

Introduction to Scientific Computing

Indian Institute of Technology, Madras

Assignment 5

Maximum Marks: 100

Assigned: April 24, 2024

Deadline: **May 5**, 2024

General Instructions

- You are expected to use the VM for this assignment. Create a directory in your home directory called `assignment_5`. Use this directory to work on the assignment.
- For each question (for question i), create a bash file called `question_i.sh` in the `assignment_5` directory. This bash file should contain the necessary code or commands to solve the respective question.
- We will be using an evaluation script to assess and evaluate your submission. Therefore, kindly ensure that the naming convention (as mentioned in usage section of each question) is strictly adhered to, and that the output which you get from running a script, matches the structure of the sample output.
- For submission, upload the MD5 checksum of the `assignment_5` directory on Moodle. You can use the following command. Make sure that you are in `HOME` directory for this command to work as intended.

```
find ./assignment_3/* -exec md5sum {} \; | cut -f 1 -d " " | md5sum
```

- After submitting the MD5 checksum on Moodle, **do not** update any file(s). Doing so will change your checksum, and your submission will not be evaluated.
- You are free to read through various resources. However, please ensure that you cite your sources to avoid plagiarism. Any detected instances of plagiarism will result in penalties.
- Please contact your assigned TA for any doubts or queries regarding this assignment.
- The **soft deadline** for this assignment is **11:59 PM** on **May 5**, 2024. Submissions after this deadline will face a linearly increasing penalty of 10 marks per late day.
- The **hard deadline** for this assignment is **11:59 PM** on **May 8**, 2024. Submissions after this deadline will not be evaluated.

[30 marks] 1. **Transform your chances:**

Context

1. The ability of Fourier Transform to approximate a signal as an infinite sum of sinusoidal waves of integral multiples of some frequency, the ability to transform a time-series data into the frequency domain, has a variety of applications in Solving of Dirichlet PDEs, Von Neumann Stability Analysis of CFD schemes, Analysis of Complicated Circuits, etc.
2. Its discrete version, discrete Fourier Transform DFT, is crucial to analyzing images in terms of noise, details, structure, filters required, etc.

Task Fourier Transform is often used to model convolution operation between two functions, where each function is applied a Fourier Transform and multiplied, to get the Fourier Transform representation of the convoluted function. Your task is to create a Sage / Sympy script that takes the functions to be convolved, as arguments from a plaintext file, performs convolution using Fourier Transform, and outputs the final convolved function.¹ You may assume that the functions given in the input are defined over $x \in (-\infty, +\infty)$.

Usage:

¹Brownie points for those who convolved traditionally, and compare the performance between the two approaches in the report³.

```
./question_1.sh functions.txt
```

Sample Input:

```
$ head functions.txt
\frac{\sqrt{2}}{8} e^{-0.5 \left(\frac{x}{2} - \frac{1}{2}\right)^2} \sqrt{\pi}
\frac{\sqrt{2}}{8} e^{-0.5 \left(\frac{x}{2} - \frac{5}{2}\right)^2} \sqrt{\pi}
```

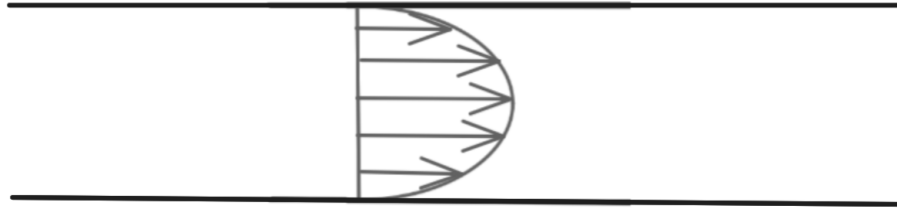
Sample Output:

```
\frac{0.0625}{\sqrt{\pi}} e^{-0.0625 (x - 6)^2}
```

[30 marks] 2. **Poised for Poiseuille flow****Context**

1. The Transport phenomenon and associated equations are derived from the continuum hypothesis, which governs the fluid flow and its related properties.
2. They are predominantly used in mechanical, chemical, metallurgy, ocean engineering, and mathematics, too, to predict fluid properties at a given time and location.

Task Given below is a typical internal fluid flow problem and given the pressure gradients, velocity profiles are related using the below flow-governing equations as:²



$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (\rho r v_r)}{\partial r} + \frac{\partial (\rho v_z)}{\partial z} + \frac{1}{r} \frac{\partial (\rho v_\theta)}{\partial \theta} = 0 \quad (1)$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = \rho \vec{g} - \nabla P + \mu \nabla^2 \vec{v} \quad (2)$$

However, governing equations are often coupled and contain partial differential terms, which cannot be solved as it is without simplifying the problem with certain logical assumptions. Some of them are as follows:

- Incompressible flow (ρ is constant)
- Fully Developed Flow (z-velocity not dependent on z)
- θ -symmetric flow (θ components and their changes can be neglected)
- Impenetrable wall (Zero radial velocity at a radius equal to pipe radius)
- Continuous and Smooth (Differentiable) flow profile.

The approach that is typically used to solve these types of coupled equations is:

- Simplify the continuity equation 1 and get a boundary condition for the pipe wall,
- Simplify z-component of the Navier-Stokes equation 2,

²All symbols depict their usual meaning

- Integrate recursively until z-velocity is obtained.

Using this knowledge of these approximations to Navier-Stokes equation 2, create a Sympy script which

- Takes as argument, a file (`press.txt`), whose only line is pressure profile along the z-direction, as a string convertible to a Sympy expression
- Computes the axial velocity **symbolically**, step-by-step using the above approach.
- Creates a C++ file (`vel.cpp`) which takes in argument as the radial distance from the axis and outputs the corresponding z-velocity to the output stream.
- Note that for this specific question, for the sake of simplicity, you can take the constants μ , R to be unity.

Usage:

```
$ ./question_1.sh press.txt
$ ./vel.cpp 0.5
```

Sample Input:

```
$ head press.txt
2*z
```

Sample Output:³

```
0.375
```

[40 marks] 3. **Reflection:**

- Write a L^AT_EX report (named `<roll_number>_A5.pdf`), which contains your observations and the overall conclusion you got from performing the tasks above. Upload the same on Moodle.
- Avoid repeating the same point multiple times in the report.
- Make use of math expressions effectively, whenever possible. Mathematically explain the significance of the assumptions stated in 2, and the corresponding equations.
- As and when possible, write your points in ordered or unordered bullet points. Images/ Screenshots of intermediate steps are appreciated.
- Remember that the more details you project in the report, the more marks you will be reflected with.

[20 marks] (a) Task 1

[20 marks] (b) Task 2

³'u' here means ' μ '