In [4]:

```
num_friends = [100, 49, 41, 40, 25, 21, 21, 19, 19, 18, 18, 16, 15, 15, 15, 15, 14, 14,
```

In [5]:

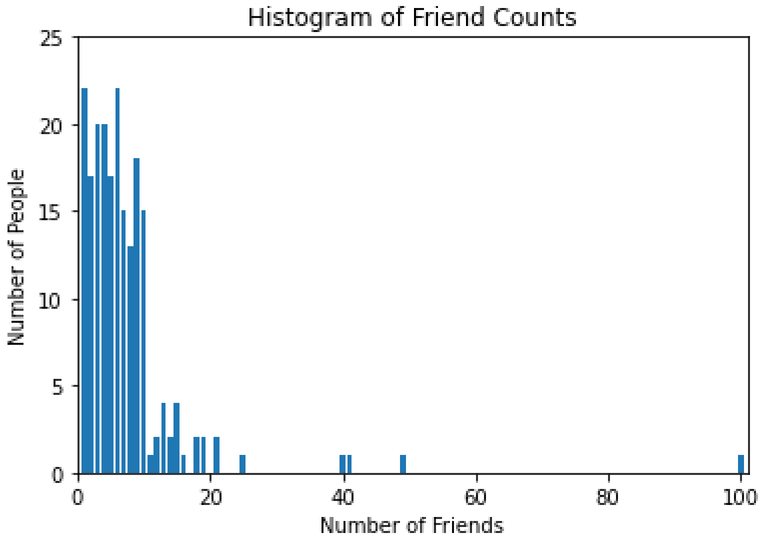
```
from collections import Counter
import matplotlib.pyplot as plt
```

In [6]:

```
friend_counts = Counter(num_friends)
xs = range(101)                         # The largest value is 100
ys = [friend_counts[x] for x in xs]     # The height is just the number of friends
plt.bar(xs, ys)
plt.axis([0, 101, 0, 25])
plt.title("Histogram of Friend Counts")
plt.xlabel("Number of Friends")
plt.ylabel("Number of People")
# plt.show()
```

Out[6]:

```
Text(0, 0.5, 'Number of People')
```



In [7]:

```
num_points = len(num_friends)               # 204
```

In [8]:

```
assert num_points == 204
```

In [9]:

```
largest_value = max(num_friends)            # 100
smallest_value = min(num_friends)           # 1
```

In [10]:

```
assert largest_value == 100
assert smallest_value == 1
```

In [11]:

```
sorted_values = sorted(num_friends)
smallest_value = sorted_values[0]           # 1
second_smallest_value = sorted_values[1]    # 1
second_largest_value = sorted_values[-2]    # 49
```

In [12]:

```
assert smallest_value == 1
```

In [13]:

```
assert second_smallest_value == 1
```

In [14]:

```
assert second_largest_value == 49
```

In [15]:

```
from typing import List
```

In [16]:

```
def mean(xs: List[float]) -> float:
    return sum(xs) / len(xs)
```

In [17]:

```
mean(num_friends)   # 7.333333
```

Out[17]:

```
7.333333333333333
```

In [18]:

```
assert 7.3333 < mean(num_friends) < 7.3334
```

In [19]:

```
# The Underscores Indicate That These Are Private Functions, as They're
# Intended to Be Called by the Median Function but Not by Other People
# Using the Statistics Library

def _median_odd(xs: List[float]) -> float:
    """If len(xs) is odd, the median is the middle element"""
    return sorted(xs)[len(xs) // 2]
```

In [20]:

```
def _median_even(xs: List[float]) -> float:
    """If len(xs) is even, it's the average of the middle two elements"""
    sorted_xs = sorted(xs)
    hi_midpoint = len(xs) // 2  # e.g. length 4 => hi_midpoint 2
    return (sorted_xs[hi_midpoint - 1] + sorted_xs[hi_midpoint]) / 2
```

In [22]:

```
def median(v: List[float]) -> float:
    """Finds the middle-most value of v"""
    return _median_even(v) if len(v) % 2 == 0 else _median_odd(v)
```

In [23]:

```
assert median([1, 10, 2, 9, 5]) == 5
```

In [24]:

```
assert median([1, 9, 2, 10]) == (2 + 9) / 2
```

In [25]:

```
assert median(num_friends) == 6
```

In [26]:

```python
def quantile(xs: List[float], p: float) -> float:
    """Returns the p-th percentile value in x"""
    p_index = int(p * len(xs))
    return sorted(xs)[p_index]
```

In [27]:

```python
assert quantile(num_friends, 0.10) == 1
```

In [28]:

```python
assert quantile(num_friends, 0.25) == 3
```

In [29]:

```python
assert quantile(num_friends, 0.75) == 9
```

In [30]:

```python
assert quantile(num_friends, 0.90) == 13
```

In [31]:

```python
def mode(x: List[float]) -> List[float]:
    """Returns a list, since there might be more than one mode"""
    counts = Counter(x)
    max_count = max(counts.values())
    return [x_i for x_i, count in counts.items()
            if count == max_count]
```

In [32]:

```python
assert set(mode(num_friends)) == {1, 6}
```

In [33]:

```python
# "range" Already Means Something In Python, so We'll Use a Different Name

def data_range(xs: List[float]) -> float:
    return max(xs) - min(xs)
```

In [34]:

```python
assert data_range(num_friends) == 99
```

In [43]:

```python
from scratch.linear_algebra import sum_of_squares
```

In [44]:

```python
def de_mean(xs: List[float]) -> List[float]:
    """Translate xs by subtracting its mean (so the result has mean 0)"""
    x_bar = mean(xs)
    return [x - x_bar for x in xs]
```

In [45]:

```python
def variance(xs: List[float]) -> float:
    """Almost the average squared deviation from the mean"""
    assert len(xs) >= 2, "Variance requires at least two elements"

    n = len(xs)
    deviations = de_mean(xs)
    return sum_of_squares(deviations) / (n - 1)
```

In [46]:

```
assert 81.54 < variance(num_friends) < 81.55
```

In [47]:

```
import math
```

In [48]:

```
def standard_deviation(xs: List[float]) -> float:
    """The standard deviation is the square root of the variance"""
    return math.sqrt(variance(xs))
```

In [49]:

```
assert 9.02 < standard_deviation(num_friends) < 9.04
```

In [50]:

```
def interquartile_range(xs: List[float]) -> float:
    """Returns the difference between the 75th percentile and the 25th percentile"""
    return quantile(xs, 0.75) - quantile(xs, 0.25)
```

In [51]:

```
assert interquartile_range(num_friends) == 6
```

In [52]:

```
daily_minutes = [1, 68.77, 51.25, 52.08, 38.36, 44.54, 57.13, 51.4, 41.42, 31.22, 34.76
```

In [53]:

```
daily_hours = [dm / 60 for dm in daily_minutes]
```

In [54]:

```
from scratch.linear_algebra import dot
```

In [55]:

```
def covariance(xs: List[float], ys: List[float]) -> float:
    assert len(xs) == len(ys), "The xs and ys must have same number of elements"

    return dot(de_mean(xs), de_mean(ys)) / (len(xs) - 1)
```

In [56]:

```
assert 22.42 < covariance(num_friends, daily_minutes) < 22.43
```

In [57]:

```
assert 22.42 / 60 < covariance(num_friends, daily_hours) < 22.43 / 60
```

In [58]:

```
def correlation(xs: List[float], ys: List[float]) -> float:
    """Measures how much the xs and ys vary in tandem about their means"""
    stdev_x = standard_deviation(xs)
    stdev_y = standard_deviation(ys)
    if stdev_x > 0 and stdev_y > 0:
        return covariance(xs, ys) / stdev_x / stdev_y
    else:
        return 0    # If there is no variation, then the correlation is zero
```

In [59]:

```
assert 0.24 < correlation(num_friends, daily_minutes) < 0.25
```

In [60]:

```
assert 0.24 < correlation(num_friends, daily_hours) < 0.25
```

In [61]:

```
outlier = num_friends.index(100)    # Index of the outlier
```

In [62]:

```
num_friends_good = [x
                    for i, x in enumerate(num_friends)
                    if i != outlier]
```

In [63]:

```
daily_minutes_good = [x
                      for i, x in enumerate(daily_minutes)
                      if i != outlier]
```

In [64]:

```
daily_hours_good = [dm / 60 for dm in daily_minutes_good]
```

In [65]:

```
assert 0.57 < correlation(num_friends_good, daily_minutes_good) < 0.58
```

In [66]:

```
assert 0.57 < correlation(num_friends_good, daily_hours_good) < 0.58
```