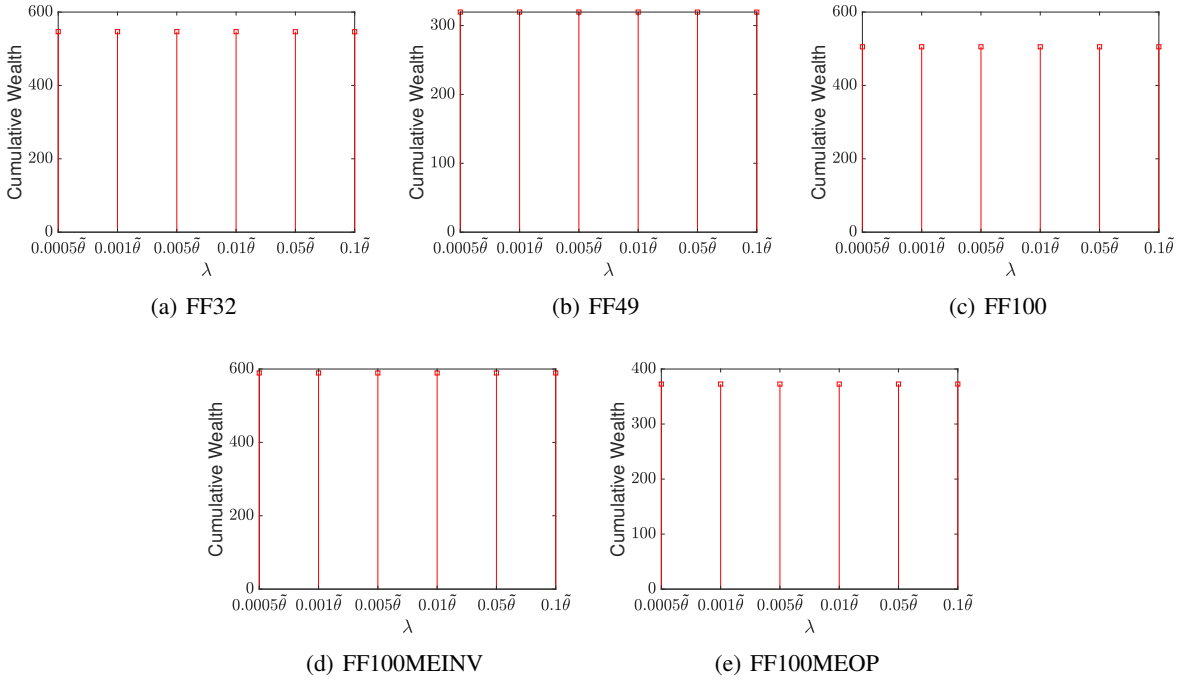
A. Three supplemental experiments for portfolio selection

We conducted three supplementary experiments for portfolio selection, including hyperparameter sensitivity analysis, hyperparameter selection cross-validation, and utilization of parameter entropy in utility maximization.

As demonstrated in Section 4.3, there are several hyperparameters in SSFS model for portfolio selection, including the parameter entropy hyperparameter κ, the ℓ_1 regularization weights $\boldsymbol{\lambda}$ and the hyperparameter γ in the matrix $\boldsymbol{A}$ in equation (11). In our experiment, we set $\kappa = \frac{p+n}{2}$ to balance with the quadratic loss, fix $\gamma = 10^{-4}$ and set $\boldsymbol{\lambda} = 0.001\tilde{\boldsymbol{\theta}}$. Now we test the sensitivity of the $\boldsymbol{\lambda}$.

A.1. The hyperparameter sensitivity

To assess the sensitivity of the hyperparameters $\boldsymbol{\lambda}$, we varied $\boldsymbol{\lambda}$ over the range $[0.0005\tilde{\boldsymbol{\theta}}, 0.001\tilde{\boldsymbol{\theta}}, 0.005\tilde{\boldsymbol{\theta}}, 0.01\tilde{\boldsymbol{\theta}}, 0.05\tilde{\boldsymbol{\theta}}, 0.1\tilde{\boldsymbol{\theta}}]$, while keeping the values of other hyperparameters constant. The results depicted in Figure 1 demonstrate that the SSFS method produces consistently stable outcomes across a wide range of $\boldsymbol{\lambda}$ values, suggesting that the remarkable performance of the SSFS method may not be highly sensitive to the choice of hyperparameters.



(a) FF32 (b) FF49 (c) FF100

(d) FF100MEINV (e) FF100MEOP

Figure 1: Final CWs of SSFS with respect to $\boldsymbol{\lambda}$ on 5 benchmark datasets.

A.2. Cross-Validation for Hyperparameter Selection

In this section, we employ a 5-fold cross-validation method to select the optimal hyperparameters $\boldsymbol{\lambda}$ in the SSFS model for portfolio selection. We generate a range of hyperparameters $\boldsymbol{\lambda}$ from $0.01\tilde{\boldsymbol{\theta}}$ to $0.5\tilde{\boldsymbol{\theta}}$ with a step size of $0.025\tilde{\boldsymbol{\theta}}$, resulting in a total of 20 values. Table 1 presents the CW and MER results for the competitors. Similar to SSFS with the default $\boldsymbol{\lambda}$ setting, SSFS with $\boldsymbol{\lambda}$ chosen by cross-validation outperforms all other competitors across all datasets. Table 2 presents the SR and IR results for the competitors. SSFS-cross-validation outperforms each competitor in at least 7 out of 10 total cases. These results indicate that SSFS with $\boldsymbol{\lambda}$ chosen by cross-validation remains effective for portfolio selection.

Table 1: Cumulative wealth and mean excess return results for portfolio selection.

Method	FF32		FF49		FF100		FF100MEINV		FF100MEOP	
	CW	MER	CW	MER	CW	MER	CW	MER	CW	MER
$1/p$	424.42	N/A	235.48	N/A	364.87	N/A	428.70	N/A	348.70	N/A
SSMP	158.98	-0.0004	186.79	0.0004	10.09	-0.0044	154.27	-0.0002	26.06	-0.0028
SPOLC	49.34	-0.0029	11.48	-0.0026	13.96	-0.0040	44.49	-0.0029	8.14	-0.0053
SSPO	10.63	-0.0048	0.77	-0.0048	1.04	-0.0066	52.87	-0.0019	4.19	-0.0055
S1	12.93	-0.0045	0.67	-0.0050	1.87	-0.0057	54.00	-0.0020	4.65	-0.0052
S2	47.85	-0.0025	0.96	-0.0048	2.86	-0.0053	46.16	-0.0024	7.27	-0.0047
S3	37.61	-0.0029	0.90	-0.0051	1.81	-0.0061	33.83	-0.0030	6.59	-0.0048
e-value	439.06	0.0001	215.57	-0.0001	392.59	0.0001	429.21	0.0000	366.33	0.0001
SAA-EUM-log	311.39	-0.0008	176.22	-0.0008	143.56	-0.0018	305.65	-0.0008	243.34	-0.0010
SAA-EUM-exp	306.99	-0.0008	162.24	-0.0008	98.70	-0.0024	321.70	-0.0007	238.85	-0.0010
SSFS	**546.57**	**0.0004**	**319.59**	**0.0005**	**505.32**	**0.0005**	**589.31**	**0.0005**	**372.65**	**0.0001**
SSFS-cross-validation	**546.54**	**0.0004**	**319.57**	**0.0005**	**505.31**	**0.0005**	**589.30**	**0.0005**	**372.66**	**0.0001**

Table 2: Sharpe ratio and information ratio results (monthly) for portfolio selection.

Method	FF32		FF49		FF100		FF100MEINV		FF100MEOP	
	SR	IR	SR	IR	SR	IR	SR	IR	SR	IR
$1/p$	0.2234	N/A	0.2057	N/A	0.2087	N/A	0.2151	N/A	0.2075	N/A
SSMP	0.1535	-0.0112	0.1658	0.0086	0.0883	-0.1027	0.1448	-0.0037	0.1084	-0.0673
SPOLC	0.1373	-0.0958	0.0898	-0.0404	0.0946	-0.0864	0.1276	-0.0800	0.0860	-0.1291
SSPO	0.0906	-0.1284	0.0501	-0.0557	0.0471	-0.1076	0.1239	-0.0456	0.0700	-0.1101
S1	0.0955	-0.1260	0.0480	-0.0587	0.0572	-0.0940	0.1254	-0.0496	0.0724	-0.1006
S2	0.1275	-0.0809	0.0516	-0.0598	0.0637	-0.0960	0.1233	-0.0659	0.0782	-0.0962
S3	0.1220	-0.0969	0.0498	-0.0642	0.0552	-0.1131	0.1169	-0.0858	0.0790	-0.0988
e-value	0.2236	0.0290	0.2027	-0.0419	0.2096	0.0270	0.2125	0.0085	0.2068	**0.0185**
SAA-EUM-log	0.2345	-0.0383	**0.2168**	-0.0270	0.1993	-0.0772	0.2240	-0.0389	**0.2200**	-0.0440
SAA-EUM-exp	0.2328	-0.0380	0.2103	-0.0295	0.1832	-0.0904	0.2238	-0.0315	0.2171	-0.0401
SSFS	**0.2347**	**0.0668**	0.2133	**0.0380**	**0.2223**	**0.0513**	**0.2263**	**0.0635**	0.2101	0.0120
SSFS-cross-validation	**0.2347**	**0.0668**	0.2133	**0.0380**	**0.2223**	**0.0513**	**0.2263**	**0.0635**	0.2101	0.0120

A.3. Utilization of parameter entropy in utility maximization

In this section, we replace the quadratic loss with a utility function to test the effectiveness of the parameter entropy in the utility maximization model. The model is formulated as follows:

$$\hat{\boldsymbol{\theta}} = \text{argmin}_{\boldsymbol{\theta}\in\Delta_p} \ -\frac{1}{n}\sum_{t=1}^{n} U(\boldsymbol{\theta}^\top \boldsymbol{x}_t) + \kappa \mathcal{W}_{\hat{\boldsymbol{\theta}}}(\boldsymbol{\theta}) + \boldsymbol{\lambda}^\top \boldsymbol{\theta}, \tag{1}$$

where U denotes a utility function. In this experiment, we specify the utility function as the exponential utility or the logarithm utility, denoted by SSFS-exp and SSFS-log, respectively. Table 1 presents the CW and MER results for the competitors. As shown in Table 1, both SSFS-exp and SSFS-log outperform all other competitors across all datasets. Table 2 presents the SR and IR results for the competitors. Both SSFS-exp and SSFS-log outperform each competitor in at least 7 out of 10 total cases. These results indicate that the proposed parameter entropy remains effective in the utility maximization model, demonstrating its wide application range.

Table 3: Cumulative wealth and mean excess return results for portfolio selection.

Method	FF32		FF49		FF100		FF100MEINV		FF100MEOP	
	CW	MER	CW	MER	CW	MER	CW	MER	CW	MER
$1/p$	424.42	N/A	235.48	N/A	364.87	N/A	428.70	N/A	348.70	N/A
SSMP	158.98	-0.0004	186.79	0.0004	10.09	-0.0044	154.27	-0.0002	26.06	-0.0028
SPOLC	49.34	-0.0029	11.48	-0.0026	13.96	-0.0040	44.49	-0.0029	8.14	-0.0053
SSPO	10.63	-0.0048	0.77	-0.0048	1.04	-0.0066	52.87	-0.0019	4.19	-0.0055
S1	12.93	-0.0045	0.67	-0.0050	1.87	-0.0057	54.00	-0.0020	4.65	-0.0052
S2	47.85	-0.0025	0.96	-0.0048	2.86	-0.0053	46.16	-0.0024	7.27	-0.0047
S3	37.61	-0.0029	0.90	-0.0051	1.81	-0.0061	33.83	-0.0030	6.59	-0.0048
e-value	439.06	0.0001	215.57	-0.0001	392.59	0.0001	429.21	0.0000	366.33	0.0001
SAA-EUM-log	311.39	-0.0008	176.22	-0.0008	143.56	-0.0018	305.65	-0.0008	243.34	-0.0010
SAA-EUM-exp	306.99	-0.0008	162.24	-0.0008	98.70	-0.0024	321.70	-0.0007	238.85	-0.0010
SSFS	**546.57**	**0.0004**	**319.59**	**0.0005**	**505.32**	**0.0005**	**589.31**	**0.0005**	**372.65**	**0.0001**
SSFS-EXP	**496.68**	**0.0002**	**301.14**	**0.0004**	**487.54**	**0.0004**	**590.23**	**0.0005**	**390.21**	**0.0002**
SSFS-LOG	**495.07**	**0.0002**	**300.25**	**0.0004**	**486.56**	**0.0004**	**588.51**	**0.0005**	**390.46**	**0.0002**

Table 4: Sharpe ratio and information ratio results (monthly) for portfolio selection.

Method	FF32		FF49		FF100		FF100MEINV		FF100MEOP	
	SR	IR	SR	IR	SR	IR	SR	IR	SR	IR
$1/p$	0.2234	N/A	0.2057	N/A	0.2087	N/A	0.2151	N/A	0.2075	N/A
SSMP	0.1535	-0.0112	0.1658	0.0086	0.0883	-0.1027	0.1448	-0.0037	0.1084	-0.0673
SPOLC	0.1373	-0.0958	0.0898	-0.0404	0.0946	-0.0864	0.1276	-0.0800	0.0860	-0.1291
SSPO	0.0906	-0.1284	0.0501	-0.0557	0.0471	-0.1076	0.1239	-0.0456	0.0700	-0.1101
S1	0.0955	-0.1260	0.0480	-0.0587	0.0572	-0.0940	0.1254	-0.0496	0.0724	-0.1006
S2	0.1275	-0.0809	0.0516	-0.0598	0.0637	-0.0960	0.1233	-0.0659	0.0782	-0.0962
S3	0.1220	-0.0969	0.0498	-0.0642	0.0552	-0.1131	0.1169	-0.0858	0.0790	-0.0988
e-value	0.2236	0.0290	0.2027	-0.0419	0.2096	0.0270	0.2125	0.0085	0.2068	**0.0185**
SAA-EUM-log	0.2345	-0.0383	**0.2168**	-0.0270	0.1993	-0.0772	0.2240	-0.0389	**0.2200**	-0.0440
SAA-EUM-exp	0.2328	-0.0380	0.2103	-0.0295	0.1832	-0.0904	0.2238	-0.0315	0.2171	-0.0401
SSFS	**0.2347**	**0.0668**	0.2133	**0.0380**	**0.2223**	**0.0513**	**0.2263**	**0.0635**	0.2101	0.0120
SSFS-EXP	0.2295	**0.0779**	0.2129	**0.0445**	**0.2195**	**0.0576**	**0.2252**	**0.0821**	0.2108	**0.0281**
SSFS-LOG	0.2294	**0.0773**	0.2129	**0.0443**	**0.2195**	**0.0579**	**0.2252**	**0.0824**	0.2109	**0.0285**