Supplementary material of Variance reduction in online A/B test by Inference on Ex-post Residual

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1 Derivation of the variance gap in Section 4.2

We derive the variance gap between estimators and the optimal estimator in the Section 4.2 of the main paper. In this section, denote the adjusting function $\hat{f} = \frac{N_c}{N}\hat{f}_T + \frac{N_t}{N}\hat{f}_C$ and suppose it is independent of the samples to be used to estimate the ATE. Denote $\frac{N_t}{N}$ as p and the variance lower bound (4) of the main paper as $Var(\Delta_{\text{opt}})$. Denote $\hat{\Delta}$ as the estimator which with the adjusting function $\hat{f}$, i.e.,

$$\hat{\Delta} = \frac{1}{N_t}\sum_{i\in T}\big(Y_i - \hat{f}(X_i)\big) - \frac{1}{N_c}\sum_{i\in C}\big(Y_i - \hat{f}(X_i)\big). \tag{1}$$

Then the variance of $\hat{\Delta}$ is

$$\begin{aligned} Var(\hat{\Delta}) =& \frac{1}{N_t}Var\big(f_T(X)+\varepsilon_T-\hat{f}(X)\big)+\frac{1}{N_c}Var\big(f_C(X)+\varepsilon_C-\hat{f}(X)\big)\\ =& \frac{1}{N}\frac{1}{p}Var\big(f_T(X)-\hat{f}(X)\big)+\frac{1}{N}\frac{1}{1-p}Var\big(f_C(X)-\hat{f}(X)\big)+\frac{1}{N_t}\sigma_T^2+\frac{1}{N_c}\sigma_C^2\ , \end{aligned} \tag{2}$$

Denote $f_{\text{opt}} = \frac{N_c}{N}f_T + \frac{N_t}{N}f_C$ be the optimal adjusting function. Omit the notation X and plug in $\hat{f}$ into (2), we have

$$\begin{aligned} N\cdot Var(\hat{\Delta}) \;=\;& \frac{1}{p}Var\big(\,f_T-f_{\text{opt}}+f_{\text{opt}}-\hat{f}\,\big)+\frac{1}{1-p}Var\big(\,f_C-f_{\text{opt}}+f_{\text{opt}}-\hat{f}\,\big)+\frac{1}{p}\sigma_T^2+\frac{1}{1-p}\sigma_C^2\\ =\;& \frac{1}{p}Var\big(\,f_T-f_{\text{opt}}\,\big)+\frac{1}{p}Var\big(\,f_{\text{opt}}-\hat{f}\,\big)+\frac{2}{p}\mathrm{Cov}\big(\,f_T-f_{\text{opt}},f_{\text{opt}}-\hat{f}\,\big)\\ &+\frac{1}{1-p}Var\big(\,f_C-f_{\text{opt}}\,\big)+\frac{1}{1-p}Var\big(\,f_{\text{opt}}-\hat{f}\,\big)+\frac{2}{1-p}\mathrm{Cov}\big(\,f_C-f_{\text{opt}},f_{\text{opt}}-\hat{f}\,\big)\\ &+\frac{1}{p}\sigma_T^2+\frac{1}{1-p}\sigma_C^2\\ =\;& \frac{Var\big(\,f_{\text{opt}}-\hat{f}\,\big)}{p(1-p)}+\frac{1}{p}Var\big(\,f_T-f_{\text{opt}}\,\big)+\frac{1}{1-p}Var\big(\,f_C-f_{\text{opt}}\,\big)+\frac{1}{p}\sigma_T^2+\frac{1}{1-p}\sigma_C^2. \end{aligned} \tag{3}$$

The last equality follows from $\frac{1}{p}(f_T - f_{\text{opt}}) = -\frac{1}{1-p}(f_C - f_{\text{opt}})$ so the two covariances can be eliminated. Recall that the variance lower bound is

$$Var(\Delta_{\text{opt}}) = \frac{1}{N_t}Var\big(\,f_T-f_{\text{opt}}\,\big)+\frac{1}{N_c}Var\big(\,f_C-f_{\text{opt}}\,\big)+\frac{1}{N_t}\sigma_T^2+\frac{1}{N_c}\sigma_C^2, \tag{4}$$

Then the variance gap in (13) of the main paper follows from plugging (4) into (3).

2 Details of the experiments

2.1 Details of the time series model in Section 5.2

The AR(2) model used in Section 5.2 of the main paper is:

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \varepsilon_t, \ \ t \in \mathbb{Z},$$

where $\{\varepsilon_t\}$ is the white noise WN(0,0.4); $a_1 = 0.75, a_2 = -0.1$.

The GARCH(1,2) model used in Section 5.2 is:

$$X_t = \sigma_t \varepsilon_t,$$

$$\sigma_t^2 = a_0 + a_1 X_{t-1}^2 + b_1 \sigma_{t-1}^2 + b_2 \sigma_{t-2}^2, \ \ t \in \mathbb{Z},$$

where $\{\varepsilon_t\}$ is the white noise WN(0,1); $a_0 = 0.1, a_1 = 0.3, b_1 = 0.3, b_2 = 0.25$.

2.2 Description of the data generation process in Section 5.2

The simulation in Section 5.3 of the main paper uses a simplified form of the 'Vita Model' in [4] to generate data. When a user 'enters the app', we initialize two parameters for her: a loyalty parameter L describing how quickly she will leave the app, and an obsession parameter O describing how she behaves while active each day. For each user, we initialize loyalty L as $1 + \mathcal{E}(q)$, where $\mathcal{E}(q)$ represents an exponentially distributed random variable with parameter q, and q is a given parameter describing the quality of an app. Obsession O is initialized as an independent exponentially distributed random variable with parameter q. In addition, each user starts with a vita value of 1. On each subsequent day, update vita as follows:

$$\text{vita}^{(t+1)} = \text{vita}^{(t)} - I\big(\frac{1}{L} > \mathcal{U}(0,1)\big) \cdot \mathcal{U}(-0.2, 1.2) \cdot \text{vita}^{(t)}.$$

$\mathcal{U}(a,b)$ denotes the uniform distribution between a and b and $I(\cdot)$ is the indicator function. If vita goes less than 0, then we consider this user leaves the app and no future information will be generated. If vita goes larger that 1, replace it with 1. If $\text{vita}^{(t)} > 0$ on day t, we assume she will be active in this day with probability $\text{vita}^{(t)}$. If she is active on day t, we first generate an intermediate variable $h^{(t)} = O \cdot \sqrt{\text{vita}^{(t)}} \cdot \mathcal{E}(1)$. Then let $a^{(t)} = h^{(t)} \cdot \mathcal{N}(10,1)$ and $b^{(t)} = h^{(t)} \cdot \mathcal{N}(50,25)$ where $\mathcal{N}(\mu, \sigma^2)$ represents a normal random variable with mean μ and variance σ^2. All of the parameters are pre-selected to imitate the real data. $a^{(t)}$ and $b^{(t)}$ are the metrics we will use in simulation, which represent the the time spent viewing videos and the number of videos viewed of a user on day t. Starting from the first day in our simulation, 2000 new users enters the app each day and evolve independently as the process described above until the 366th day. We will impose a certain kind of treatment in the 339th day to influence the value of $a^{(t)}$ and $b^{(t)}$ in the following days. The average viewing time in the last 28 days, i.e., $Y = \frac{1}{28}\sum_{t=339}^{366} b^{(t)}$ is the metric of interest in the simulation. The data in the 28 days before the 339th day, i.e., $X = \big\{(a^{(t)}, b^{(t)}), 311 \le t \le 338\big\}$ are the 56-dimensional covariates used in ATE estimation algorithms. Note that we only consider those users who enter the app before the 311th day and have been active in at least one day in the following 56 days.

We consider 3 kinds of treatments in the simulation. For $339 \leq t \leq 366$, a certain part of the data generating process is changed for the treatment group:

- T1-treatment: $h^{(t)} = [\, O + \frac{1}{2}\mathcal{E}(1)\,] \cdot \sqrt{\text{vita}^{(t)}} \cdot \mathcal{E}(1)$
- T1-control: $h^{(t)} = \; O \cdot \sqrt{\text{vita}^{(t)}} \cdot \mathcal{E}(1)$

- T2-treatment: $b^{(t)} = h^{(t)} \cdot \mathcal{N}(50,25) + 0.1 \cdot h^{(t)} \cdot \mathcal{N}(50,25)$
- T2-control: $b^{(t)} = h^{(t)} \cdot \mathcal{N}(50,25)$

- T3-treatment: $\text{vita}^{(t)} = \text{vita}^{(t-1)} - I\big(\frac{1}{L} > \mathcal{U}(0,1)\big) \cdot \mathcal{U}(-0.2,1.2) \cdot \text{vita}^{(t-1)} - 0.02$
- T3-control: $\text{vita}^{(t)} = \text{vita}^{(t-1)} - I\big(\frac{1}{L} > \mathcal{U}(0,1)\big) \cdot \mathcal{U}(-0.2,1.2) \cdot \text{vita}^{(t-1)}$

The first two kinds of treatments aim to increase the viewing time of each user, and the third treatment aims to decrease users' engagement.

We generate data in the treatment group and the control group using the Vita Model for each kind of treatment. In addition, we also consider a fourth situation where no treatment is imposed on the users in the treatment group. Note that $(a^{(t)}, b^{(t)})$ is missing if the user is not active in this day. We supply the missing value as -99 if $311 \leq t \leq 338$ and supply it as 0 if $339 \leq t \leq 366$.

3 Supplementary simulation: CI coverage rate

This section aims to verify that the confidence interval generated by IER is close to the true confidence interval through a simple simulation. This section uses a similar data generation process as [1–3], because this is a simple example that the linear adjusting function does not work well. Instead of comparing the variance reduction effect, the main goal of this appendix is to show that the estimated confidence interval will cover the true ATE of the generated model with a close probability to the corresponding confidence level. We first generate 10000 i.i.d. samples $X_i \sim \mathcal{N}(0, I_{10})$, treatment indicators $T_i \sim \text{Bern}(0.5)$ which are independent of X_i. The independent random error term is $\varepsilon_i \sim \mathcal{N}(0,1)$ and independent of other random variables. Then we generate the outcome variables as follows,

$$Y_i = b(X_i) + T_i \cdot \tau(X_i) + \varepsilon_i \cdot 5X_{i,1}^2, \;\; i = 1,2,\cdots,N,$$

where $b(x) = 10\sin(\pi \cdot x_1 x_2) + 20(x_3 - 0.5)^2 + 10x_4 + 5x_6$ and $\tau(x) = 10x_1 + 5\log(1 + \exp x_1)$, $x = (x_1, \cdots, x_{10})$.

We use Monte Carlo integration to count the expectation of $\tau(x)$ and find that the true ATE is around 4.031. We then use IER to obtain a 0.95-confidence interval and test whether it covers 4.031. Repeating this procedure 200 times and 4.031 falls into the confidence interval 188 times. This shows that the empirical coverage rate is close to the confidence level we choose. So IER is valid for hypothesis testing in this simulation. For variance reduction effect, using the same LSTM structure as Section 5.3 to obtain the adjusting function, the average sample variance of 200 experiments is 12.1% of the D-i-M estimator while using the linear adjusting function is 62.6%. The results using GBDT or elastic net in [2, 3] are similar with using LSTM. This simulation also shows that the LSTM-based method is also suitable for the non-time series data. Note that this simple simulation aims only to verify the coverage rate, so we did not carefully design the LSTM structure or adjust the hyperparameters. Nevertheless, our method still has competitive results compared with other methods in this simulation.

References

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