Family Name .... *Name* ....*Student ID(Matricola):* ....

Solve the problems adding to the replies short and essential explenations. *Please write the solutions in the designed areas. NO EXTRA SHEETS WILL BE ACCEPTED.* 1 Problem = 4 marks. Duration: 2 hours. No questions allowed in the first hour and in the last 20 minutes.

1	2	3a	3b	4a	4b	4c	5	6	TOTAL

1. Prove that there exists infinitely many integers n such that $3n^2 + 2$ is divisible both by 5 and by 7.

2. Determine the continued fraction of $\sqrt{6/7}$.

3. For $n \in \mathbf{N}$, let $f(n) = 0$ if n is even and $f(n) = (-1)^{(n-1)(n+5)/8}$ if n is odd.

   a. Show that $f(n)$ is a multiplicative function. Is it completely multiplicative?

   b. Prove that $\sum_{n \leq T} f(n) = O(1)$.

4. Let $n \in \mathbf{N}$ and let $\lambda(n) = \prod_{p|n} (-1)^{v_p(n)}$

   a. prove that λ is multiplicative. Is it completely multiplicative?

   b. prove that $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a perfect prower} \\ 0 & \text{otherwise} \end{cases}$.

c. Determine a function g such that $g \star \lambda(n) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$.

5. Determine all integers x, y, z, t such that $0 < x \leq y \leq z \leq t$ and $x^2 + y^2 + z^2 + t^2 = 7$ or $x^2 + y^2 + z^2 + t^2 = 28$. Deduce that for any fixed $n \geq 1$, $x^2 + y^2 + z^2 + t^2 = 7 \cdot 4^n$ has at least three solutions.

6. Let p and q be distinct odd prime numbers. How many integers a are there such that $0 \leq a < pq$ and $X^2 \equiv a \bmod pq$ has a least one solution?