Causal Inference Workshop

Week 7 - Synthetic Control, Synthetic Difference-in-Differences

Causal Inference Workshop

March 1, 2024

Workshop outline

A. Causal inference fundamentals
   - Modeling assumptions matter too
   - Conceptual framework (potential outcomes framework)
B. Design stage: common identification strategies
   - IV + RDD [coding]
   - DiD, DiDiD, Event Studies, New TWFE Lit [coding]
   - Synthetic Control / Synthetic DiD [coding]
C. Analysis stage: strengthening inferences
   - Limitations of identification strategies, pre-estimation steps
   - Estimation [controls] and post-estimation steps [supporting assumptions]
D. Other topics in causal inference and sustainable development
   - Inference (randomization inference, bootstrapping)
   - Weather data regressions, other common/fun SDev topics [coding]
   - Remote sensing data, other common/fun SDev topics

Causal inference roadmap

What is causal inference?

- Process by which we use data to make claims about causal relationships
- *Potential outcomes* [framework]
  - Causal effect is difference between two potential outcomes
- *Identification* [application/implementation] [today]
  - Identifying assumptions needed for a statistical estimate to have causal interpretation
  - Removing selection bias in regressions
  - E.g., RD, IV, ...
- *Estimation* [application/implementation]
  - (Usually) use linear regression model

Overview of event studies, DiD+, and SCM

- General setup:
  - **Objective:** estimate the impact of some treatment at a certain time
  - **Data**: panel data (multiple units, multiple time periods)

  $\rightarrow$ to estimate **causal** effect of event, we want the counterfactual of it occurring (this is the unit's outcome had the event not occurred)

- Event study, DiD+, and synthetic control methods all use some counterfactual
  - **Event study**: All units treated, assume group's *past* value is CF (*within* variation)
  - **DiD, DiDiD, TWFE**: Some units never get treated, assume CF *trends* of treated and untreated are parallel, use control units to remove trends in Y in treated (*within* and *between* variation)
  - **Synthetic control, synthetic DiD**: Units large aggregates, most (some) units never get treated, combine untreated units to construct optimal synthetic unit that captures CF trajectory of treated unit(s)

Outline

Workshop outline

Synthetic control method

Synthetic difference-in-differences

Coding example

Synthetic control method, overview and original setting

Estimating the effects of *aggregate* interventions

- Event or intervention occurs at the level of aggregate entities (e.g., states) at $t = \tau$

    $\rightarrow$ when there are a few aggregate entities, may be hard to find single or few unaffected unit that provide a suitable comparison at the time of the intervention

Synthetic control method, overview and original setting

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- **Synthetic control method idea**: instead, take a weighted average of unaffected units, which may approximate the characteristics of the affected units better

Synthetic control method, overview and original setting

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- **Synthetic control method idea**: instead, take a weighted average of unaffected units, which may approximate the characteristics of the affected units better

- Developed in Abadie and Gardeazabal (2003), *AER*

  Effect of terrorism on aggregate income in the Basque region in Spain

- Elaborated upon in Abadie et al. (2010), *Journal of the American Statistical Association*

  Effect of the 1988 Californian tobacco control program (cigarette tax), “Proposition 99”

Synthetic control method, DGP

- Event or intervention occurs at the level of aggregate entities (e.g., states, countries) at time $t = \tau$. A single unit $j = 1$ is affected, the other units $j = 2, \ldots, J + 1$ are unaffected. As the units of observation are a small number of aggregate entities, no single unit alone may provide a good comparison for the exposed unit.
- We observe k predictors of the outcome, $X_{1j}, \ldots, X_{kj}$, which are unaffected by intervention (and may include pre-intervention values of Y)
- We consider a **"synthetic control unit"**, a weighted average of different J units in the donor pool that should reproduce the behavior of Y for the treated unit in the absence of the treatment
  - Set of optimal weights $W^* = (w_2^*, \ldots, w_{J+1}^*)$ is the "synthetic control"

SCM, identifying assumption, estimand, and estimator

- Identifying assumptions

A1. synthetic control unit reproduces behavior of Y for treated unit in the absence of treatment

- Estimand

$$\beta_{SCM,t} = Y_{1t} - \hat{Y}_{SU,t} = \underbrace{Y_{1t}^{1} - Y_{1t}^{0}}_{TET}$$

- Estimator

$$\forall t, \hat{\beta}_{SCM,t} = Y_{1t} - \sum_{j=2}^{J+1} w_j Y_{jt}$$

$\rightarrow$ synthetic control estimator is not regression-based, so we don't automatically get a standard error or p-value (need to do additional steps)

SCM, examples

- Abadie and Gardeazabal (2003), *AER*
  - construct synthetic Basque Country from other regions of Spain in order to study the effects of terrorism/political instability on economic prosperity
    $\rightarrow$ after the outbreak of terrorism, per capita GDP declined by $\sim$10% relative to the synthetic control region

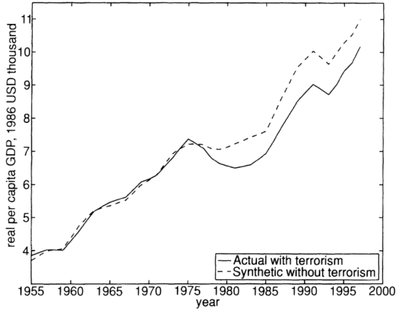

FIGURE 1. PER CAPITA GDP FOR THE BASQUE COUNTRY

SCM, examples

- Abadie et al. (2010), *Journal of the American Statistical Association*
  - construct synthetic California from other US states in order to study the effects of the 1988 Proposition 99 (a large-scale tobacco control program)
    → annual per-capita cigarette sales decreased by about 26 packs by 2000

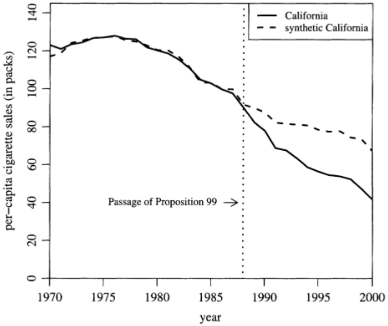

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

SCM, examples

- Andersson (2019), *AEJ: Economic Policy*
  - construct synthetic Sweden from other OECD countries to study the effects of a carbon tax and value-added tax on transport fuel on emissions
    → carbon dioxide emissions from transport declined ∼11% (mostly due to carbon tax)

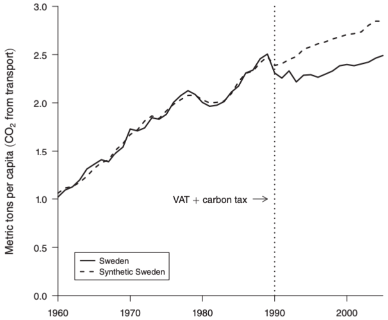

Figure 4. Path Plot of Per Capita CO_2 Emissions from Transport during 1960–2005: Sweden versus Synthetic Sweden

SCM, steps and best practices

- **Step 1.** Design phase: creating the comparison unit = determining the optimal weights
  - *Goal*: minimize the distance between the treated unit and the synthetic unit, in terms of ability of predicting the post-treatment outcome Y
  - **Duel optimization problem:** which units? + which characteristics?
  - *Calculate weights on units* ($w_2, \ldots, w_{J+1}$):

$$w_j^*(V) = \arg\min_{w_j} ||X_1 - X_0 W|| = \sqrt{(X_1 - X_0 W)' V (X_1 - X_0 W)}$$

$$\text{s.t.} \qquad w_j \geqslant 0 \quad \forall j = 2, \ldots, J+1 \quad \text{and} \quad \sum w_j = 1$$

  - *Calculate relative importance of each characteristic in predicting post-treatment outcome* ($v_1, \ldots, v_k$) (e.g., minimize mean squared prediction error (MSPE) of synthetic control)

$$\sum_{t \in \{1,2,\ldots,T_0\}} \left(Y_{1t} - w_2(V) Y_{2t} - \ldots - w_{J+1}(V) Y_{J+1t}\right)^2$$

SCM, steps and best practices

- **Step 2.** Estimation
  - Report the weights $w_2^*, ..., w_{J+1}^*$
  - Check that synthetic unit has values similar to treated unit across all matching variables
  - Show robustness to
    - Choice of predictors
    - Choice of units in the donor pool (e.g., leave-one-out analysis)

TABLE 2—COUNTRY WEIGHTS IN SYNTHETIC SWEDEN

Country	Weight	Country	Weight
Australia	0.001	Japan	0
Belgium	0.195	New Zealand	0.177
Canada	0	Poland	0.001
Denmark	0.384	Portugal	0
France	0	Spain	0
Greece	0.090	Switzerland	0.061
Iceland	0.001	United States	0.088

Note: With the synthetic control method, extrapolation is not allowed so all weights are between $0 \leq w_j \leq 1$ and $\sum w_j = 1$.

SCM, steps and best practices

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TABLE 1—CO_2 EMISSIONS FROM TRANSPORT PREDICTOR MEANS BEFORE TAX REFORM

Variables	Sweden	Synth. Sweden	OECD sample
GDP per capita	20,121.5	20,121.2	21,277.8
Motor vehicles (per 1,000 people)	405.6	406.2	517.5
Gasoline consumption per capita	456.2	406.8	678.9
Urban population	83.1	83.1	74.1
CO_2 from transport per capita 1989	2.5	2.5	3.5
CO_2 from transport per capita 1980	2.0	2.0	3.2
CO_2 from transport per capita 1970	1.7	1.7	2.8

Notes: All variables except lagged CO_2 are averaged for the period 1980–1989. GDP per capita is purchasing power parity (PPP)—adjusted and measured in 2005 US dollars. Gasoline consumption is measured in kilograms of oil equivalent. Urban population is measured as percentage of total population. CO_2 emissions are measured in metric tons. The last column reports the population-weighted averages of the 14 OECD countries in the donor pool.

SCM, steps and best practices

- **Step 2.** Estimation
  - Report the weights $w_2^*, \ldots, w_{J+1}^*$
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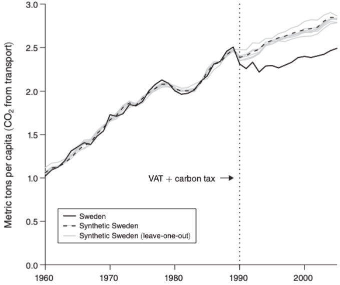
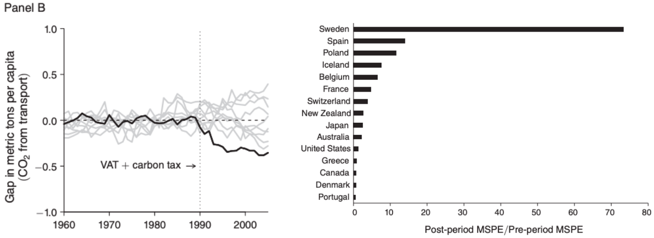


FIGURE 9. LEAVE-ONE-OUT: DISTRIBUTION OF THE SYNTHETIC CONTROL FOR SWEDEN

SCM, steps and best practices

- **Step 3.** Inference: Is the treatment effect on the exposed unit "significant"?
  $\longrightarrow$ *design-based inference*
  - Compute a permutation distribution and p-value
    - Iteratively reassign the treatment to each untreated unit, repeat the SCM procedure $\rightarrow$ distribution of placebo effects
    - Choose a test statistic, for example: the ratio of the post- to pre-treatment root mean squared prediction error $RMSPE = RMSPE_{post}/RMSPE_{pre}$
    - Sort the values of this statistic in descending order
    - Calculate the p-value $p = \frac{\text{rank}}{\text{total=J}}$ (position in the distribution)



SCM, steps and best practices

- **Falsification tests:** Move the event to points earlier in time, estimate on this earlier placebo date and check that there are zero effects

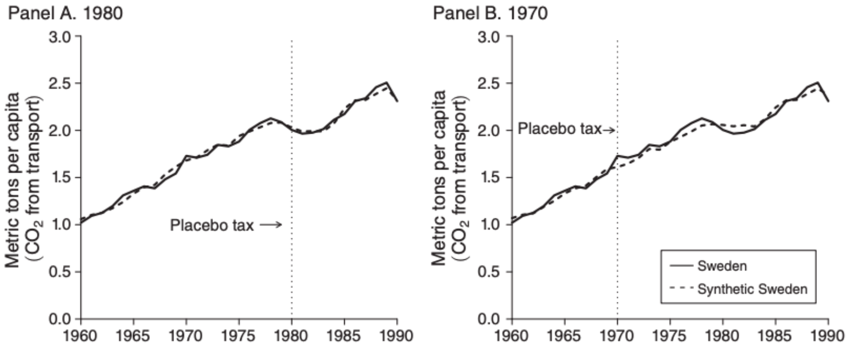


FIGURE 6. PLACEBO IN-TIME TESTS

SCM, strengths, and weaknesses

See Abadie (2021), *JEL* for more details

- \+ SCM formalizes the creation of a comparison unit using data-driven procedure, synthetic unit is optimally estimated counterfactual to the unit that received treatment
- \+ Transparency of: fit (char.'s & pre-intervention outcomes) $+$ counterfactual (weights)
- \+ Generalization of DiD that relaxes parallel trends assumption (don't need parallel trends, instead build artificial control that has best pre-trend possible)
- \+ Precludes extrapolation (synthetic control weights are nonnegative and sum to one)
- \- SCM doesn't remove subjective researcher bias (specification searching)
  - \- Distance function is still endogenously chosen by the researcher, and therefore one can cherry-pick through choice of covariates and distance function (see Ferman et al. (2020)) $\rightarrow$ important to present results under various specifications

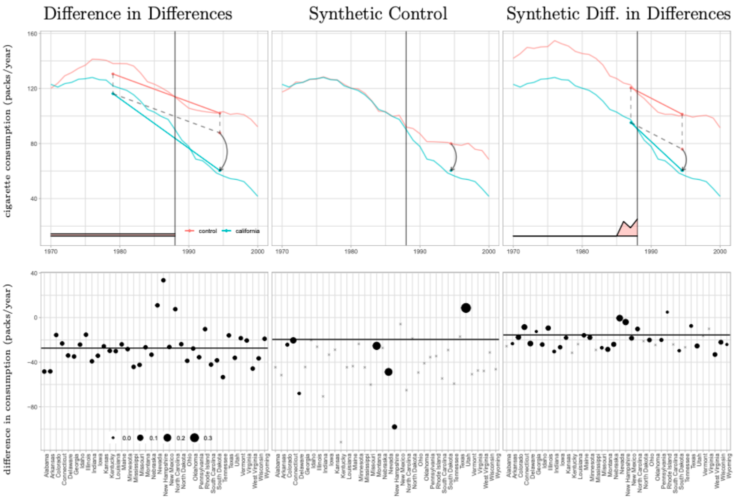
Outline

Synthetic difference-in-differences

- Extension of synthetic control method Arkhangelsky et al. (2021), *AER*
- Combine features of synthetic control and difference-in-differences
  - DiD: parallel trends assumptions; can control for selection effects by accounting for unit-specific and time-specific fixed effects
  - SCM: reweighing units to match pretrends
  - → SDiD: reweigh and match pre-exposure trends to weaken reliance on parallel trend assumptions
- Makes TWFE regression "local", by emphasizing
  - Units that are on average similar in terms of past to the treated units
  - Periods that are on average similar to the treated periods

$$\left(\hat{\tau}^{sdid}, \hat{\mu}, \hat{\alpha}, \hat{\beta}\right) = \underset{\tau,\mu,\alpha,\beta}{\arg\min} \left\{ \sum_{i=1}^{N} \sum_{t=1}^{T} \left(Y_{it} - \mu - \alpha_i - \beta_t - W_{it}\tau\right)^2 \hat{\omega}_i^{sdid} \hat{\lambda}_t^{sdid} \right\}$$

Synthetic difference-in-differences



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Synthetic difference-in-differences

Coding example

Synthetic control and synthetic diff-in-diff examples

Use: `01_synthdid.R`

- Load data (Abadie et al. (2010))
- Synthetic control estimation (`tidysynth` R package)
  - Recreates Abadie et al. (2010)
  - Can modify predictors and optimization parameters
- Synthetic difference-in-differences estimation (`sytnhdid` R package)
  - Recreates comparison in Arkhangelsky et al. (2021)

Questions? Comments?

Thank you!

References

Heavily based on Claire Palandri's 2022 version of the Causal Inference Workshop.

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