University of Ruhuna- Faculty of Technology

Bachelor of Information & Communication Technology

Level I (Semester II) Examination – December 2017
Course Unit: TMS 1233 Discrete Mathematics
Time Allowed 3 Hours

Write answers for all six (06) questions. TG / 2016 / 109

All symbols have their usual meaning.

1. 1.1 Every nonempty set S is guaranteed to have at least two subsets, the empty set and the set S itself, that is, $\emptyset \subseteq S$ and $S \subseteq S$.

   Prove that for every set S, (a) $\emptyset \subseteq S$ and
   (b) $S \subseteq S$.

   1.2 List the set E of even integers between 50 and 63. Write the set E, using the set builder notation.

   1.3 For each of the following pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

   $A \subseteq B$. $B \subseteq A$

   a) the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi.
   b) the set of people who speak English, the set of people who speak Chinese.
   c) the set of flying squirrels, the set of living creatures that can fly.

   A B .

   1.4 Show the following set using the Venn diagram. $A \subseteq B$.

   $$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

   1.5 Find the cardinality of each of these sets? A.

   a) $\{x\}$ b) $\{\{x\}\}$ c) $\{x,\{x\}\}$ d) $\{x,\{x\},\{x,\{x\}\}\}$

   1.6 Find the power set of each of the following sets, where a and b are distinct elements.

   a) $\{x\}$ b) $\{x,y\}$ c) $\{\emptyset,\{\emptyset\}\}$

2. 2.1 Prove the following Second De Morgan Law using Set-Builder Notation.

   $$\overline{A \wedge B} \subseteq \bar{A} \vee \bar{B}$$

   2.2 Use a membership table to show that $A\cap(B \cup C) = (A\cap B)\cup(A\cap C)$.

2.3 What are the truth sets of the predicates P(x) , Q(x), and R(x), where the domain is the set of integers and P(x) is "|x|=1," Q(x) is "x^2 =2," and R(x) is "|x|=x."

2.5 a) Briefly explain what is mean by one-to-one and onto functions.

b) Let f be the function from $\{u, x, y, z\}$ to $\{1,2,3\}$ defined by $f(u) = 3$, $f(x) = 2$, $f(y) = 1$, and $f(z) = 3$. Is f an onto function?

c) Is the function $f(y) = y^2$ from the set of integers to the set of integers onto?

d) Determine whether the function f(x) = x +1 from the set of real numbers to the set of real numbers is

(i) one-to one or (ii) onto.

2.6 Show that the set of odd positive integers is a countable set.

3. 3.1 Explain the difference between sentence and proposition.

3.2 Suppose that during the most recent fiscal year, the annual revenue of ACT Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of NDA Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of QUI Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) QUI Media had the largest annual revenue.

b) NDA Software had the lowest net profit and ACT Computer had the largest annual revenue.

c) ACT Computer had the largest net profit or QUI Media had the largest net profit.

d) If QUI Media had the smallest net profit, then ACT Computer had the largest annual revenue.

e) NDA Software had the smallest net profit if and only if ACT Computer had the largest annual revenue.

3.3 Let p and q be the propositions "Swimming at the Nilaweli beach is allowed" and "Sharks have been spotted near the shore," respectively. Express each of the following compound propositions as an English sentence.

a) $\neg p \lor q$ b) $\neg q \rightarrow p$ c) $\neg p \rightarrow \neg q$ d) $p \leftrightarrow \neg q$

e) $\neg p \land (p \lor \neg q)$

3.4 Let p and q be the propositions given by

p :You drive over 65 miles per hour.

q :You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

a) You drive over 65 miles per hour, but you do not get a speeding ticket. $p \wedge \neg q$

b) You will get a speeding ticket if you drive over 65 miles per hour. $p \to q$

c) If you do not drive over 65 miles per hour, then you will not get a speeding ticket. $\neg p \to$

d) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

e) You get a speeding ticket, but you do not drive over 65 miles per hour.

$q \wedge \neg p$

3.5 Construct a truth table for each of the following compound propositions.

a) $(p \vee \neg q) \to q$

b) $(p \vee q) \to (p \wedge q)$

c) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

4. 4.1 How can this English sentence be translated into a logical expression? "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

4.2 Express following specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).

a) "The message is scanned for viruses whenever the message was sent from an unknown system."

b) "The message was sent from an unknown system but it was not scanned for viruses."

c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."

d) "When a message is not sent from an unknown system it is not scanned for viruses."

4.3 Determine whether these system specifications are consistent:

"The diagnostic message is stored in the buffer or it is retransmitted."

"The diagnostic message is not stored in the buffer."

"If the diagnostic message is stored in the buffer, then it is retransmitted."

4.4 Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.

4.5 Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

5. 5.1 Explain the need of the Predicate logic instead of Propositional Logic.

5.2 a) Let Q(x,y) denote the statement "$x = y + 3$." What are the truth values of the propositions Q(1,2) and Q(3,0)?

b) Let A(c,n) denote the statement "Computer c is connected to network n." where c is a variable representing a computer and n is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What are the values of A(MATH1, CAMPUS1) and A(MATH1, CAMPUS2)?

5.3 a) What is the truth value of $\forall x P(x)$, where P(x) is the statement "$x^2 < 20$" and the domain consists of the positive integers not exceeding 5?
b) What is the truth value of $\exists x P(x)$, where P(x) is the statement "$x^2 > 20$" and the universe of discourse consists of the positive integers not exceeding 5?

5.4 a) The De Morgan's Laws for Quantifiers can be expressed as

1. $\neg\exists x\, Q(x) \equiv \forall x \neg Q(x)$
2. $\neg\forall x P(x) \equiv \exists x \neg P(x)$

Prove the above 2nd law quantifiers in for any propositional functions (P(x)) and for any domain.

b) Use the above De Morgan's Laws to find the negations of the following statements.

i) $\forall x(x^2 > x)$
ii) $\exists x(x^2 = 3)$

5.5 Express the following statements using predicates and quantifiers.
a) "Some student in this class has visited Jaffna"
b) "Every student in this class has visited either Colombo or Jaffna"

5.6 Let Q(x,y) denote "$x + y = 0$."What are the truth values of the quantifications $\exists y \forall x Q(x,y)$ and $\forall x \exists y Q(x,y)$, where the domain for all variables consists of all real numbers?

6. 6.1 Find the value of following Boolean function and translate into a logical equivalence.

$$1.0 + \overline{(0+1)}$$

6.2 a) Use a table to express the values of each of the following Boolean functions.
b) Find the sum-of-product expansion of each of the following Boolean functions.

$a)\ F(x,y,z) = \bar{x}\,y$

$b)\ F(x,y,z) = x + yz$

$c)\ F(x,y,z) = x\bar{y} + \overline{(xyz)}$

$d)\ F(x,y,z) = x\,(yz + \bar{y}\,\bar{z})$

6.3 Show that

$a)\ \bar{x} = x \downarrow x$

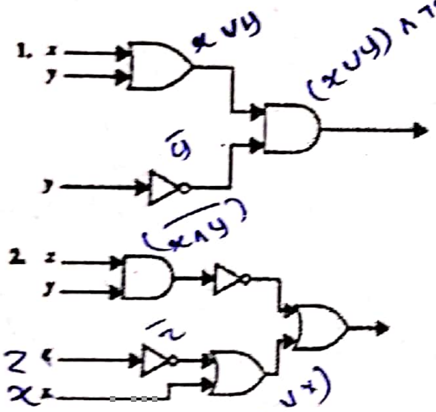
$b)\ xy = (x \downarrow x) \downarrow (y \downarrow y)$

$c)\ x + y = (x \downarrow y) \downarrow (x \downarrow y)$

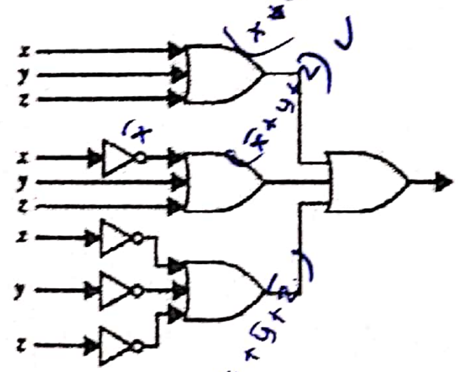
6.4 Find the output of the following given circuit.

1.

2.



3.



6.5 Draw the K-maps of the following sum-of-products expansions in two variables and simplify the sum-of-products expansion in part (c).

$a)\ x\bar{y}$ $\qquad$ $b)\ xy + x\bar{y} + \bar{x}y + \bar{x}\,\bar{y}$

$c)\ xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}\bar{z}$