The Rabbit Apocalypse

If you're reading this, it means that the Rabbits have taken over.

There is hope. The Final Solution to the rabbit scourge is almost complete. But in order to deploy it effectively, we must first know the number of rabbits that have proliferated thus far.

Thanks to the work of Leonardo Fibonacci, we know that it is possible to calculate the number of rabbits based on their litter size and the number of months since the proliferation began. The following program will help you determine this.

This program should be compatible with any computer grade 8 or above.

Provide these instructions to the interpreter along with a slide rule and a pencil. There should be a slide rule kit attached to this letter. Follow the instructions to assemble it.

Prelude

import function **sum** from grade_school, aliased as (**+**)
import function **subtract** from grade_school, aliased as (**-**)
import function **round** from grade_school
import function **multiply** from slide_rule, aliased as (*****)
import function **divide** from slide_rule, aliased as (**/**)
import function **exp** from slide_rule, aliased as (**^**)
import function **sqrt** from slide_rule

Instructions

given ***n***, the number of months since the proliferation began
given ***k***, the number of rabbit pairs per litter

let **f** be defined as **4 * k**
let **h** be defined as **f + 1**
let **g** be defined as **sqrt**(**h**)
let **a** be defined as **1 + g**
let **b** be defined as **1 - g**
let **c** be defined as **a ^ n**
let **d** be defined as **b ^ n**
let **e** be defined as **c - d**
let **m** be defined as **2 ^ n**

```
let p be defined as m * g
let r be defined as e / p
return round(r)
```

Example

```
given n = 5
given k = 3

let f be defined as 4 * k = 4 * 3 = 12
let h be defined as f + 1 = 12 + 1 = 13
let g be defined as sqrt(h) = sqrt(13) = 3.6056
let a be defined as 1 + g = 1 + 3.6056 = 4.6056
let b be defined as 1 - g = 1 - 3.6056 = -2.6056
let c be defined as a ^ n = 4.6056 ^ 5 = 2072.2
let d be defined as b ^ n = -2.6056 ^ 5 = -120.10
let e be defined as c - d = 2072.1 - -120.10 = 2192.3
let m be defined as 2 ^ n = 2 ^ 5 = 32
let p be defined as m * g = 32 * 3.6056 = 115.38
let r be defined as e / p = 2192.3 / 115.38 = 19.001
return round(r) = 19
```

Shortcut

If time is of the essence, this shorter method may be employed. It will be off by a small margin, but will do in a pinch. Hopefully you can take care of any remaining rabbits by other means. godspeed.

```
given n, the number of months since the proliferation began
given k, the number of rabbit pairs per litter

let f be defined as 4 * k
let h be defined as f + 1
let g be defined as sqrt(h)
let a be defined as 1 + g
let p be defined as a / 2
let q be defined as p ^ n
let r be defined as q / g
return round(r)
```

Shortcut Example

```
given n = 12
given k = 3
```

let **f** be defined as **4 * k** = 4 * 3 = **12**
let **h** be defined as **f + 1** = 12 + 1 = **13**
let **g** be defined as **sqrt**(**h**) = sqrt(13) = **3.6056**
let **a** be defined as **1 + g** = 1 + 3.6056 = **4.6056**
let **p** be defined as **a / 2** = 4.6056 / 2 = **2.3028**
let **q** be defined as **p ^ n** = 2.3028 ^ 12 = **22236**
let **r** be defined as **q / g** = 22236 / 3.6056 = **6167.1**
return round(**r**) = **6167**

12 months after the proliferation began, the rabbits were still countable. There were 6160 rabbits counted, so this estimate would still have us safely within a 0.1% margin of error.

Derivation

The formula employed by this by program is a closed-form solution which relies on the fact that the rabbit's reproduction may be defined by a linear recurrence with constant coefficients.
There is a proof, but unfortunately this margin is too small to contain it. It is thus left as an exercise for the reader.

Lib - slide_rule

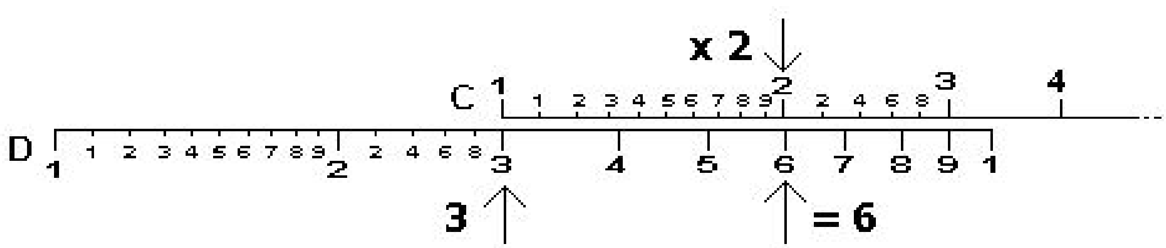
By the Year of our Rabbits 002 the rabbits had already chewed through all of the wiring, effectively crippling human civilization. Without technology to lean upon, humanity descended into chaos. The slide rule is ideal for such times as it will work underwater, in a vacuum, in the sun, in the snow, in a nuclear winter. It never runs out of batteries, and you can drop it off a building and it will still work. It is the ideal tool for the discerning scientist of the post-collapse-of-civilization world.

Your slide rule is made of three bars that are fixed together. The sliding center bar is sandwiched by the outer bars which are fixed with respect to each other. The "window" is inserted over the slide rule to act as a placeholder. A cursor is fixed in the center of the "window" to allow for accurate readings.
The scales (A-D) are labeled on the left-hand side of the slide rule. The number of scales on a slide rule vary depending on the number of mathematical functions the slide rule can perform. Multiplication and division are performed using the C and D scales. Square and square root are performed with the A and B scales. The numbers are marked according to a logarithmic scale. Therefore, the first number on the slide rule scale (also called the index) is 1 because the log of zero is one.

multiply

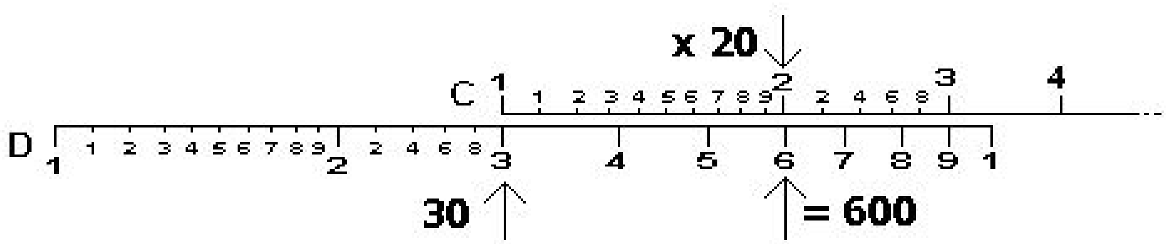
<u>Example - To multiply 3 by 2:</u>

1. Move the sliding middle section with scale "C" so that the 1 on the scale matches the 3 on the lower fixed section with scale "D".
2. Now read along the top scale to the 2 and see what it says on the bottom scale.
3. The bottom scale should read approximately 6 which is the answer.
4. We have just added a log of 3 distance on the bottom to a log of 2 distance on the top to get a log of 6 distance on the bottom.



Calculating 3 x 2 on a Slide Rule

The slide rule also works for numbers larger than what is represented on the scale of the ruler. For example, double digit numbers can be represented by mentally "moving" the decimal places.



Calculating 30 x 20 on a Slide Rule

divide

To perform division, simply reverse the steps for multiplication:

1. Set the divisor on the C scale opposite the dividend on the D scale
2. Read the result of the D scale under the C scale index (where the scale reads 1)

sqrt

To find the square root of a number:

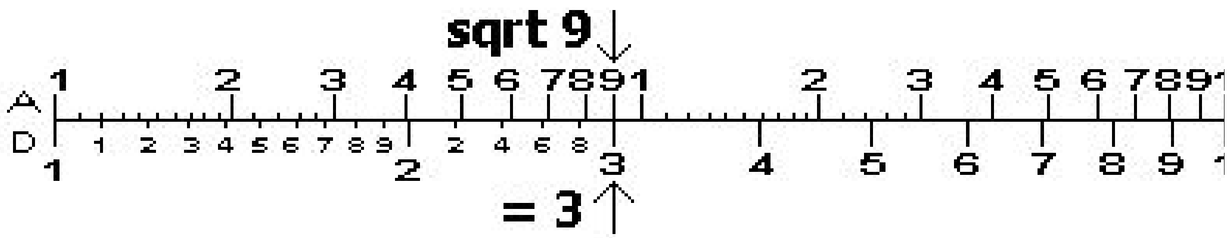
1. The A and B scales are the squares of the D and C scales, respectively. Example: To determine the square root of 9, look on the A scale for 9.

2. Find the answer, 3, on the D scale below the A scale.

exp

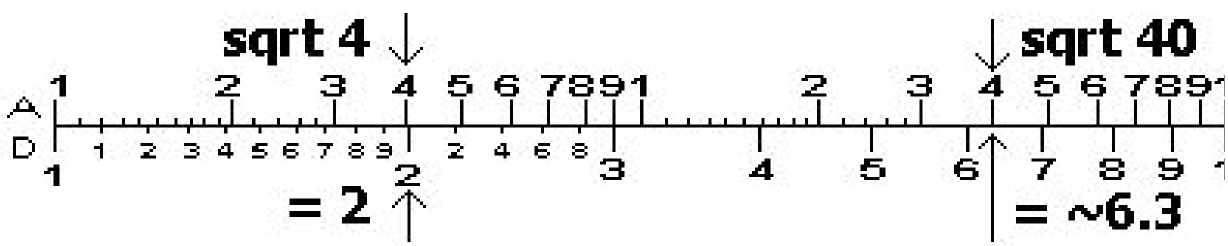
To find the square:

1. Reverse the square root process to find the square.



Calculating $9^{1/2}$

IMPORTANT NOTE: There may be confusion about which side of the A scale to use. For example, the square root of 4 is 2 and the square root of 400 is 20 - both calculations can be found using the left side of the A scale. However, to find the square root of 40, it is necessary to use the right side of the scale to get the correct answer of ~6.3. The simplest method to determine which side of the A scale to use is to write the number in standard scientific notation form (ie n.nnn x 10^exp). If the power of ten was even (exp = even), use the left side to find the square/square root (and the resulting exponent of ten was one half the original exponent). For odd powers of ten, shift the decimal place of the number one place to the right and decreased the exponent of ten by one. Then use the right side (and again used one half the exponent of ten for the resulting exponent).



References

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[virtual slide rule]
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