

# Face Recognition(Isometry Invariant) using GMDS

GENERALIZED MULTI-DIMENSIONAL SCALING

## Abstract

The face recognition method we use, has 12 models, 3 for each person, and a probe is passed into the system, to look for the best match and also decide which person this probe belongs to.

We perform expression invariant three dimensional face matching, using a method which captures the distinction of the model and the probe intrinsic properties associated with the metric structure of the surface, while ignoring the extrinsic properties that describe the way the surface is immersed into the ambient space and that often change while the surface bends.

Our results show a clear distinction between the faces (is accurate), and performs well (is computationally efficient)

## Face Recognition Method

The face recognition method using is Generalized Multi-Dimensional Scaling Algorithm. It measures the minimum possible distortion when trying to isometrically embed one surface into another. By using GMDS, we can handle full as well as partial surface matching; this ability is one advantage over a straightforward use of the GH distance. A crucial part of the GMDS problem is the computation of geodesic distances. GMDS uses the “fast marching method” (FMM), which computes geodesic distances on surfaces by solving the eikonal equation on general triangulated meshes.

Other methods that can perform isometric invariant surface matching include MDS(Multi-Dimensional Scaling) Algorithm, but this produces inappropriate results for Partial surface Matching.

One of the earliest attempts of isometry-invariant surface matching is the classical “iterative closest point” (ICP) algorithm (2). It addresses a particular case of the partial matching problem where only rigid (Euclidean) isometries are allowed. An efficient method for the construction of near-isometry-invariant representations of surfaces [called “canonical forms” (CFs)] based on Euclidean embeddings was presented in ref. 3, as a generalization of ref. 4. This approach used a multidimensional scaling (MDS) algorithm (5). MDS is closely related to dimensionality reduction (6, 7) and can be performed in a computationally efficient manner. Euclidean embeddings are used in theoretical computer science for representing metric spaces usually arising from geometry of graphs (8).

The geodesic distances  $d_Q(q_i, q_j)$ , between the fixed sample points of the models( $Q$ ) can be precomputed using Fast Marching method , however the co-ordinates( $\mathbf{u}_i$ ) representing  $\psi(q_i)$  on  $\mathcal{S}$  (the probe) change during the iterations of the numerical minimization algorithm. Thus, the distances  $d_{\mathcal{S}}(\mathbf{u}_i, \mathbf{u}_j)$  have to be reevaluated at every iteration. This computation is critical for the GMDS.

The 4k\_mesh points are used for our data and probe.

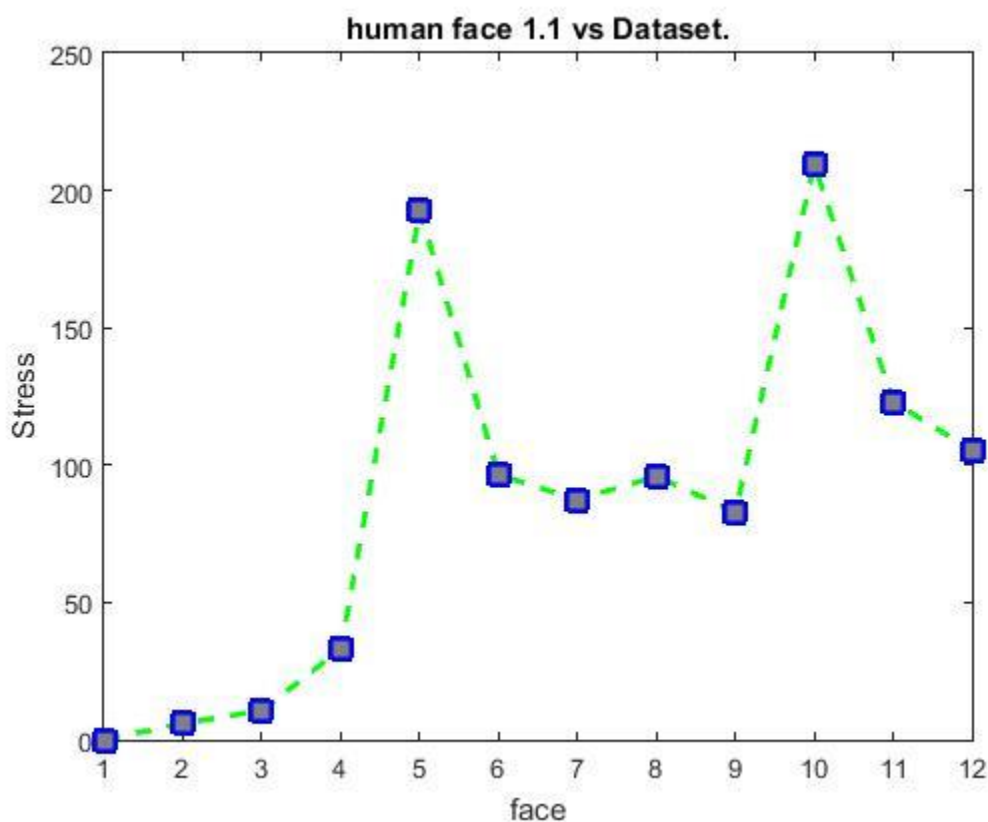
Our approach favorably compares with previous attempts to perform isometry-invariant surface matching. First, our PE distance naturally allows for isometry-invariant matching of partially missing surfaces. Secondly, the properties of our distance and its computation are completely deterministic. Thirdly, GMDS used for the PE distance computation is a continuous optimization problem and can be solved very efficiently by using standard optimization methods.

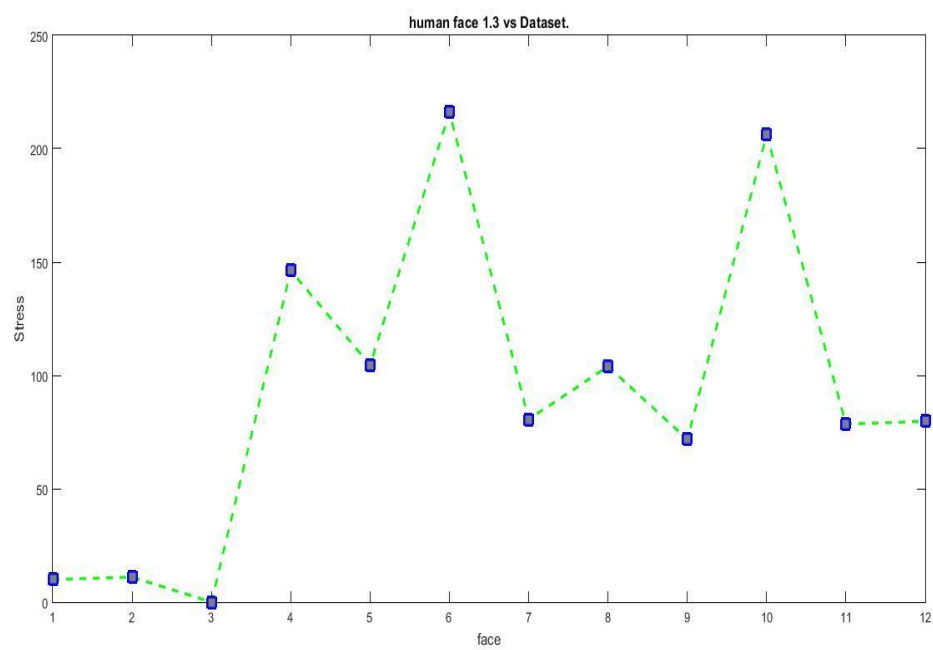
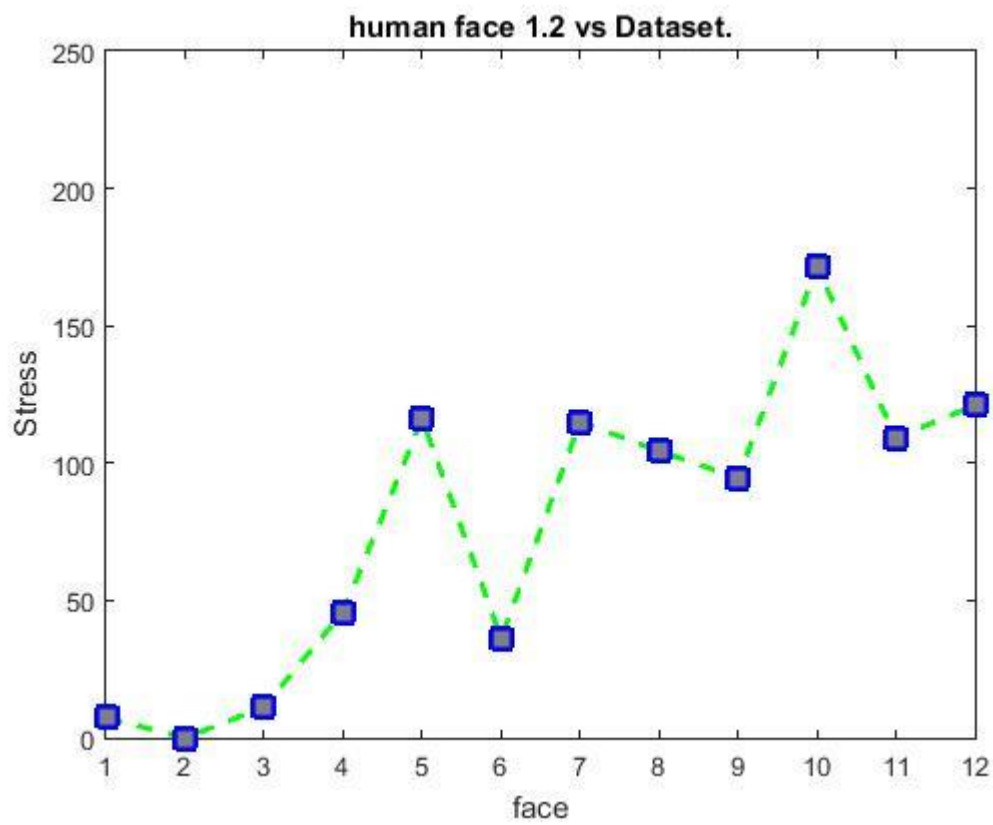
## Performance Evaluation Method

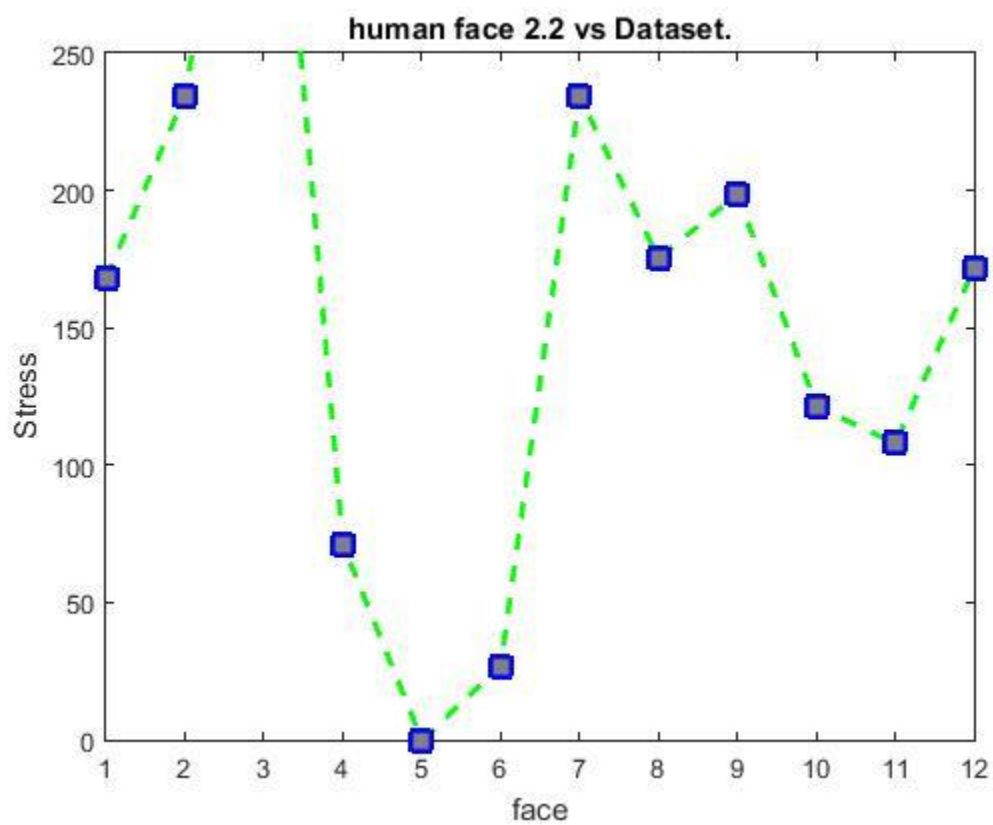
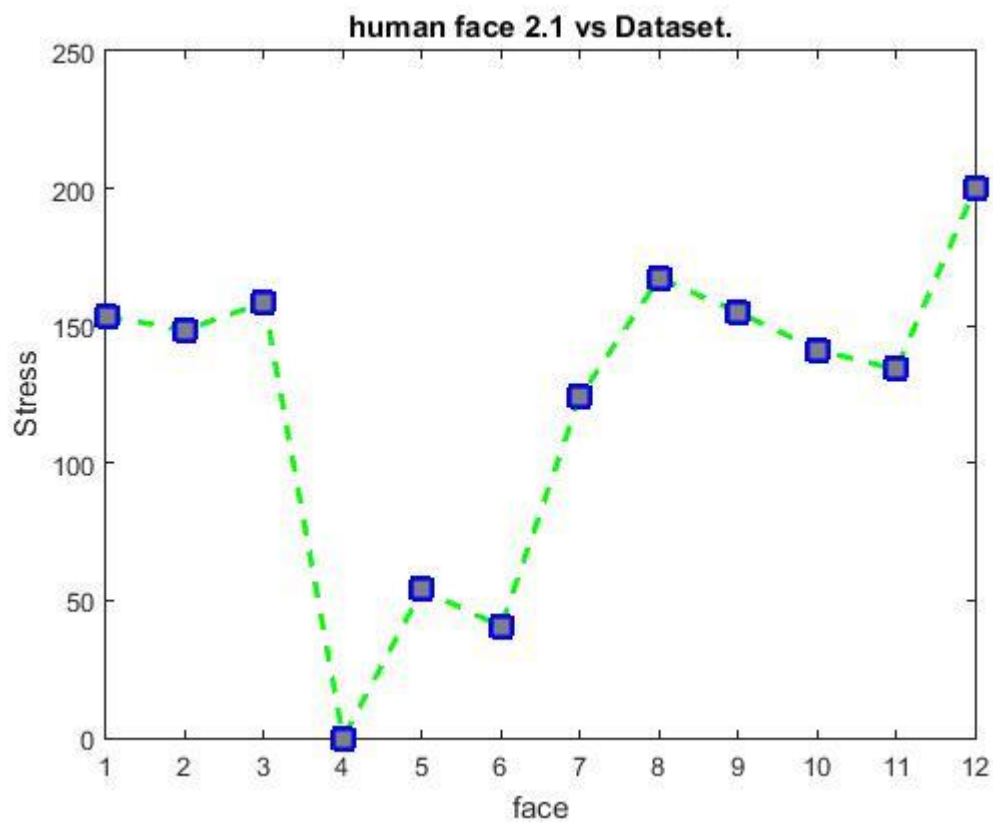
The input value (the probe file), is taken into the face recognition system, and compared with the existing datasets. The stress values are then calculated between the probe and each model (one-by-one) and compared.

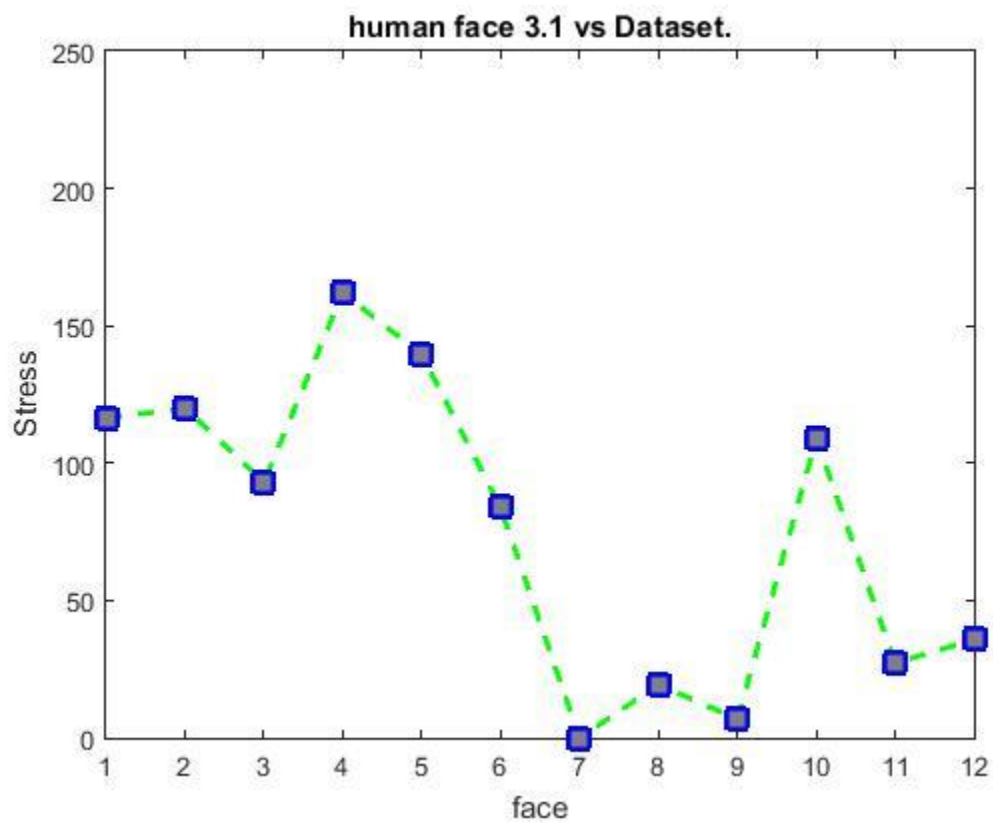
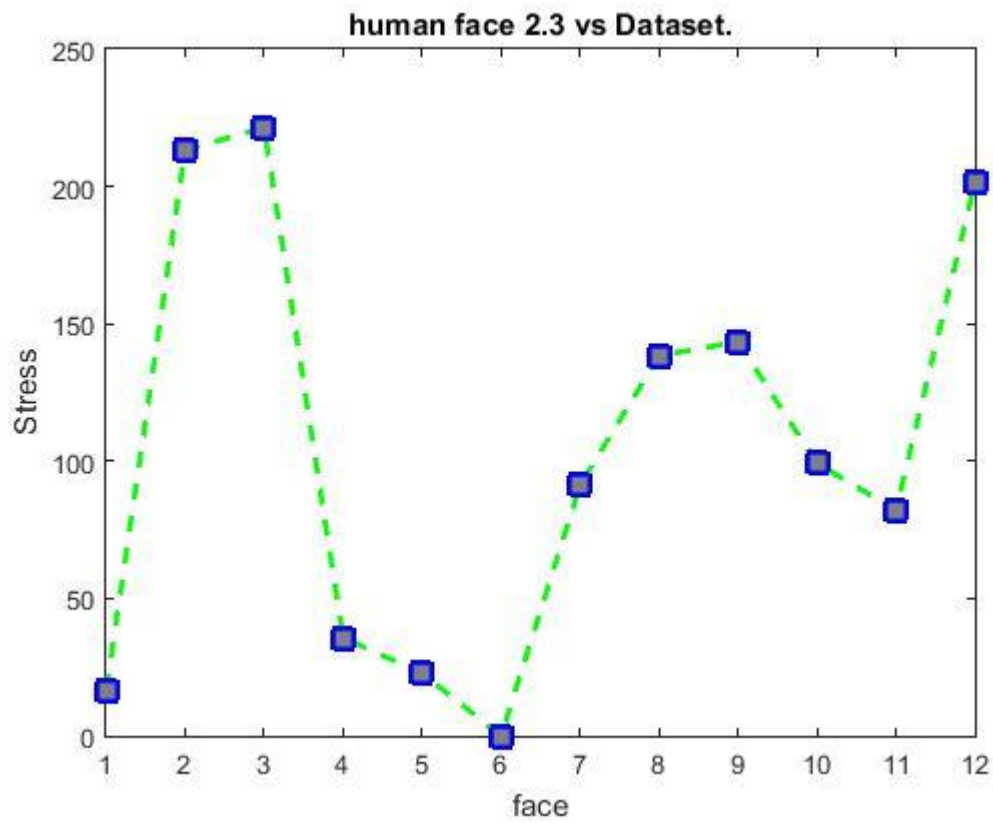
We plot 12 graphs for each of the twelve models denoting the stress levels between each pair. The plots clearly show the results, distinguishing the faces belonging to the same person, from different ones.

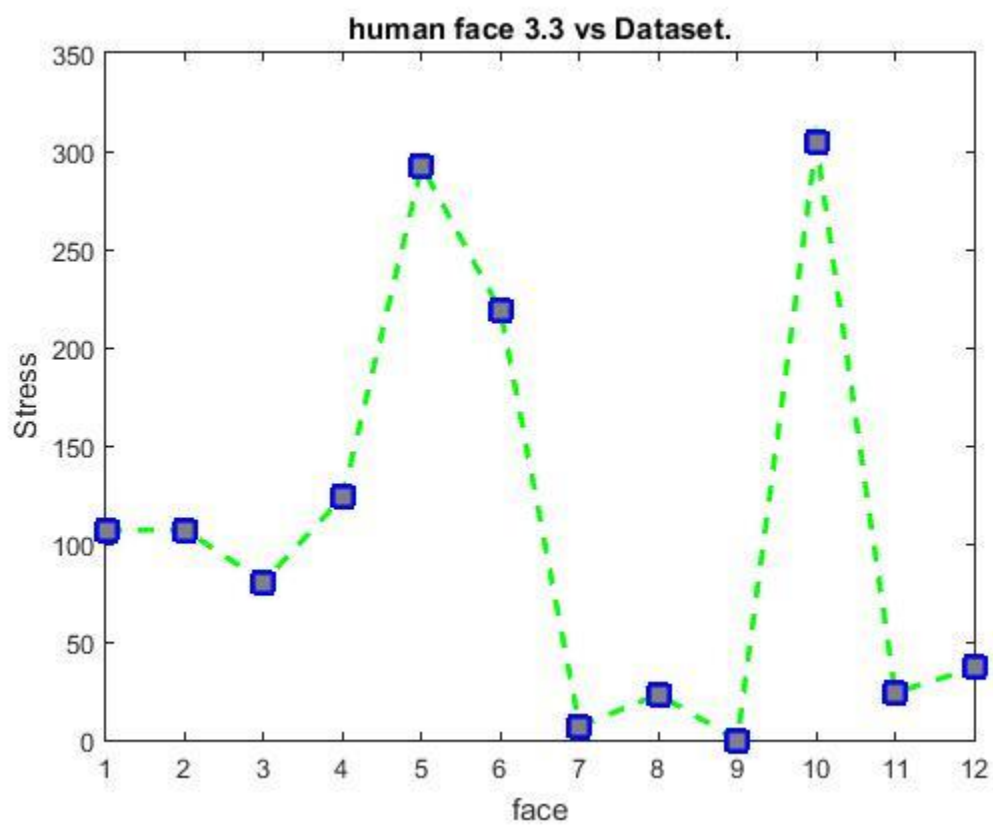
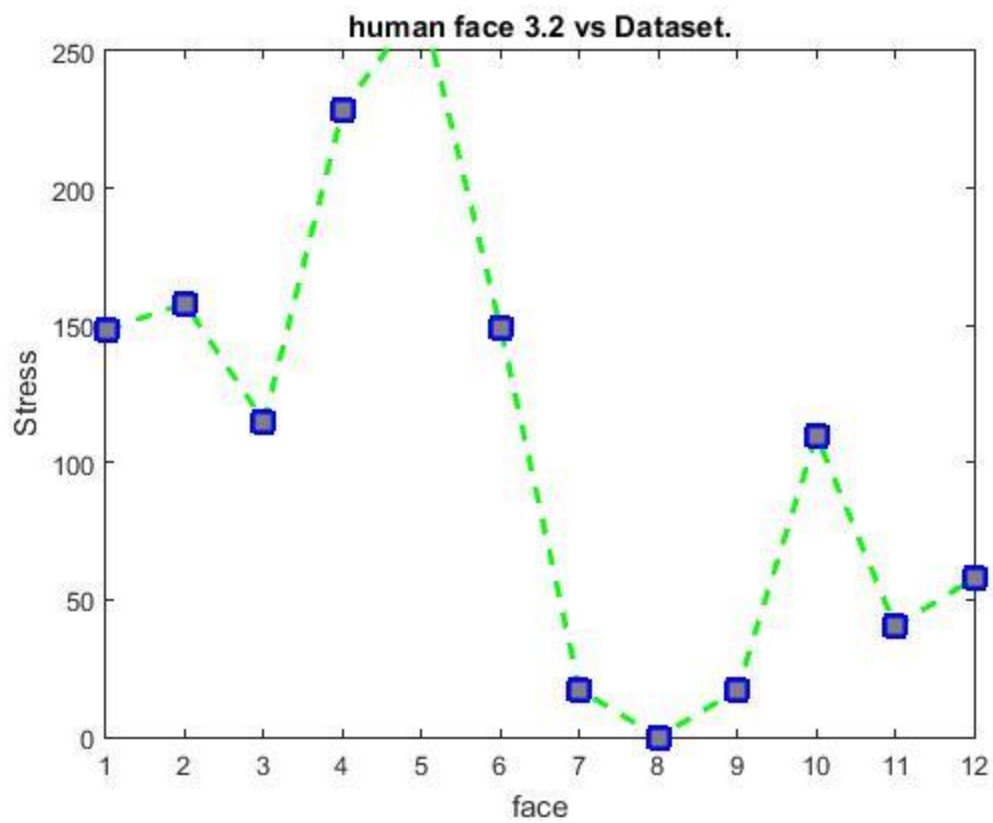
The below graphs shows the stress value of the input face vs. the dataset, the faces with low stress are more similar to the input face, which shows the level of isometric invariance of the algorithm to the faces.



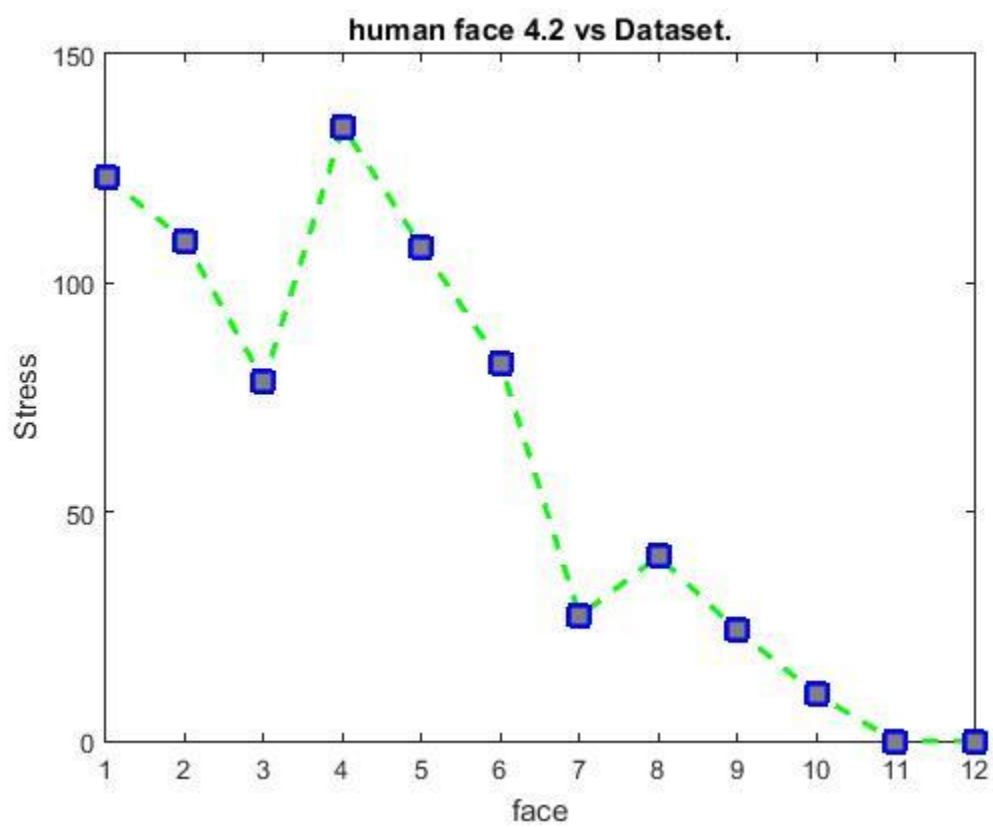
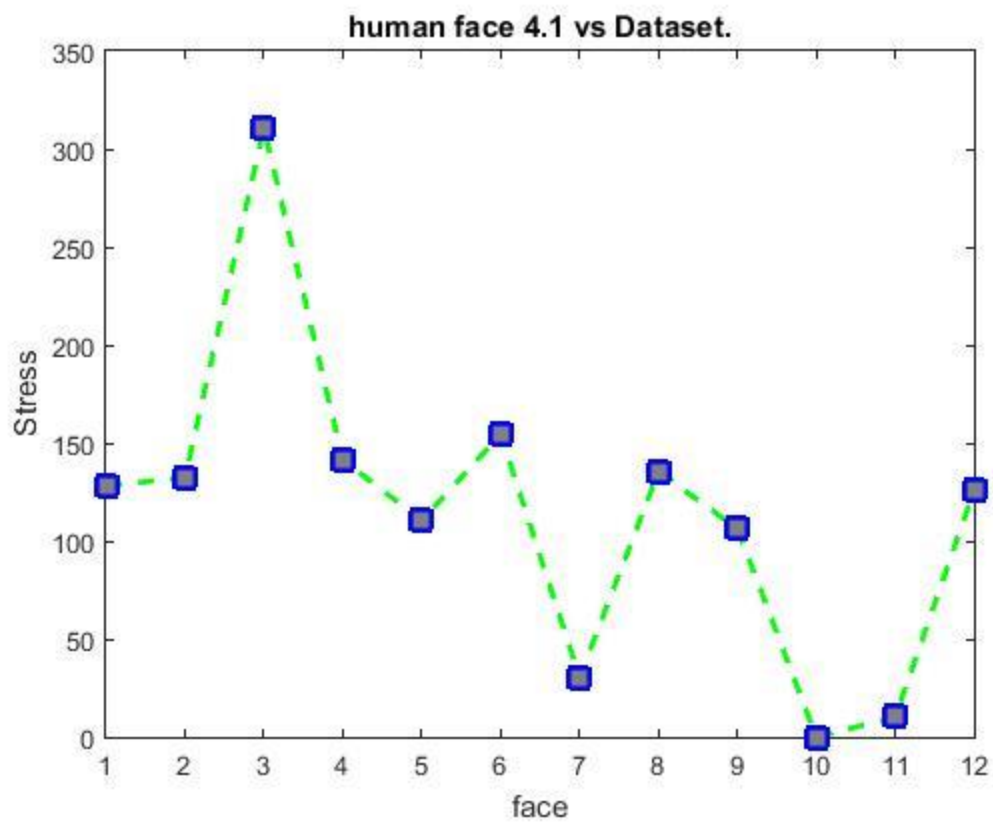


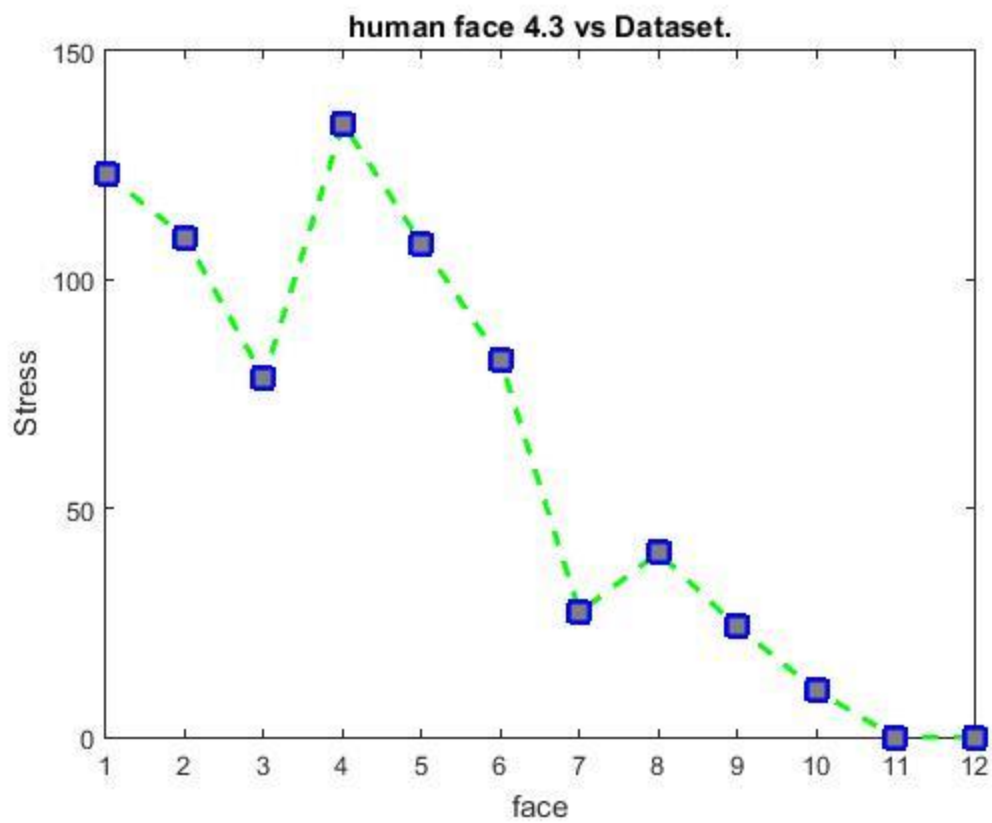












These are the 12 results for 12 comparisons(one-by-one) of faces with the dataset.

## Experimental Results

### Face 1\_1 vs Models

The following figures show the final stress values between face 1\_1 and the models represented by appending the row and column entry : (Maximum stress between face1 and probe ~11)

Final Stress(f)	1	2	3
Face 1_	7.5125e-29	6.07268	10.7074
Face 2_	33.1987	192.6678	96.5027
Face 3_	87.4409	95.9419	83.0556
Face 4_	209.5504	123.1524	105.4298

As expected the final stress values are really low for the faces belonging to the same person, and Face 1\_1 matches the most to face1\_1, Face2\_2, and Face3\_3, out of off the faces.

### Face 1\_2 vs Models

The following figures show the final stress values between face 1\_2 and the models represented by appending the row and column entry : (Maximum stress between face1 and probe ~11.5)

Final Stress(f)	1	2	3
Face 1_	7.4149	1.0340e-29	11.3749
Face 2_	44.8896	116.1668	36.3176
Face 3_	115.0986	104.4673	94.5973
Face 4_	171.4107	109.2993	121.0722

Again, we can see here that Face1\_2 matches best with the faces belonging to person 1 (Face1\_1, Face1\_2, Face1\_3), and can be clearly distinguished based on stress levels.

### Face 1\_3 vs Models

The table shows the final stress values between Face1\_3 and all the 12 models: (Maximum stress between face1 and probe ~11)

Final Stress(f)	1	2	3
Face 1_	9.9346	10.9989	1.3603e-28
Face 2_	146.2932	104.5626	216.0896
Face 3_	80.6462	103.9473	71.7712
Face 4_	206.2760	78.5174	79.7574

Hence we can again clearly conclude that the face matches best with the faces of person 1.

**Face 2\_1 vs Models (Maximum stress between face2 and probe ~54)**

Final Stress(f:2_1)	1	2	3
Face_1	153.4353	148.4855	158.5732
Face_2	2.9081	53.9625	40.5393
Face_3	124.3283	167.3298	155.0979
Face_4	141.3575	134.1656	199.8248

**Face 2\_2 vs Models (Maximum stress between face2 and probe ~71.3)**

Final Stress(f:2_2)	1	2	3
Face_1	168.0773	234.5768	405.4089
Face_2	71.2594	8.6434	26.9066
Face_3	234.3049	175.4894	198.5038
Face_4	121.6856	107.8792	171.5984

**Face 2\_3 vs Models (Maximum stress between face2 and probe ~1.4)**

Final Stress(f:2_3)	1	2	3
Face_1	16.7542	213.4743	221.3043
Face_2	35.6563	22.8768	1.3570
Face_3	91.2647	138.3442	143.5655
Face_4	99.1405	82.3222	201.4976

**Face 3\_1 and 3\_2 vs Models (Maximum stress between face3 and probe ~19.4, ~ 17.4 resp.)**

Final Stress(f:3_1)	1	2	3
Face_1	116.4675	120.0195	93.1911
Face_2	162.0727	139.5881	83.8418
Face_3	4.6416	19.3626	6.9007
Face_4	108.8204	127.3322	35.7880

Final Stress(f: 3_2)	1	2	3
Face 1_	148.3541	158.0237	114.9666
Face 2_	228.8330	267.5252	148.9455

Face 3_	17.3301	1.7287	17.0759
Face 4_	109.9715	40.2915	57.8296

Face 3\_3 vs Models (Maximum stress between face3 and probe ~23.26)

Final Stress(f: 3_3)	1	2	3
Face 1_	106.8522	107.1008	80.5768
Face 2_	124.0488	292.4827	218.4633
Face 3_	6.9419	23.2611	6.4462
Face 4_	304.3276	24.1091	36.7689

Face 4\_1 vs Models (Maximum stress between face4 and probe ~125.9)

Final Stress(f: 4_1)	1	2	3
Face 1_	128.0934	132.4526	310.7897
Face 2_	141.5271	110.5242	154.4562
Face 3_	30.5185	135.3957	106.3895
Face 4_	1.4960	10.2389	125.8838

Face 4\_2 vs Models (Maximum stress between face4 and probe ~10.3)

Final Stress(f: 4_2)	1	2	3
Face 1_	123.1523	109.2992	78.5173
Face 2_	134.1656	107.8792	82.3222
Face 3_	27.3322	40.2915	24.1090
Face 4_	10.2389	0	0

Face 4\_3 vs Models (Maximum stress between face2 and probe ~54)

Final Stress(f: 4_3)	1	2	3
Face 1_	123.1523	109.2992	78.5173
Face 2_	134.1656	107.8792	82.3222
Face 3_	27.33222	40.2915	24.1090
Face 4_	10.2389	0	0

## Conclusion & Contribution

Hence, we conclude that the face recognition system works well for majority of the cases with the Generalized Multi Dimensional Scaling Method. If we had the complete facial data points, then we expect the results to be more accurate.

We both have contributed equally to the project, including discussions, code executions, report writing.

## References

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