Causal Inference Workshop

Week 2 - Potential Outcomes Framework

Causal Inference Workshop

January 26, 2024

Workshop outline

A. Causal inference fundamentals
  - Modeling assumptions matter too
  - Conceptual framework (potential outcomes framework)

B. Design stage: common identification strategies
  - IV + RDD [coding]
  - DiD, DiDiD, Event Studies, New TWFE Lit [coding]
  - Synthetic Control / Synthetic DiD [coding]

C. Analysis stage: strengthening inferences
  - Limitations of identification strategies, pre-estimation steps
  - Estimation [controls] and post-estimation steps [supporting assumptions]

D. Other topics in causal inference and sustainable development
  - Inference (randomization inference, bootstrapping)
  - Weather data regressions, other common/fun SDev topics [coding]
  - Remote sensing data, other common/fun SDev topics

Causal inference roadmap

- *Potential outcomes* [framework] [today]
  - Causal effect is difference between two potential outcomes
- *Identification* [application/implementation]
  - Identifying assumptions needed for a statistical estimate to have causal interpretation
  - Removing selection bias in regressions
  - E.g., RD, IV, ...
- *Estimation* [application/implementation] [last week]
  - (Usually) use linear regression model

Outline

Causal inference roadmap

- *Potential outcomes (PO)* [framework]
  - Causal effect is difference between two potential outcomes
  - Lingua franca for expressing causal statements in economics / social sciences
  - This is one "approach to causality" (Imbens 2020)
    - Builds on Neyman 1923
    - Extended to observational studies by Rubin 1974
  - PO framework is not the *only* approach
    - Directed Acyclic Graph (DAG) approach is another alternative

- *Identification* [application/implementation]

- *Estimation* [application/implementation]

Outline

Potential outcomes framework and treatment effects

- We have:
  - A population, of which we observe sample of units $i = 1, ..., N$
  - A binary treatment of interest $D_i \in \{0, 1\} \rightarrow$ want to estimate the causal effect of D on Y
  - Let unit i's potential outcomes be: Y_i^1 if received treatment, Y_i^0 otherwise
  - Let unit i's observable outcome be: Y_i

Potential outcomes framework and treatment effects

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- Note the difference between *potential* outcomes (Y_i^1, Y_i^0) and *observable* or "actual" outcomes (Y_i); can relate them according to: $Y_i = D_i Y_i^1 + (1 - D_i) Y_i^0$

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- Define:

individual treatment effects (TEs)	$Y_i^1 - Y_i^0 \ \forall i$	*ideally estimate;* *unknowable*
average treatment effect (ATE)	$\mathbb{E}[Y_i^1 - Y_i^0]$	*reasonably estimate;* *unknowable, but can be estimated*
average treatment effect on the treated (ATT)	$\mathbb{E}[Y_i^1 - Y_i^0 \mid D_i = 1]$	*reasonably estimate;* *unknowable, but can be estimated*
difference in average observed outcomes	$\mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0]$	*what we can estimate*

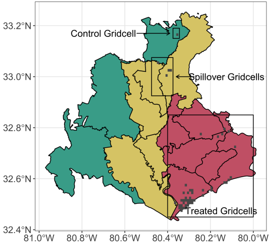
Potential outcomes framework and treatment effects: assumptions

- Assume additive treatment effects and no interference between units
- **Stable Unit Treatment Value Assumption (SUTVA)**: treatment received by one unit does not affect potential outcomes for other units
  - Each unit has only two possible potential outcomes Y_i^1, Y_i^0, which implies:
    - No spillovers
    - No general equilibrium effects

Potential outcomes framework and treatment effects: assumptions

Example of possible SUTVA violation:

- What are the effects of plastic bag laws on plastic litter in the environment?
- Use data on $\sim$100k shoreline cleanups
- Aggregate outcome data to 0.01 lat/lon gridcells
- Treatment at the zip code level (is there a policy in zip code?)
- Why may SUTVA be violated?



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  - Each unit has only two possible potential outcomes Y_i^1, Y_i^0, which implies:
    - No spillovers
    - No general equilibrium effects
  - Often not realistic in economics studies
  - Many papers on SUTVA as nuisance
    - Can change how treatment is defined (e.g., within-household spillover)
    - Change level at which you interpret results
  - Some papers on SUTVA as substance (modeling the impact of the interference between units), e.g., spillovers:
    - (Hong and Raudenbush 2006; Hudgens and Halloran 2008; Aronow and Samii 2017; Rosenbaum 2007)

Potential outcomes framework and the selection bias problem

- Back to the various parameters:

individual treatment effects (TEs)	$Y_i^1 - Y_i^0 \; \forall i$	*ideally estimate;* unknowable
average treatment effect (ATE)	$\mathbb{E}[Y_i^1 - Y_i^0]$	*reasonably estimate;* unknowable, but can be estimated
average treatment effect on the treated (ATT)	$\mathbb{E}[Y_i^1 - Y_i^0 \| D_i = 1]$	*reasonably estimate;* unknowable, but can be estimated
difference in average observed outcomes	$\mathbb{E}[Y_i \| D_i = 1] - \mathbb{E}[Y_i \| D_i = 0]$	*what we can estimate*

- We never observe causal effects

- What we can do is compute the difference in average observed outcomes:

$$\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] = \ldots$$

$$= \underbrace{\mathbb{E}[Y_i^1 - Y_i^0|D_i = 1]}_{\text{ATT}} + \underbrace{\mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0]}_{\text{selection bias}}$$

▸ math details

Potential outcomes framework and the selection bias problem

- What we can do is compute the difference in average observed outcomes:

$$\mathbb{E}[Y_i|D_i=1]-\mathbb{E}[Y_i|D_i=0]=\dots$$
$$=\underbrace{\mathbb{E}[Y_i^1-Y_i^0|D_i=1]}_{\text{ATT}}+\underbrace{\mathbb{E}[Y_i^0|D_i=1]-\mathbb{E}[Y_i^0|D_i=0]}_{\text{selection bias}}$$

- Selection bias is the average difference in Y_i^0 between the treated and untreated

▸ math details

Independence assumption and selection bias

- When treatment is independent of POs $\rightarrow$ no selection bias *in expectation*
  - $(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i$; **independence assumption** (IA)
  - Selection bias is eliminated and $\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] = \mathbb{E}[Y_i^1 - Y_i^0|D_i = 1]$ or difference in average observed outcomes equals the ATT (in expectation)
  - Holds in expectation for experiments, not for (virtually any) observational study

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- Let's consider various assignment mechanisms:
  - Random assignment (e.g., experiments)
  - Selection on observables
  - Selection on unobservables

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  - Holds in expectation for experiments, not for (virtually any) observational study

- Let's consider various assignment mechanisms:
  - Random assignment (e.g., experiments)
    - If treatment is randomly assigned, IA holds and identifies ATT (no selection bias in expectation, **NOT** for any single trial)
  - Selection on observables
  - Selection on unobservables

Independence assumption and selection bias

- When treatment is independent of POs $\rightarrow$ no selection bias *in expectation*
  - $(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i$; **independence assumption** (IA)
  - Selection bias is eliminated and $\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] = \mathbb{E}[Y_i^1 - Y_i^0|D_i = 1]$ or difference in average observed outcomes equals the ATT (in expectation)
  - Holds in expectation for experiments, not for (virtually any) observational study

- Let's consider various assignment mechanisms:
  - Random assignment (e.g., experiments)
  - Selection on observables
    - If conditional on some pre-treatment characteristic X_i, we have $(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i|X_i$, we can once again eliminate selection bias in expectation (**conditional independence assumption**, CIA)
    - Compare outcomes within each stratum of X_i
  - Selection on unobservables

Independence assumption and selection bias

- When treatment is independent of POs $\rightarrow$ no selection bias *in expectation*
  - $(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i$; **independence assumption** (IA)
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  - Holds in expectation for experiments, not for (virtually any) observational study

- Let's consider various assignment mechanisms:
  - Random assignment (e.g., experiments)
  - Selection on observables
  - Selection on unobservables
    - Will need other identification strategies to eliminate selection bias

Identifying assumptions

- We can recover an **unbiased** estimator of a causal effect iff an **identifying/independence assumption** holds:
  - if IA holds ($(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i$) $\rightarrow$ estimate ATT
  - if $\cancel{\text{IA}}$, but CIA ($(Y_i^0, Y_i^1) \perp\!\!\!\perp D_i | X_i$) $\rightarrow$ can estimate ATT in each stratum (and then combine)
  - if $\cancel{\text{CIA}}$, need relevant exogenous source of variation in D_i (e.g., $(Y_i^0, Y_i^1) \perp\!\!\!\perp Z_i$; $Z_i \not\!\perp\!\!\!\perp D_i$) $\rightarrow$ estimate a LATE

- Need an **identification strategy** that convinces us that IA holds

- Bottom-line:
  - Econometrics / regression controls won't bring causality $\rightarrow$ need identification strategy
  - BUT, even with good identification strategy, no reason to expect balance for all relevant pre-treatment characteristics $\rightarrow$ control for relevant pre-treatment variables

Outline

Treatment effects as a linear regression

- *Potential outcomes (PO)* [framework] [just now]
- *Identification* [application/implementation]
- *Estimation* [application/implementation] [last week]

Treatment effects as a linear regression

- Suppose heterogeneous TE: $Y_i^1 - Y_i^0 = \beta_i$
  - $\rightarrow \beta$ ATT $\mathbb{E}[\beta_i|D_i = 1]$)

- Then we can write

$$
\begin{aligned}
Y_i &= Y_i^0 + (Y_i^1 - Y_i^0)D_i \\
&= Y_i^0 + \beta_i D_i \\
&= Y_i^0 + (\beta_i - \beta + \beta)D_i + \mathbb{E}[Y_i^0] - \mathbb{E}[Y_i^0] \\
&= \mathbb{E}[Y_i^0] + \beta D_i + Y_i^0 - \mathbb{E}[Y_i^0] + (\beta_i - \beta)D_i \\
&= \alpha + \beta D_i + e_i
\end{aligned}
$$

Treatment effects as a linear regression

- What more can we say given the linear regression?

$$Y_i = \alpha + \beta D_i + e_i$$

  - β_{OLS} simplifies to $\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0]$ (difference in avg. observed outcomes)
  - Also, from regression, $\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] = \beta + \mathbb{E}[e_i|D_i = 1] - \mathbb{E}[e_i|D_i = 0]$
  - Lastly, $\mathbb{E}[e_i|D_i = 1] - \mathbb{E}[e_i|D_i = 0] = \mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0]$

▸ math details, part 1 ▸ math details, part 2 ▸ math details, part 3

Treatment effects as a linear regression

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- To summarize:

$$\begin{aligned}\beta_{OLS} &= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] \\ &= \beta + \mathbb{E}[e_i|D_i = 1] - \mathbb{E}[e_i|D_i = 0] \\ &= \underbrace{\beta}_{\text{ATT}} + \underbrace{\mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0]}_{\text{selection bias}}\end{aligned}$$

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Treatment effects as a linear regression

- To summarize:

$$\begin{aligned}\beta_{OLS} &= \mathbb{E}[Y_i|D_i=1] - \mathbb{E}[Y_i|D_i=0] \\ &= \quad \beta \quad + \mathbb{E}[e_i|D_i=1] - \mathbb{E}[e_i|D_i=0] \\ &= \underbrace{\quad \beta \quad}_{\text{ATT}} + \underbrace{\mathbb{E}[Y_i^0|D_i=1] - \mathbb{E}[Y_i^0|D_i=0]}_{\text{selection bias}}\end{aligned}$$

- Which means that:
  - $\hat{\beta}_{OLS}$ is unbiased for the ATT iff:
    - there is no selection bias (identification problem; independence)
    - e is uncorrelated with D (regression problem, endogeneity)

Outline

Workshop outline

Potential outcomes framework

- The original selection bias problem
- Treatment effects as a linear regression
- When does IA/CIA not hold?

An alternative framework, the DAG

- A very basic overview of DAGs
- Comparative strengths and weaknesses of the PO and DAG approaches

Summary

Endogeneity

- In simple linear regression model $y_i = \alpha + \beta x_i + e_i$, variable x_i is:
    - **endogenous** if it is correlated with the error term, or $cov[x_i, e_i] \neq 0$
    - **exogenous** otherwise, if $cov[x_i, e_i] = 0$ (A3. of CLRM)

- If x is endogenous, then OLS estimator of β will be biased and inconsistent for β

- In our setting (potential outcomes framework), if treatment D_i is endogenous ($cov[D_i, e_i] \neq 0$), there is imbalance in potential outcomes across treatment groups
    - $\rightarrow$ CIA doesn't hold (again, identification problem $\leftrightarrow$ regression problem)

Sources of endogeneity

- **Reverse causality or simultaneity**
  - If y also affects D, this is captured by e, making e correlated with D
- **Measurement error** in D that is **correlated** with y
- **Omitted variable bias (OVB)**
  - If omitted variable w is correlated with D, e is correlated with D (w is a "confounding variable")

$\rightarrow$ in observational studies, excluding confounder creates bias, so must adjust for all confounders; but we can rarely be certain to have measured all confounders, which is why we turn to alternative **"identification" strategies**

Outline

Outline

Directed acyclic graphs (DAGs)

- An alternative to the potential outcomes framework is the causal graph framework or work on directed acyclic graphs (DAGs) (Pearl 2009)
  - PO and DAG frameworks are *not* contradicting; both define causality using counterfactuals
  - Each framework has its own benefits (see Imbens 2020 for a review of these) and are therefore complementary perspectives

Directed acyclic graphs (DAGs)

- Relationships between random variables are encoded with nodes and directed edges
  - Nodes are random variables (solid for observed variables, hollow for unobserved)
  - Arrows represent possible direct causal relationships
  - Paths are sequences of edges
  - DAG is a *complete* encoding of assumptions about causal relationships

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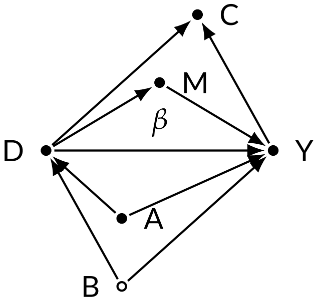
  - Types of elementary paths:
    - **Mediating path:** $D \rightarrow M \rightarrow Y$
    - **Confounding paths:** $D \leftarrow A \rightarrow Y$ (closed); $D \dashleftarrow B \dashrightarrow Y$ (open)
    - **Colliding path:** $D \rightarrow C \leftarrow Y$

  - Identification strategies:
    - Blocking back-door paths (adjusting for all confounders)
    - Instruments (alternative identification strategies)

    $\rightarrow$ same conclusion as with potential outcomes framework



Outline

Strengths and weaknesses of the PO and DAG approaches

- See Imbens 2020 for a review of the relevance of DAGs for empirical economics
  - **Experiments and manipulability:**
    $\rightarrow$ PO framework elevates randomized experiments as "gold standard", while DAG doesn't deem experiments special ($\sim$ notion of manipulability)
  - **Parts of causal analysis addressed: (pre-identification, identification, post-identification)**
    $\rightarrow$ DAGs only consider step 2, while steps 2 and 3 are considered jointly in PO
  - **Representation of identifying assumptions and identification strategies**
    $\rightarrow$ Identifying assumptions explicit in graphical versions and often much clearer than algebraic versions, BUT many other assumptions not easily captures in DAG framework; accounting for treatment heterogeneity difficult with DAGs

- Bottom-line:
  - Can be very helpful for thinking about or communicating research designs
  - May be helpful to know how to represent your analysis in both frameworks

Outline

Workshop outline

Potential outcomes framework

An alternative framework, the DAG

Summary

Causal inference roadmap

- *Potential outcomes* [framework] [today]
  - Causal effect is the difference between two potential outcomes
  - We can't observe this difference, but can see differences in average observed outcomes
  - If **(conditional) independence assumption** holds, can estimate unbiased ATT
- *Identification* [application/implementation] [up next!]
  - In most empirical settings, IA and CIA do not hold, which is why we need an **identification strategy**
  - Want to eliminate selection bias (identification problem)
- *Estimation* [application/implementation] [last week]
  - (Usually) use linear regression model
  - $\hat{\beta}_{OLS}$ unbiased estimator for ATT if e is uncorrelated with treatment (regression problem)

Questions? Comments?

Thank you!

References

Heavily based on Claire Palandri's 2022 version of the Causal Inference Workshop.

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Appendix

$$
\begin{aligned}
\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] &= \mathbb{E}[Y_i^1|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0] \\
&= \mathbb{E}[Y_i^1|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 1] + \mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0] \\
&= \underbrace{\mathbb{E}[Y_i^1 - Y_i^0|D_i = 1]}_{\text{ATT}} + \underbrace{\mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0]}_{\text{selection bias}}
\end{aligned}
$$

back

Appendix

$$Y_i = \alpha + \beta D_i + e_i$$

→ OLS slope estimand simplifies to the difference in average observed outcomes

$$\beta_{OLS} = \frac{cov[Y_i, D_i]}{Var[D_i]} = \frac{\mathbb{E}[Y_i D_i] - \mathbb{E}[Y_i]\mathbb{E}[D_i]}{\mathbb{E}[D_i^2] - \mathbb{E}[D_i]^2} =$$

$$= \frac{\mathbb{E}[Y_i|D_i = 1]P(D_i = 1) - \left(\mathbb{E}[Y_i|D_i = 0]P(D_i = 0) + \mathbb{E}[Y_i|D_i = 1]P(D_i = 1)\right) \times \frac{1}{2}}{\left(\frac{1}{2} \times 1^2 + \frac{1}{2} \times 0^2\right) - \left(\frac{1}{2} \times 1 + \frac{1}{2} \times 0\right)^2}$$

$$= \frac{\mathbb{E}[Y_i|D_i = 1] \times \frac{1}{2} - \left(\mathbb{E}[Y_i|D_i = 0] \times \frac{1}{2} + \mathbb{E}[Y_i|D_i = 1] \times \frac{1}{2}\right) \times \frac{1}{2}}{\frac{1}{4}}$$

$$= \frac{\mathbb{E}[Y_i|D_i = 1] - \left(\mathbb{E}[Y_i|D_i = 0] \times \frac{1}{2} + \mathbb{E}[Y_i|D_i = 1] \times \frac{1}{2}\right)}{\frac{1}{2}}$$

$$= \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0]$$

back

Appendix

Since:

- $\mathbb{E}[Y_i|D_i = 1] = \alpha + \beta + \mathbb{E}[e_i|D_i = 1]$ and
- $\mathbb{E}[Y_i|D_i = 0] = \alpha + \mathbb{E}[e_i|D_i = 0]$

We then have:

$$\mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0] = \beta + \mathbb{E}[e_i|D_i = 1] - \mathbb{E}[e_i|D_i = 0]$$

back

Appendix

Since:

$$e_i = Y_i^0 - \mathbb{E}[Y_i^0] + (\beta_i - \beta)D_i$$

We have:

$$\begin{aligned}
\mathbb{E}[e_i|D_i = 1] - \mathbb{E}[e_i|D_i = 0] &= \mathbb{E}[\beta_i - \beta|D_i = 1] + e\mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0] - \mathbb{E}[Y_i^0|D_i = 0] + \mathbb{E}[Y_i^0] \\
&= \mathbb{E}[\beta_i|D_i = 1] - \beta + \mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0] \\
&= \mathbb{E}[Y_i^0|D_i = 1] - \mathbb{E}[Y_i^0|D_i = 0]
\end{aligned}$$

back