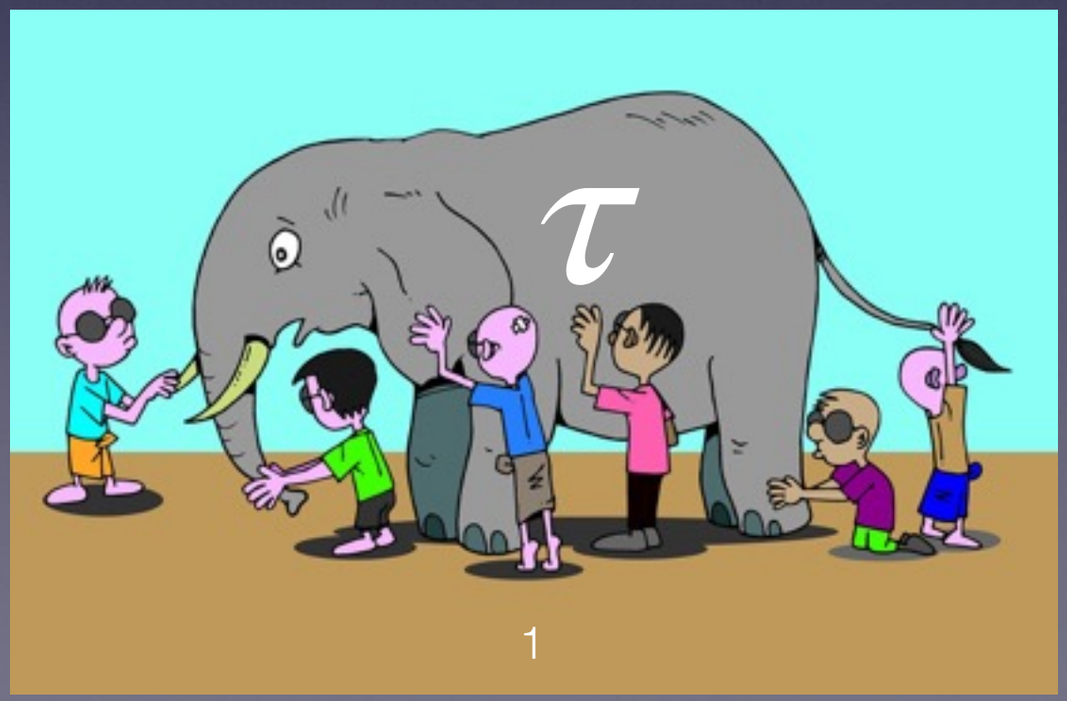
What type of thing is a type?

Ronald Garcia
University of British Columbia



Type Discipline (two practitioner views)

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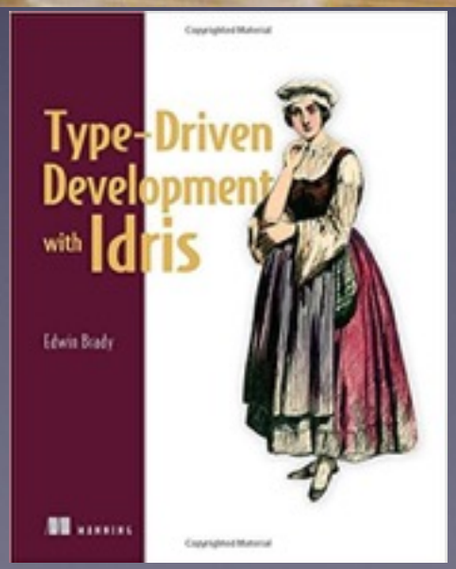



```
(* ... in a function later in the file ... *)
        add vList1 s
```

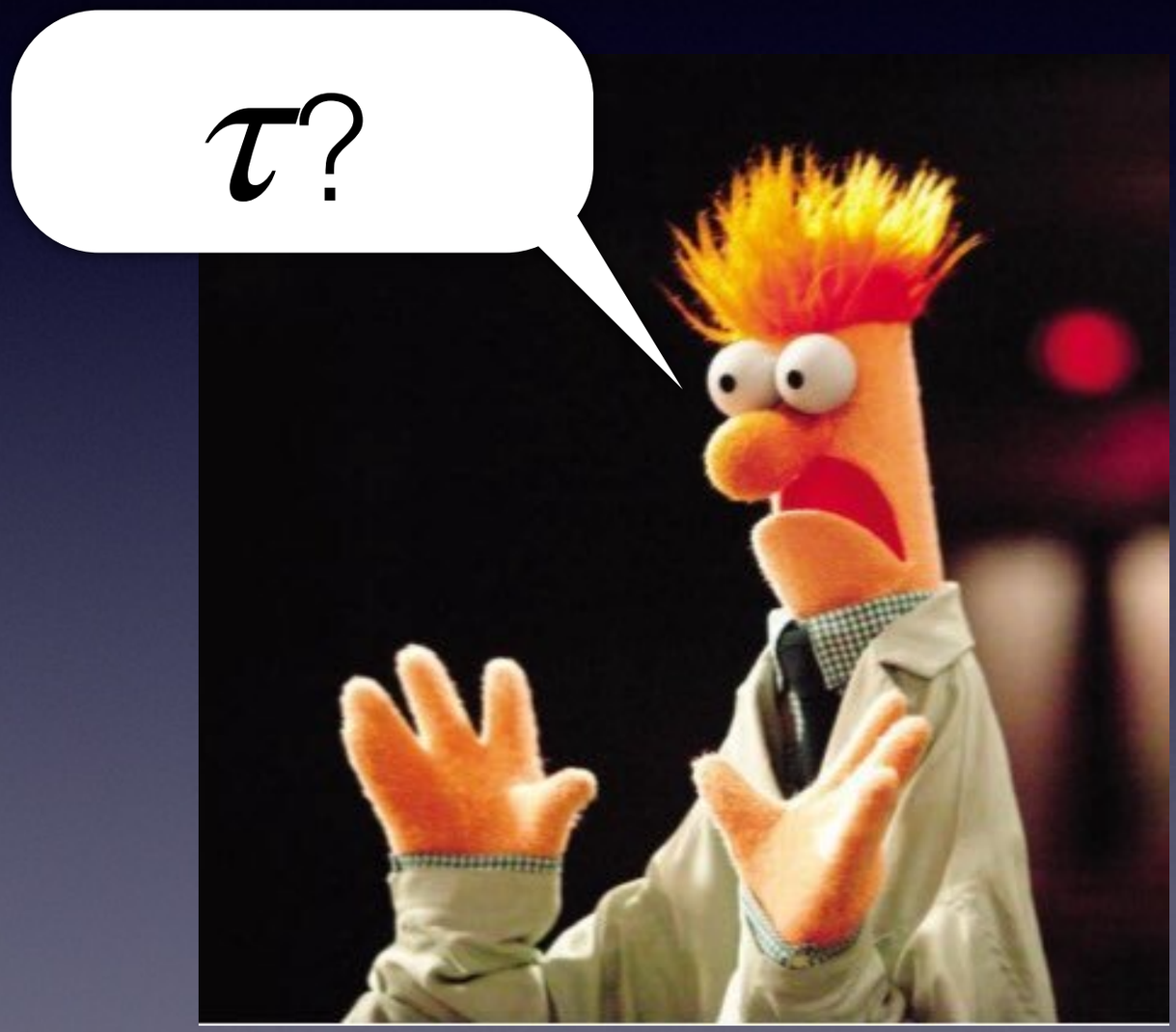
Type-checker:

```
The expression s has type string
but is here used with type string list list
```

Our approach:

```
Try replacing add vList1 s
with add s vList1
...
```

This talk



A theoretician's view

About Me

About Me

Dynamic Typing

Static Typing

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A Chronology (4 decades)

References:

Types Are Not Sets
by James H. Morris (1973)

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Themes

Syntax

Themes

Semantics

Themes

Pragmatics

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A Conceptual Blueprint for Analyzing Types!

Look Ma, a Language!

$$e ::= x \mid (ee') \mid \textit{if } e \textit{ then } e' \textit{ else } e'' \mid \lambda x \cdot e \mid \textit{fix } x \cdot e \mid \textit{let } x = e \textit{ in } e'.$$

$$\mathscr{E}[\![x]\!]\eta = \eta[\![x]\!]$$

$$\mathscr{E}[\![(e_1 e_2)]\!]\eta = v_1 \,\mathsf{E}\, F \to (v_2 \,\mathsf{E}\, W \to \text{wrong}, (v_1 \mid F)v_2), \text{ wrong}$$
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Basically a Data Type

```
(require plai/datatype) ;; for define-type and type-case

(define-type Exp
  [var (x symbol?)]
  [app (e Exp?) (e^ Exp?)]
  [if-then-else (e Exp?) (e^ Exp?) (e^^ Exp?)]
  [lam (x symbol?) (e Exp?)]
  [fix (x symbol?) (e Exp?)]
  [let-in (x symbol?) (e Exp?) (e^ Exp?)])
```

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Semantics

Closely Resembles an Interpreter(*)

$\mathscr{E}[\![e]\!]\eta$

```
((eval e) env)
```

$$\mathscr{E} \in \text{Exp} \to \text{Env} \to V,$$

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It's a Dynamic Language!

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It's a Dynamic Language!
(all closed programs run)

Look Pa, Types!

Syntax

The syntax of types is as follows.

(1) ι_0 , ι_1 ,... are (basic) types; one for each B_i .

(2) There is a denumerable set of type variables, which are types. We use α, β, γ,... to range over type variables.

(3) If ρ and σ are types, so is $\rho \rightarrow \sigma$.

Basically
a
Data Type

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$$\mu, \nu \quad ::= \quad \iota_1 \mid \iota_2 \mid \cdots \mid \mu \to \nu \quad \text{“monotypes”}$$

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$$\text{INT}$$

$$\text{INT} \to \text{BOOL}$$

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$$\text{INT}$$

$$\text{INT} \to \text{BOOL}$$

$$\alpha \to \alpha$$

$$\alpha \to \beta \to \alpha$$

$$\alpha \to \text{INT} \to \alpha$$

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Semantics: when do values have monotypes?

We first give the semantics of monotypes; that is, we give the conditions under which a value $v \in V$ *possesses* a monotype μ, which we write $v : \mu$.

(i) $v : \iota_i$ iff $v = \bot_V$ or $v \mathsf{E} B_i$

(ii) $v : \mu \to \nu$ iff *either* $v = \bot_V$, *or* $v \mathsf{E} F$ and $(v \mid F)u : \nu$ whenever $u : \mu$.

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Nontermination has any type
(yes, nontermination is a value!!!)

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Basic values have their (obvious) basic type (i.e. true is a Bool, etc.)

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Any function that
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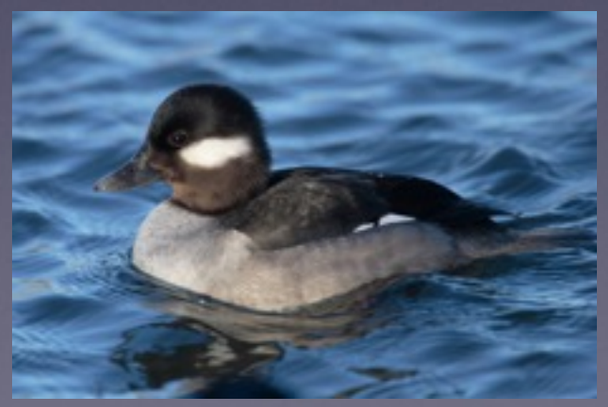
Behavioural Definition!



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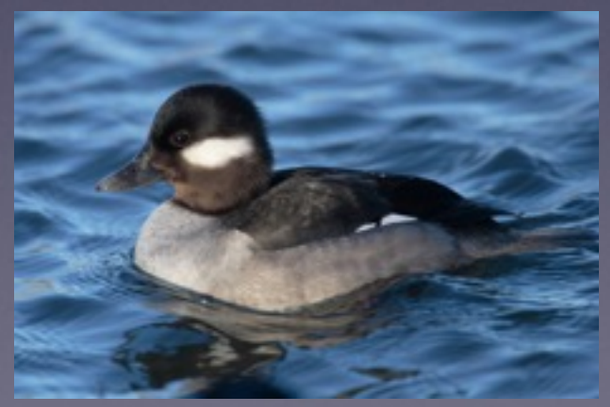
Behavioural Definition!

And of course $\perp_V$ has every type



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Semantics: what about polytypes?

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In English: a value v has a polytype σ if it has any monotype μ that you get by substituting for σ's type variables.

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$$v : \sigma \text{ iff } \forall \mu \leqslant \sigma \cdot v : \mu.$$

In English: a value v has a polytype σ if it has any monotype μ that you get by substituting for σ's type variables.

Example:

$$v : \alpha \to \alpha \quad \textit{iff } v : \text{INT} \to \text{INT}$$
$$\textit{and } v : \text{BOOL} \to \text{BOOL}$$
$$\textit{and } v : (\text{INT} \to \text{INT}) \to (\text{INT} \to \text{INT})$$
$$\textit{and } \ldots$$

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Semantics: what about types for *expressions?*

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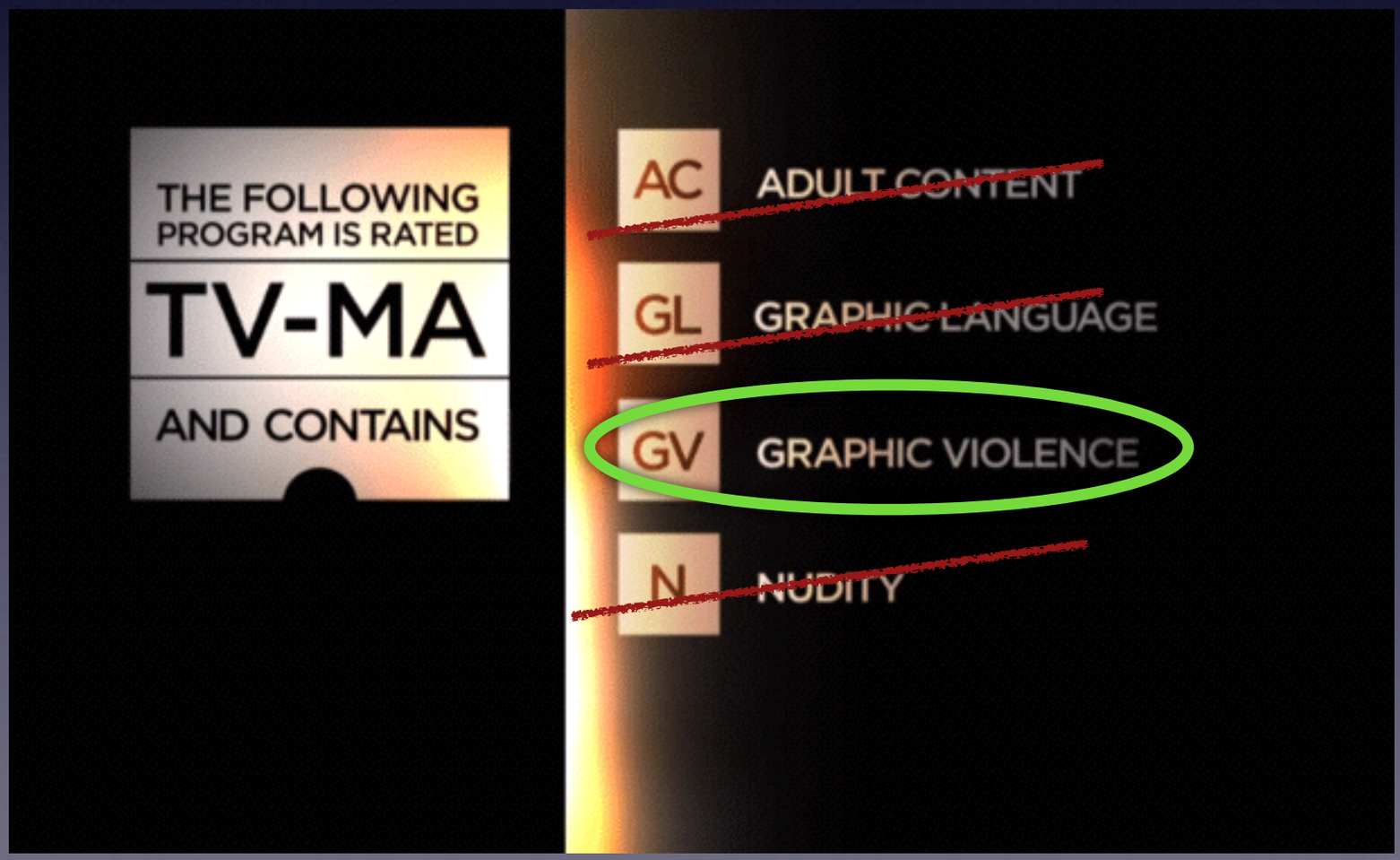

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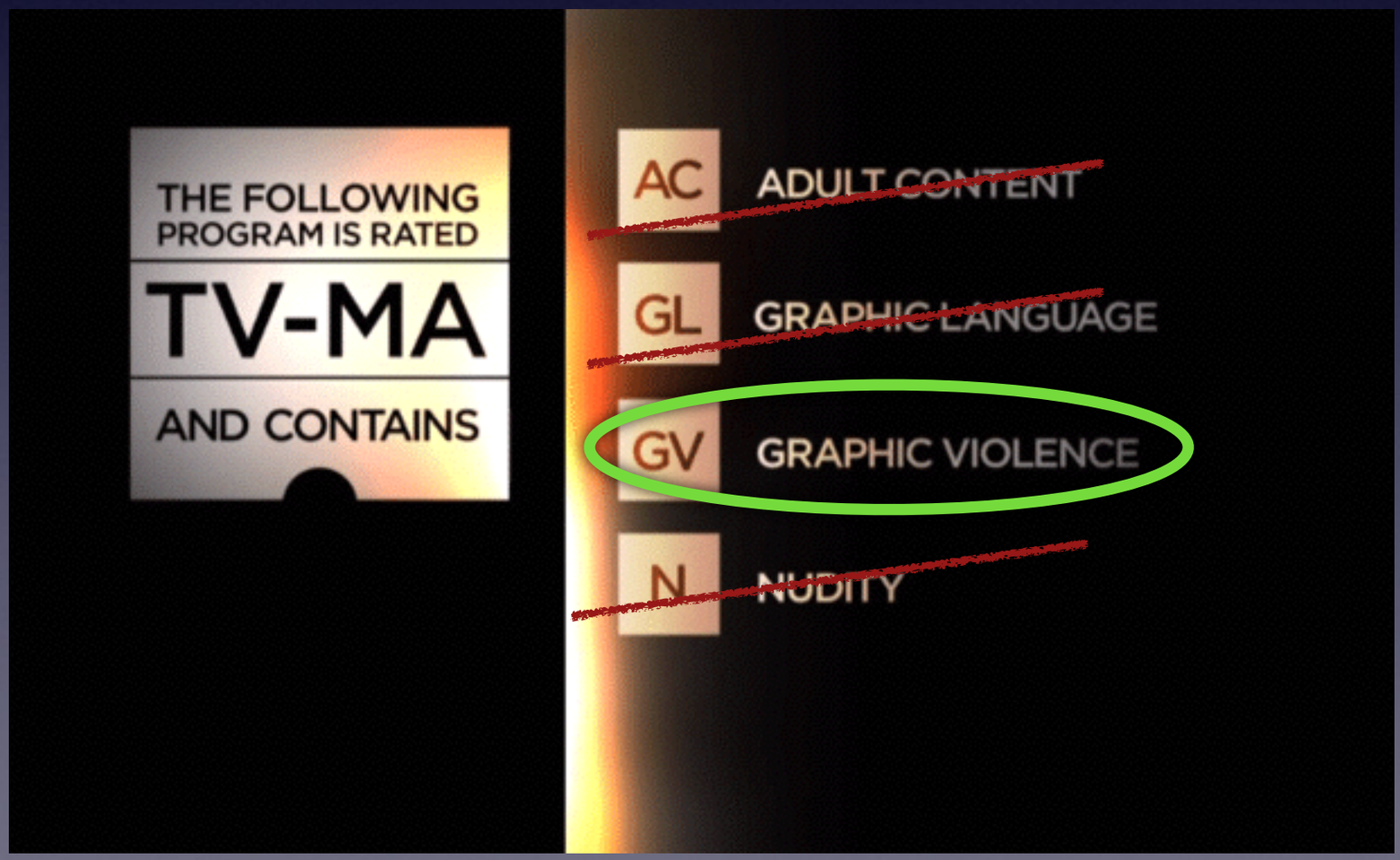

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A Theory of Type Polymorphism in Programming

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Computer Science Department, University of Edinburgh, Edinburgh, Scotland

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The aim of this work is largely a practical one. A widely employed style of programming, particularly in structure-processing languages which impose no discipline of types, entails defining procedures which work well on objects of a wide variety. We present a formal type discipline for such polymorphic procedures in the context of a simple programming language, and a compile time type-checking algorithm $\mathscr{W}$ which enforces the discipline. A Semantic Soundness Theorem (based on a formal semantics for the language) states that well-type programs cannot "go wrong" and a Syntactic Soundness Theorem states that if $\mathscr{W}$ accepts a program then it is well typed. We also discuss extending these results to richer languages; a type-checking algorithm based on $\mathscr{W}$ is in fact already implemented and working, for the metalanguage ML in the Edinburgh LCF system.

1. INTRODUCTION

The aim of this work is largely a practical one. A widely employed style of programming, particularly in structure-processing languages which impose no discipline of types (LISP is a perfect example), entails defining procedures which work well on objects of a wide variety (e.g., on lists of atoms, integers, or lists). Such flexibility is almost essential in this style of programming; unfortunately one often pays a price for it in the time taken to find rather inscrutable bugs—anyone who mistakenly applies CDR to an atom in LISP, and finds himself absurdly adding a property list to an integer, will know the symptoms. On the other hand a type discipline such as that of ALGOL 68 [22] which precludes the flexibility mentioned above, also precludes the programming style which we are talking about. ALGOL 60 was more flexible—in that it required procedure parameters to be specified only as "*procedure*" (rather than say "*integer to real procedure*") —but the flexibility was not uniform, and not sufficient.

An early discussion of such flexibility can be found in Strachey [19], who was probably the first to call it polymorphism. In fact he qualified it as "parametric" polymorphism, in contrast to what he called "adhoc" polymorphism. An example of the latter is the use of "+" to denote both integer and real addition (in fact it may be further extended to denote complex addition, vector addition, etc.); this use of an identifier at several distinct types is often now called "overloading," and we are not concerned with it in this paper.

In this paper then, we present and justify one method of gaining type flexibility, but also retaining a discipline which ensures robust programs. We have evidence that this

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$\bar{p}$ is an assignment of a type to each element of p, (In modern notation: Γ)

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Type Environments Assign Types to Value Environments!

η *respects* $\bar{p}$ iff, whenever *let* x_ρ or λx_ρ or *fix* x_ρ is active in $\bar{p}$, $\eta[\![x]\!] : \rho$.

Suggestive notation: $\eta : \bar{p}$

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$\bar{p} \models d : \tau$ iff for all η, *If* η *respects* $\bar{p}$ *then* $\mathscr{E}[\![d]\!]\eta : \tau$.

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Static analogue to $v = \mathscr{E}[\![d]\!]\eta$

- Relates a *class* of inputs to a *class* of outputs!

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But it's so so so undecidable!

Syntactic Type System

a.k.a. Type Assignment System

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$\bar{p} \mid \bar{e}_\sigma$

(i) $\bar{p} \mid x_\tau$ is wt iff it is standard, and *either*

(a) λx_τ or *fix* x_τ is active in $\bar{p}$, *or*

(b) *let* x_σ is active in $\bar{p}$, and τ is a generic instance of σ.

(ii) $\bar{p} \mid (\bar{e}_\rho \bar{e}'_\sigma)_\tau$ is wt iff $\bar{p} \mid \bar{e}$ and $\bar{p} \mid \bar{e}'$ are both wt, and $\rho = \sigma \to \tau$.

(iii) $\bar{p} \mid (\textit{if } \bar{e}_\rho \textit{ then } \bar{e}'_\sigma \textit{ else } \bar{e}''_\sigma)_{\tau'}$ is wt iff $\bar{p} \mid \bar{e}$, $\bar{p} \mid \bar{e}'$ and $\bar{p} \mid \bar{e}''$ are all wt, $\rho = \iota_0$, and $\sigma = \tau = \tau'$.

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$$\text{TAUT: } \frac{}{A \vdash x : \sigma} \ (x : \sigma \in A) \qquad \text{INST: } \frac{A \vdash e : \sigma}{A \vdash e : \sigma'} \ (\sigma > \sigma')$$

$$\text{GEN: } \frac{A \vdash e : \sigma}{A \vdash e : \forall \alpha \sigma} \ (\alpha \text{ not free in } A) \qquad \text{COMB: } \frac{A \vdash e : \tau' \to \tau \qquad A \vdash e' : \tau'}{A \vdash (e\, e') : \tau}$$

$$\text{ABS: } \frac{A_x \cup \{x : \tau'\} \vdash e : \tau}{A \vdash (\lambda x.e) : \tau' \to \tau} \qquad \text{LET: } \frac{A \vdash e : \sigma \qquad A_x \cup \{x : \sigma\} \vdash e' : \tau}{A \vdash (\texttt{let}\, x = e\, \texttt{in}\, e') : \tau}$$

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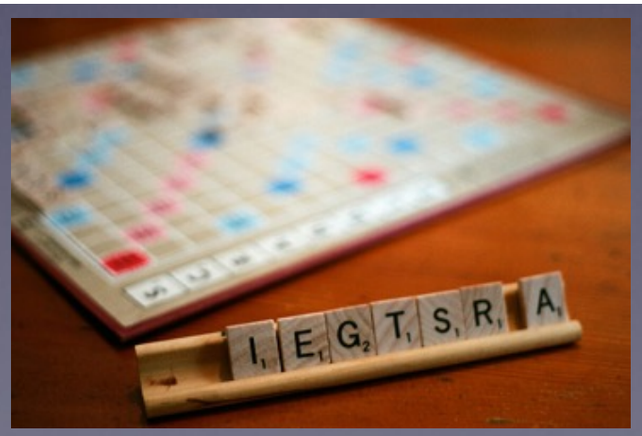

Basically a Data Type!
(proofs of type assignment)

Comparison

Semantic

We first give the semantics of monotypes; that is, we give the conditions under which a value $v \in V$ *possesses* a monotype μ, which we write $v : \mu$.

(i) $v : \iota$ iff $v = \perp_V$ or $v \mathsf{E} B_i$

(ii) $v : \mu \to \nu$ iff *either* $v = \perp_V$, *or* $v \mathsf{E} F$ and $(v \mid F)u : \nu$ whenever $u : \mu$.

Syntactic (but suggestive!)

$$\text{ABS: } \frac{A_x \cup \{x : \tau'\} \vdash e : \tau}{A \vdash (\lambda x.e) : \tau' \to \tau} \qquad \text{COMB: } \frac{A \vdash e : \tau' \to \tau \qquad A \vdash e' : \tau'}{A \vdash (e\, e') : \tau}$$

Semantic Soundness

Connects Type Assignment to Type Semantics

THEOREM 1 (Semantic Soundness). *If* η *respects* $\bar{p}$ *and* $\bar{p} \mid \bar{d}_\tau$ *is well typed then* $\mathscr{E}[\![d]\!]\eta : \tau$.

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i.e., If $\bar{p} \vdash d : \tau$ then $\bar{p} \models d : \tau$

Data Structure

Behaviour

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Data Structure

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Every proof of type assignment
says something meaningful about code

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Syntax

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Syntax

Semantics

As a corollary, under the conditions of the theorem we have

$$\mathscr{E}[\![d]\!]\eta \neq \text{wrong},$$

since wrong has no type.

Big-Picture Payoff!

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Syntax

Semantics

Every proof of type assignment
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How can we construct such proofs for our code?

Type Checker

4. A Well-Typing Algorithm and Its Correctness

4.1. *The Algorithm $\mathcal{W}$*

In this section we tackle the question of finding a well typing for a prefixed expression. We present an algorithm $\mathcal{W}$ for this. We would like to prove that $\mathcal{W}$ is both syntactically sound and (in some sense) complete. By syntactic soundness, we mean that whenever $\mathcal{W}$ succeeds it produces a wt; by completeness, we mean that whenever a wt exists, $\mathcal{W}$ succeeds in finding one which is (in some sense) at least as general.

well typing = type assignment (proof)

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Algorithm $\mathscr{W}$

$\mathscr{W}(\bar{p}, f) = (T, \bar{f})$, where

(i) If f is x, then:

if λx_σ or *fix* x_σ is active in $\bar{p}$ then
$T = I, \bar{f} = x_\sigma$;
if *let* x_σ is active in $\bar{p}$ then
$T = I, \bar{f} = x_\tau$
where $\tau = [\beta_i/\alpha_i]\sigma$, α_i are the generic variables of σ, and β_i are new variables.

(ii) If f is (de), then:

let $(R, \bar{d}_\rho) = \mathscr{W}(\bar{p}, d)$, and $(S, \bar{e}_\sigma) = \mathscr{W}(R\bar{p}, e)$;
let $U = \mathscr{U}(S\rho, \sigma \to \beta)$, β new;
then $T = USR$, and $\bar{f} = U(((S\bar{d})\bar{e})_\beta)$.

(iii) If f is (*if* d *then* e *else* e'), then:

let $(R, \bar{d}_o) = \mathscr{W}(\bar{p}, d)$ and $U_0 = \mathscr{U}(\rho, \iota_0)$;
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$\bar{f} = U((\textit{if } S'\bar{e} \textit{ then } S'SU_0\bar{d} \textit{ else } \bar{e}')_\sigma)$.

(iv) If f is $(\lambda x \cdot d)$, then:

let $(R, \bar{d}) = \mathscr{W}(\bar{p} \cdot \lambda x_\beta, d)$, where β is new;
then $T = R$, and $\bar{f} = (\lambda x_{R\beta} \cdot \bar{d}_\rho)_{R\beta \to \rho}$.

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then $T = SR$, and $\bar{f} = (\textit{let } x_{S\sigma} = S\bar{d} \text{ in } \bar{e})_\sigma$. ∎

Some Computational Procedure or Other
(see paper for details)

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Some Computational Procedure or Other
(see paper for details)

Is it any good?

Syntactic Soundness

YES!

THEOREM 2 (Syntactic Soundness). *Let $\bar{p}$ be a standard prefix, and $p \mid f$ a (closed) pe. Then, if $\mathscr{W}(\bar{p}, f) = (T, \tilde{f}_\tau)$,*

(A) $T\bar{p} \mid \tilde{f}$ *is wt,*

(B) $\mathrm{Inv}(T) \subseteq \mathrm{Spec}(\bar{p}) \cup New,$

and

(C) $\mathrm{Vars}(\tau) \subseteq \mathrm{Spec}(\bar{p}) \cup New,$

where New is the set of new type variables used by $\mathscr{W}$.

Connects the results of
Type Checking to the definition of Type Assignment

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If *Algorithm W* returns a type assignment,

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If Algorithm W returns a type assignment, then / can build the corresponding proof

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If Algorithm W returns a type assignment,
then / can build the corresponding proof
(the algorithm essentially does but discards it)

Synopsis

Computation — Type Checker

Syntactic Soundness

Syntax — Type Assignment

Semantic Soundness

Semantics — Semantics of Types

“Well-typed Programs Don’t Go Wrong”

Synopsis

Computation — Type Checker *Often "Obvious"*

Syntactic Soundness

Syntax — Type Assignment

Semantic Soundness

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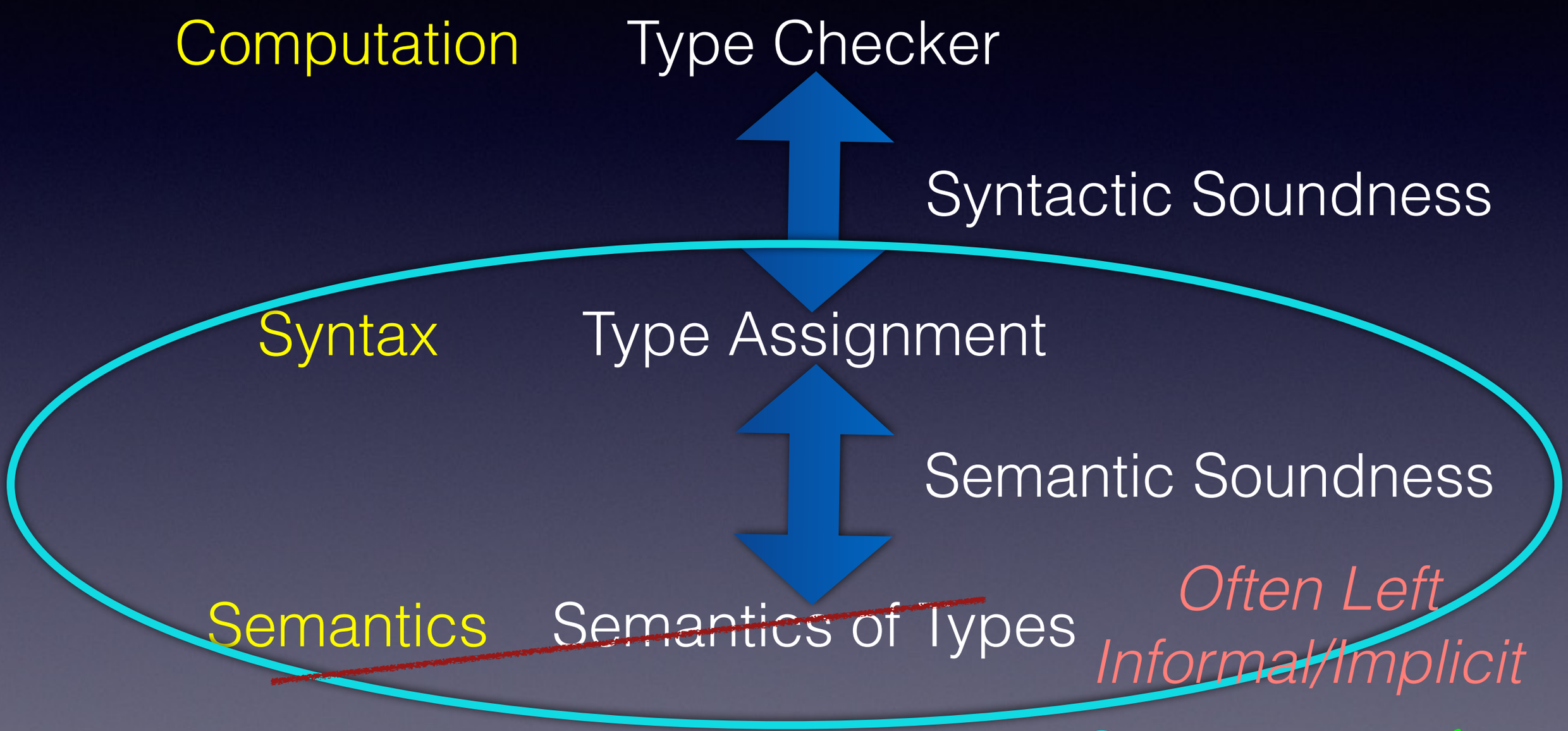
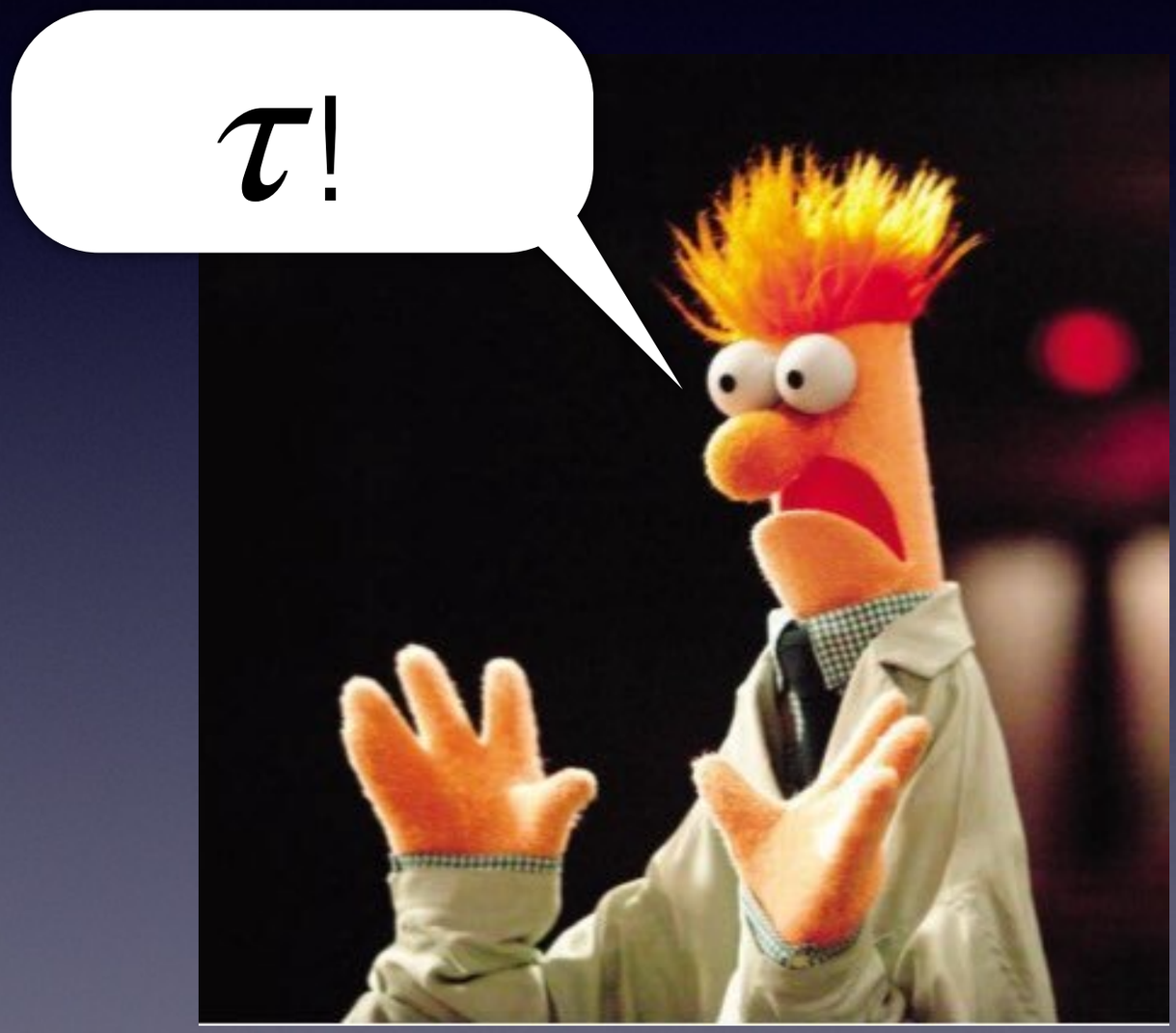
"Well-typed Programs Don't Go Wrong"

Synopsis


Computation
Type Checker
Syntactic Soundness
Syntax
Type Assignment
Semantic Soundness
Semantics
Semantics of Types
Often Left
Informal/Implicit
"Well-typed Programs Don't Go Wrong"


τ!

What are Types?

- Programming Languages have:
- Syntax
- Semantics
- Pragmatics

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A type system *is a language…*

What are Types?

- *Types* have:
  - Syntax
  - Semantics
  - Pragmatics

A type system *is a language* for "talking about" properties of programs and program fragments!

A Whirlwind Tour

A Chronology (4 decades)

References:

Types Are Not Sets
by James H. Morris (1973)

A Theory of Type Polymorphism in Programming
by Robin Milner (1978)

Types, Abstraction, and Parametric Polymorphism
by John Reynolds (1983)

Co-induction in Relational Semantics
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Typing First-class Continuations in ML
by Bruce Duba, Robert Harper, and David MacQueen (1991)

A Syntactic Approach to Type Soundness
by Andrew Wright and Matthias Felleisen (1994)

Semantics of Types for Mutable State
by Amal Ahmed (2004)

A Theory of Type Polymorphism in Programming

ROBIN MILNER

Computer Science Department, University of Edinburgh, Edinburgh, Scotland

Received October 10, 1977; revised April 19, 1978

Turing Award (1991)

Did not have a PhD!

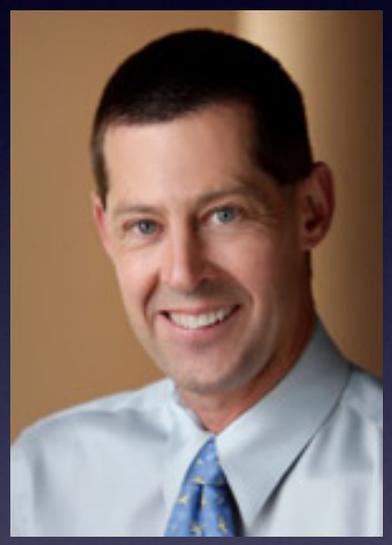
"The Hindley/Milner Type Inference Paper"

A Conceptual Blueprint for Analyzing Types!

A Syntactic Approach to Type Soundness

ANDREW K. WRIGHT AND MATTHIAS FELLEISEN*

Department of Computer Science, Rice University,
Houston, Texas 77251-1892



A *Practical* Blueprint for Analyzing Types!

"Well-typed Programs Don't Go Wrong" ✔

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TYPES ARE NOT SETS*

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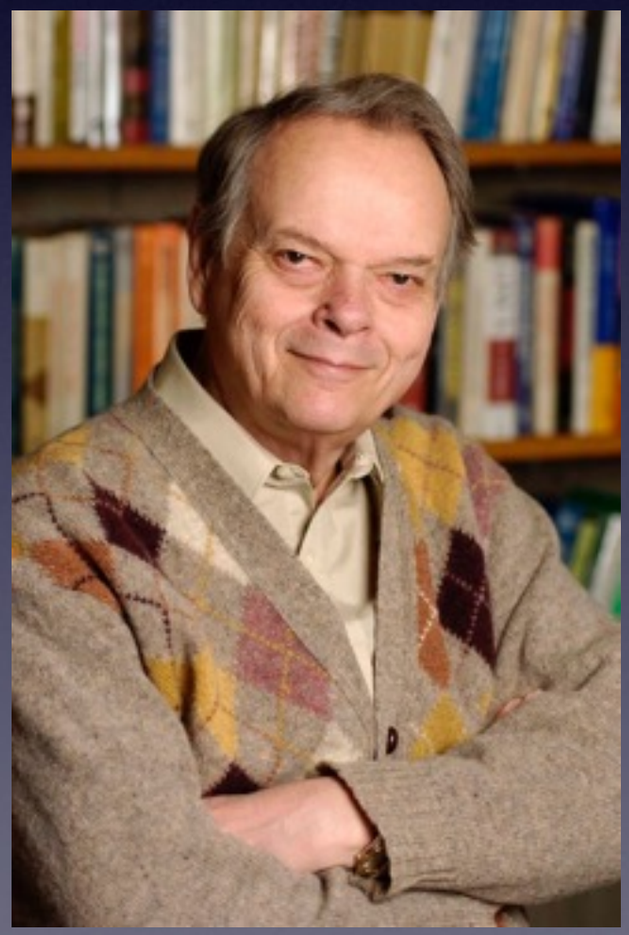
TYPES, ABSTRACTION AND PARAMETRIC POLYMORPHISM†

John C. REYNOLDS
Syracuse University
Syracuse, New York, USA



Types can describe and enforce strong behavioural properties (e.g. information-hiding)

A Chronology (4 decades)

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Semantics of Types for Mutable State

Amal Jamil Ahmed

Practical Blueprint for a Semantic Approach to Analyzing Types

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  - Syntax
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  - Pragmatics

A type system *is a language* for "talking about" properties of programs and program fragments!

Photo Attribution

Dave and Margie Hill: Basai-dai
Hans Olofson: The virgin chastising the Christ child (Max Ernst)
Lerner et al. Searching For Type-Error Messages
Matt Thorpe: Reading