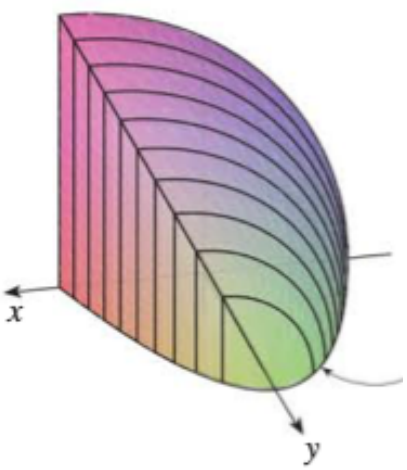
(6-2b) Volumes of Solids with Known Cross Sections

Finding Volume of 3-D Solids Whose Cross-Sections are Not Disks/Washers

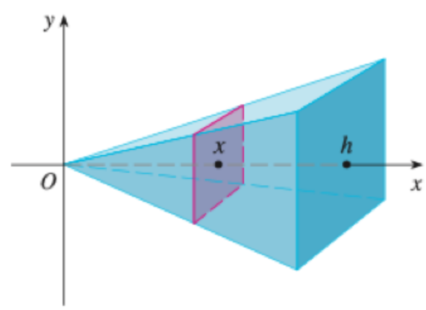
Cross Sections are Quarter Circles



Cross-sections perpendicular to the y-axis are quarter circles.

You could also say, cross-sections parallel to the x-axis are quarter circles.

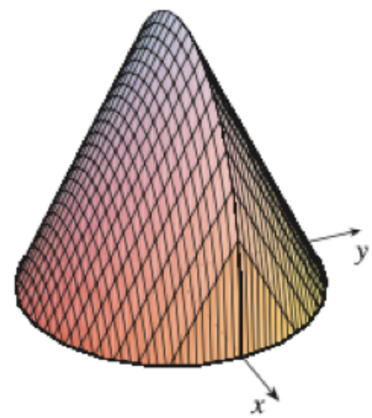
Cross Sections are Squares



Cross-sections perpendicular to the x-axis are squares.

You could also say, cross-sections parallel to the y-axis are squares.

Cross Sections are Triangles



Cross-sections perpendicular to the x-axis are triangles.

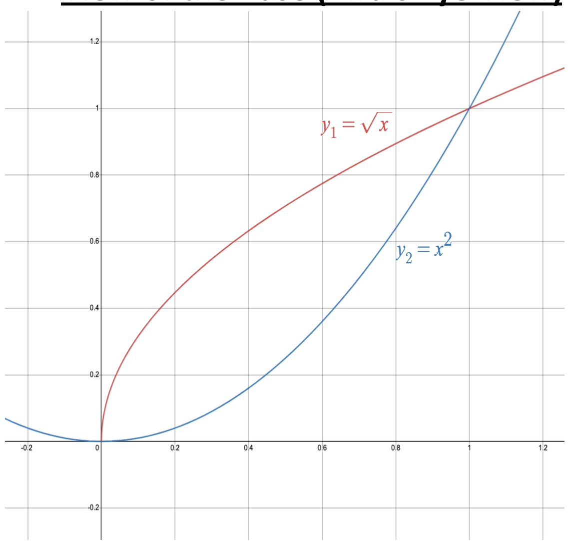
You could also say, cross-sections parallel to the y-axis are triangles.

Area of One Cross-Section $A(x)$ or $A(y)$			
Volume of One Slice $A(x)dx$ or $A(y)dy$			
Total Volume of 3-D Solid			

<u>Example 1a:</u> The base of S is the region in the xy-plane bounded by $y = \sqrt{x}\ and\ y = x^2$. Find the volume when cross-sections perpendicular to the x-axis are squares.

<u>Step 1</u>: Graph the base and try to visualize the cross-sections of the 3-D Solid

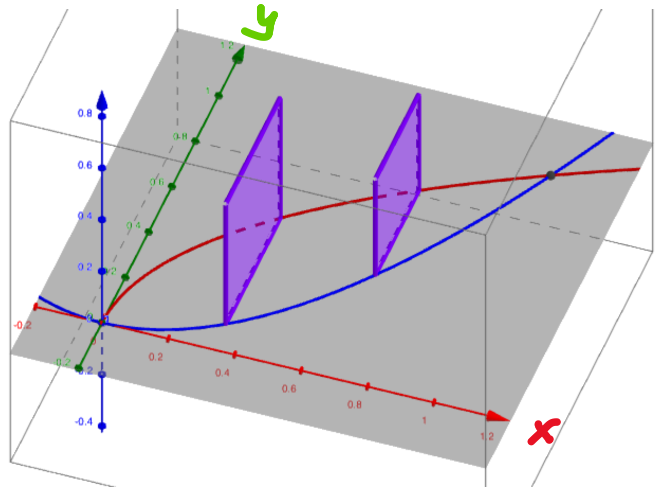
<u>View of the Base (Bird's Eye View)</u>



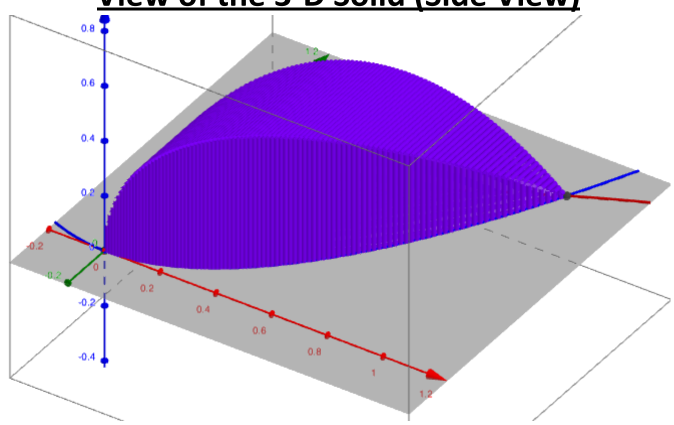
<u>Step 2</u>: Setup an expression for $A(x)\ or\ A(y)$, the area of the cross-section.

<u>Step 3a</u>: Set up an expression for volume, either $A(x)dx\ or\ A(y)dy$. This is determined by the width (dx/dy) of the rectangle in the drawing.

<u>View of Two Square Cross-Sections (Side View)</u>



<u>View of the 3-D Solid (Side View)</u>

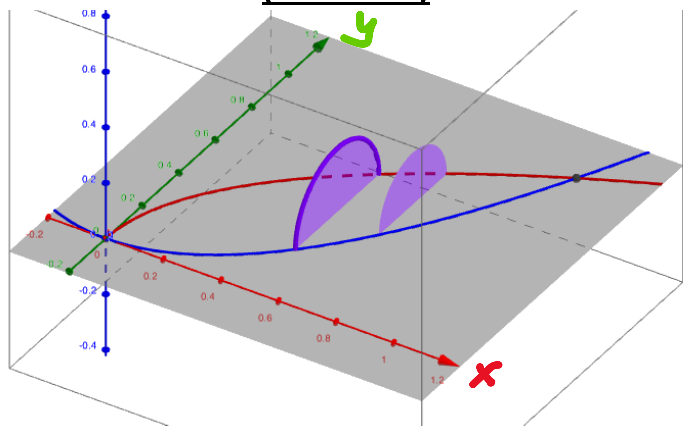


<u>Step 4</u>: Setup and evaluate an integral to total/sum the volume of <u>all</u> cross-sections.

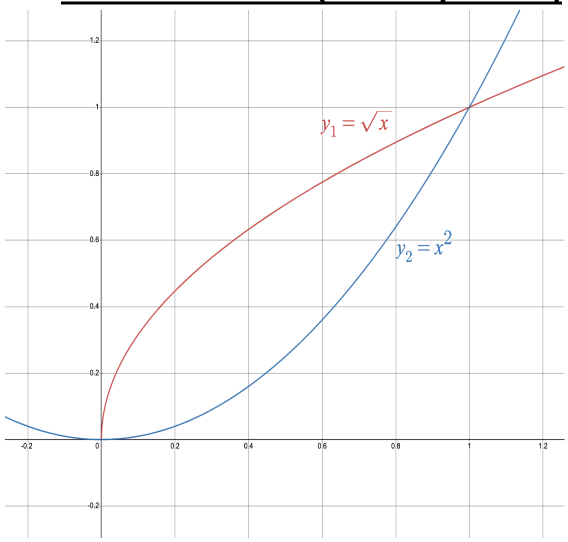
Example 1b: The base of S is the region in the xy-plane bounded by $y = \sqrt{x}$ *and* $y = x^2$. Find the volume when cross-sections perpendicular to the x-axis are semi-circles.

Step 1: Graph the base and try to visualize the cross-sections of the 3-D Solid

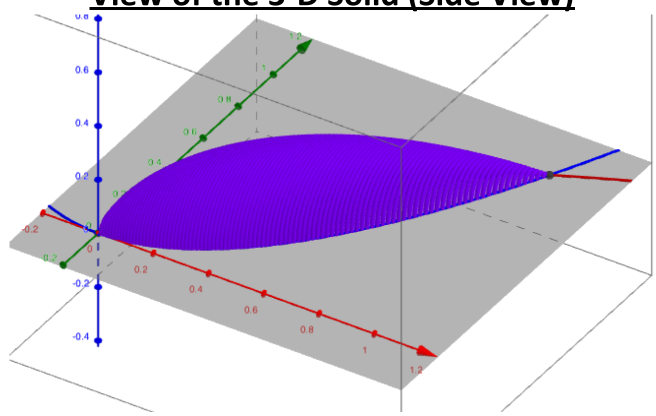
View of Two Semi-Circular Cross-Sections (Side View)



View of the Base (Bird's Eye View)



View of the 3-D Solid (Side View)



Step 2: Setup an expression for $A(x)$ *or* $A(y)$, the area of the cross-section.

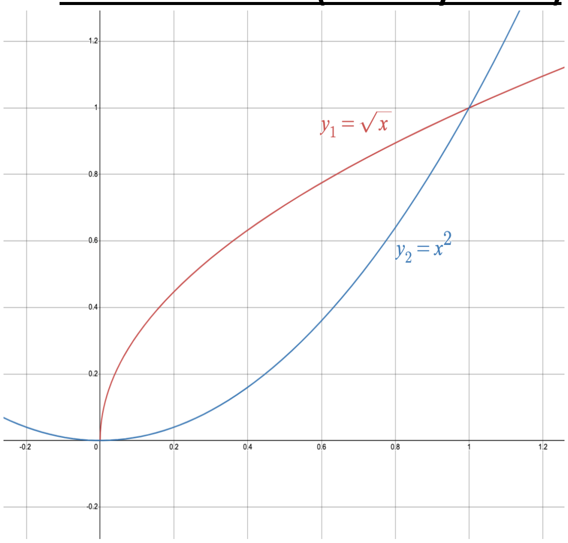
Step 3a: Set up an expression for volume, either $A(x)dx$ *or* $A(y)dy$. This is determined by the width (dx/dy) of the rectangle in the drawing.

Step 4: Setup and evaluate an integral to total/sum the volume of all cross-sections.

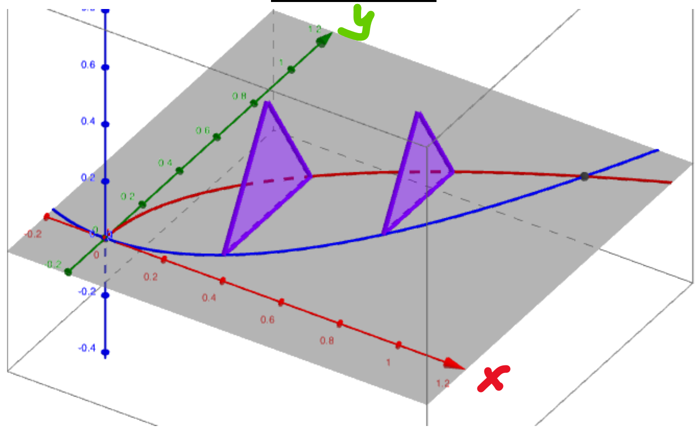
Example 1c: The base of S is the region in the xy-plane bounded by $y = \sqrt{x}$ and $y = x^2$. Find the volume when cross-sections perpendicular to the x-axis are equilateral triangles.

Step 1: Graph the base and try to visualize the cross-sections of the 3-D Solid

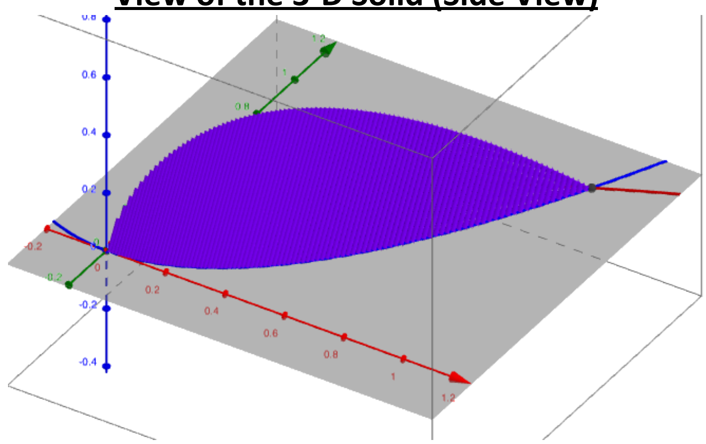
View of the Base (Bird's Eye View)



View of Two Triangular Cross-Sections (Side View)



View of the 3-D Solid (Side View)



Step 2: Setup an expression for $A(x)$ or $A(y)$, the area of the cross-section.

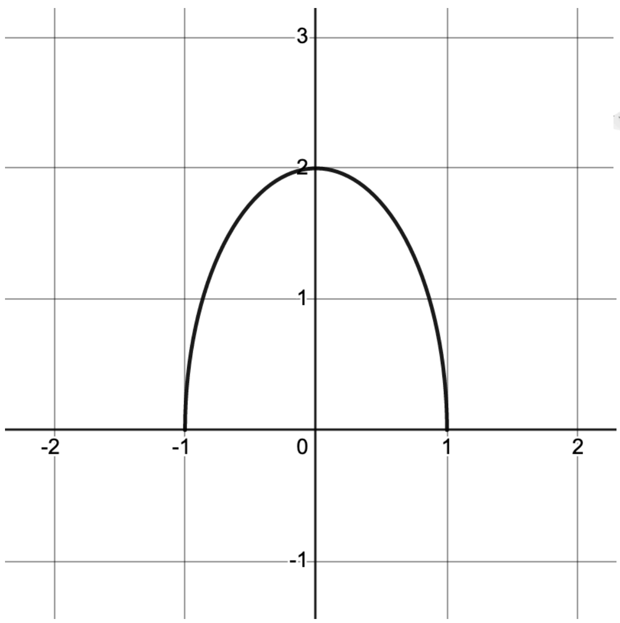
Step 3a: Set up an expression for volume, either $A(x)dx$ or $A(y)dy$. This is determined by the width (dx/dy) of the rectangle in the drawing.

Step 4: Setup and evaluate an integral to total/sum the volume of all cross-sections.

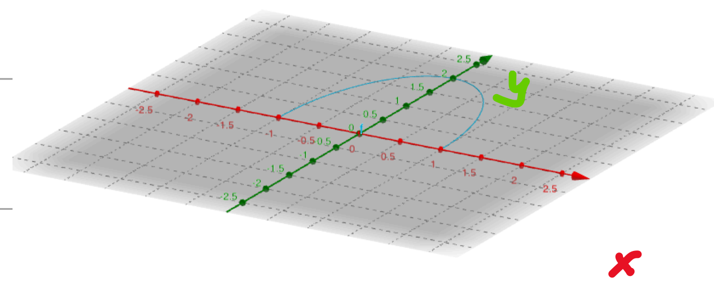
Example 2a: A solid has it's base in the xy-plane bounded by portion of the ellipse $4x^2 + y^2 = 4$ in the first and second quadrant. Every cross-section perpendicular to the y-axis is an isoceles right triangle, with hypotenuse in the base. Find the volume of the solid.

Step 1: Graph the base and try to visualize the cross-sections of the 3-D Solid

View of the Base (Bird's Eye View)



View of Two Isosceles Right Triangle Cross-Sections (Side View)



Step 2: Setup an expression for $A(x)\ or\ A(y)$, the area of the cross-section.

Step 3a: Set up an expression for volume, either $A(x)dx\ or\ A(y)dy$. This is determined by the width (dx/dy) of the rectangle in the drawing.

Step 4: Setup and evaluate an integral to total/sum the volume of all cross-sections.

Step 3b: If necessary, use the equation of the base to rewrite the volume expression using the correct variable.