TG/2017/265

University of Ruhuna- Faculty of Technology

Bachelor of Information & Communication Technology

Level I (Semester II) Examination – April 2019
Course Unit: TMS 1233 Discrete Mathematics

Write answers for all five (05) questions. **Time Allowed 3 Hours**

All symbols have their usual meaning.

1. 1.1 Draw the Venn Diagram for following sets given in set builder notation.
   a) $\{x|x \in A \wedge x \in B\}$
   b) $\{x|x \in \text{A} \wedge x \notin \text{B}\}$

   1.2 List the set "S" of even integers between 30 and 60. Express the set S using;
   a) set builder notation
   b) interval notation.

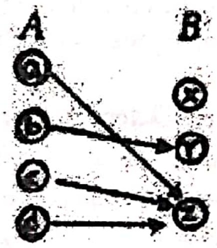
   1.3 Write down the cartesian products ($A \times B \times$ C), where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$.

   1.4 Let $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8\}$ be two subsets of the universal set $U = \{0,1,2,3,4,5,6,7,8,9,10\}$. Find the *symmetric difference* of A and B ($A \oplus B$) by drawing Venn diagram.

   1.5 State clearly and prove second De Morgan Law using set-builder notation.

   1.6 Consider the two sets A and B, and the function $f: A \rightarrow B$ given below. Find

   i. $f(b)$
   ii. The image of d
   iii. The domain of f
   iv. The codomain of f
   v. The preimage of y
   vi. $f(A)$
   vii. The preimage(s) of z



   1.7 Let f be the function from $\{a,b,c,d\}$ to $\{1,2,3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an onto function? Explain your answer.

   1.8 Let f be the function from $\{a,b,c\}$ to $\{1,2,3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible and if so what is its inverse?

2. 2.1 Write-down the corresponding truth values (T or F) of the expressions given below.

P	Q	Expression	Truth Value
T	T	$P \vee Q$	
T	F	$P \wedge \neg Q$	
F	T	$P \rightarrow Q$	
F	F	$\neg P \leftrightarrow Q$	

2.2 a) Prove the following Second De Morgan Law using the truth table;

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

b) Use De Morgan's Laws to write the negation of the following expression, and translate the negation in English

" Kamal is Mathematics major but not ICT major ".

2.3 Translate the following sentences into propositional expressions:

P = "Ms. Kate is healthy"
Q = "Ms. Kate is wealthy"
R = "Ms. Kate is wise"

$T \Leftrightarrow F$

a) Ms. Kate is healthy and wealthy but not wise,
b) Ms. Kate is not wealthy but he is healthy and wise,
c) Ms. Kate is neither healthy nor wealthy nor wise,
d) If Ms. Kate is healthy then she is not wealthy or not wise.

2.4 Show that $\neg(p \vee (\neg p \wedge q)$ is logically equivalent to $\neg p \wedge \neg q$ by developing a series of logical equivalences.

2.5 Determine the satisfiability of the following compound propositions and identify the truth values for each parameter p, q and r:

a) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$,

b) $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$,

c) $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$.

3. 3.1 Let P(x) denotes the statement " $x > 5$". What are the truth values of P(6) and P(3) ?

3.2 State clearly and prove the De Morgan's Laws for Quantifiers.

3.3 Translate the following sentences in quantified expressions of predicate logic, write down the negated expression and then translate the negated expression in English. The predicates to be used are given in parentheses;

a) All students that study discrete math are good at logic, (student(x), study_discrete_math(x), good_at_logic(x))

b) International students are not eligible for government loans, (international_student(x), eligible(x))

3.4 Translate the following expression into English statement

$$\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0)).$$

3.5 Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry,
b) Everybody loves somebody,
c) There is somebody whom everybody loves,
d) Nobody loves everybody,
e) Everyone loves himself or herself.

4. 4.1 Explain the rule of inference called modus ponens.

4.2 Determine whether the following arguments are valid or invalid by clearly explaining the reason with corresponding rule of inference.

a) **Premises:**
i) If Priya don't study hard, he will not pass this course
ii) If Priya don't pass this course he cannot graduate this year.
Conclusion: If Priya don't study hard, Priya won't graduate this year.

b) **Premises:**
i) If I read the newspaper in the kitchen, my glasses would be on the kitchen table.
ii) I did not read the newspaper in the kitchen.
Conclusion: My glasses are not on the kitchen table.

c) **Premises:**

i) If it rains today, then we will not have a barbecue today.

ii) If we do not have a barbecue today, then we will have a barbecue tomorrow.

Conclusion: Therefore, if it rains today, then we will have a barbecue tomorrow.

4.3 Prove that for an integer n, if n^2 is odd, then n is odd by using the contraposition.

4.4 a) Find the *floor* function and *ceiling* function of the following numbers

i. $\lfloor 4.5 \rfloor$

ii. $\lceil 1.5 \rceil$

iii. $\lceil 3.5 \rceil$

iv. $\lceil -1.5 \rceil$

v. $\lfloor -1.5 \rfloor$

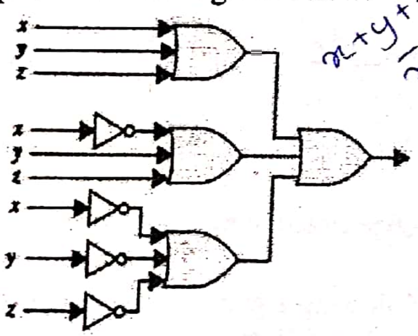
b) Prove that x is a real number, then

$$\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$$

5. 5.1 Construct logic circuits that produce the following output:

$$(x+y+z)(\bar{x}\,\bar{y}\,\bar{z})$$

5.2 Find the output of the circuit given below:



$x+y+z$

$\bar{x}+y+z$

$\bar{x}+\bar{y}+\bar{z}$

5.3 a) Explain the concept of functionally completeness operators.

b) Express each of the following Boolean functions using the operators “+” and “•”;

i. $x+y+z$

ii. $x+\bar{y}(\bar{x}+z)$

iii. $\overline{x+\bar{y}}$

iv. $\bar{x}(x+\bar{y}+\bar{z})$

c) Show that the set of operators $\{+, \cdot\}$ is not functionally complete.

5.4 Prove the absorption law $x(x+y) = x$ using the sequences of identities of Boolean algebra.

5.5 Following truth table 1 is an output a of display unit. Answer the following questions using the table 1.

i) Form the sum of the minterms corresponding to each row "a" of the table1.
ii) Draw the K-maps of the following truth table 1 and find the corresponding function.
iii) Find the corresponding logic gate circuit for the function in (ii).

A	B	C	D	a	
0	0	0	0	1	$\bar{A}\bar{B}\bar{C}\bar{D}$ ✓
0	0	0	1	0	
0	0	1	0	1	$\bar{A}\bar{B}C\bar{D}$ ✓
0	0	1	1	1	$\bar{A}\bar{B}CD$ ✓
0	1	0	0	0	
0	1	0	1	1	$\bar{A}B\bar{C}D$ ✓
0	1	1	0	1	$\bar{A}BC\bar{D}$ ✓
0	1	1	1	1	$\bar{A}BCD$ ✓
1	0	0	0	1	$A\bar{B}\bar{C}\bar{D}$ ✓
1	0	0	1	1	$A\bar{B}\bar{C}D$ ✓
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	

Table 1