Completely Randomized Designs

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CRD

Create the Data Set

```r
r1 <- c(9.8, 8.8, 8.4, 9.5, 9.2)
r2 <- c(8.2, 6.9, 7.5, 7.1, 6.5)
r3 <- c(6.8, 6.6, 5.9, 7.3, 7.2)
r4 <- c(4.8, 5.2, 5.4, 5.9, 4.6)
times <- c(r1, r2, r3, r4)
trt <- rep(1:4, each = 5)
dat <- data.frame(y = times, trt = as.factor(trt))
```

Summary Statistics by Treatments

```r
(means <- tapply(dat$y, dat$trt, mean))
```

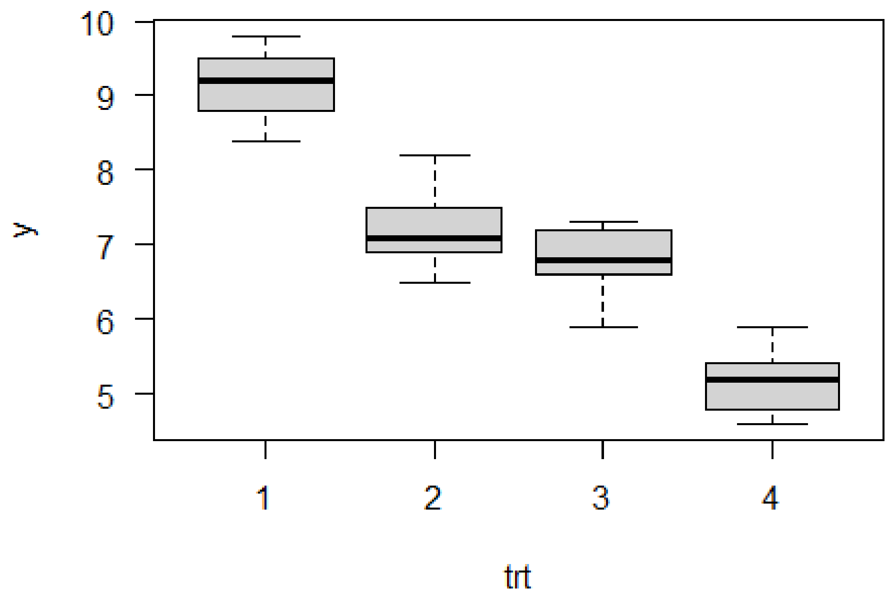
```
##    1    2    3    4 
## 9.14 7.24 6.76 5.18
```

```r
(vars <- tapply(dat$y, dat$trt, var))
```

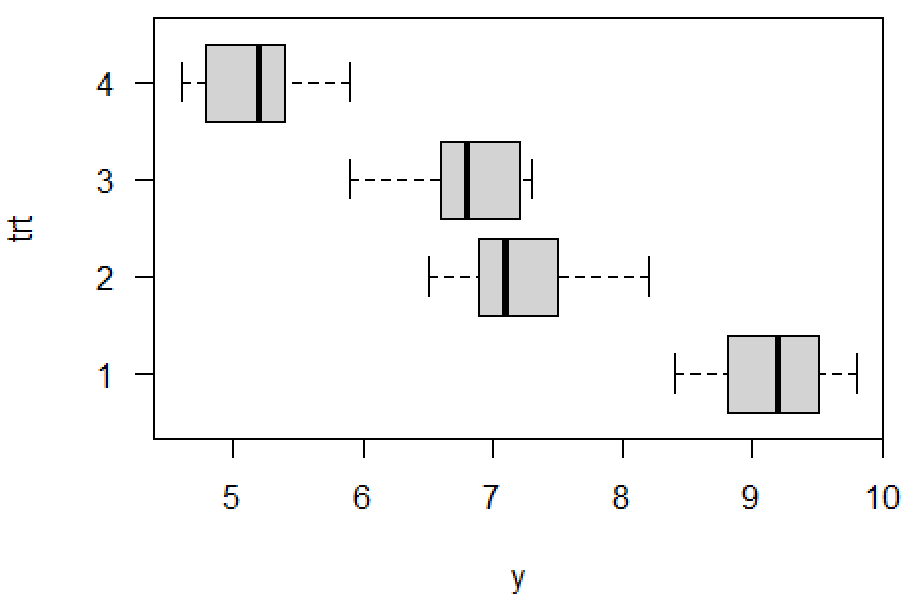
```
##     1     2     3     4 
## 0.308 0.418 0.313 0.262
```

Plot the Data

```r
boxplot(y ~ trt, data = dat, las = 1)
```




```
boxplot(y ~ trt, data = dat, las = 1, horizontal = T)
```



ANOVA Table

```
AOV <- aov(y ~ trt, data = dat)
summary(AOV)

##             Df Sum Sq Mean Sq F value   Pr(>F)    
## trt          3  39.91  13.303    40.9 9.92e-08 ***
## Residuals   16   5.20   0.325                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Multiple Comparisons

```
# LSD
# install.packages("agricolae")
library(agricolae)
LSD_bon <- LSD.test(AOV ,"trt", p.adj = "bonferroni")
LSD_bon$groups

##      y groups
## 1 9.14      a
## 2 7.24      b
## 3 6.76      b
## 4 5.18      c

# HSD
HSD <- TukeyHSD(AOV, conf.level = 0.95)
HSD$trt

##      diff       lwr       upr        p adj
## 2-1 -1.90 -2.931952 -0.868048 4.024593e-04
## 3-1 -2.38 -3.411952 -1.348048 3.310735e-05
## 4-1 -3.96 -4.991952 -2.928048 4.112087e-08
## 3-2 -0.48 -1.511952  0.551952 5.577630e-01
## 4-2 -2.06 -3.091952 -1.028048 1.708962e-04
## 4-3 -1.58 -2.611952 -0.548048 2.363679e-03
```

Model Assumptions

Example: Balloon Experiment (taken from Dean and Voss Exercise 3.12)

The experimenter (Meily Lin) had observed that some colors of birthday balloons seem to be harder to inflate than others. She ran this experiment to determine whether balloons of different colors are similar in terms of the time taken for inflation to a diameter of 7 inches. Four colors were selected from a single manufacturer. An assistant blew up the balloons and the experimenter recorded the times with a stop watch. The data, in the order collected, are given in Table 3.13, where the codes 1, 2, 3, 4 denote the colors pink, yellow, orange, blue, respectively.

Source: Table 3.13 of Dean and Voss Exercise 3.12

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Read the Data Into R

```
balloon <- read.csv("cr_assumptions.csv", header = T)
head(balloon)

##   ORDER COLOR TIME
## 1     1     1 22.0
## 2     2     3 24.6
## 3     3     1 20.3
## 4     4     4 19.8
## 5     5     3 24.3
## 6     6     2 22.2

summary(balloon)

##      ORDER           COLOR           TIME      
##  Min.   : 1.00   Min.   :1.00   Min.   :14.00  
##  1st Qu.: 8.75   1st Qu.:1.75   1st Qu.:17.40  
##  Median :16.50   Median :2.50   Median :19.70  
##  Mean   :16.50   Mean   :2.50   Mean   :20.24  
##  3rd Qu.:24.25   3rd Qu.:3.25   3rd Qu.:22.60  
##  Max.   :32.00   Max.   :4.00   Max.   :28.80

head(balloon, 10)

##    ORDER COLOR TIME
## 1      1     1 22.0
## 2      2     3 24.6
## 3      3     1 20.3
## 4      4     4 19.8
## 5      5     3 24.3
## 6      6     2 22.2
## 7      7     2 28.5
## 8      8     2 25.7
## 9      9     3 20.2
## 10    10     1 19.6
```

Convert Variable COLOR to a Factor

```
attach(balloon)
colorf <- as.factor(COLOR)
colorf

##  [1] 1 3 1 4 3 2 2 2 3 1 2 4 4 4 3 1 2 1 4 3 1 4 4 2 2 4 2 3 3 1 1 3
## Levels: 1 2 3 4
```

Model Fitting and Residuals

```
mod1 <- lm(TIME ~ colorf)
summary(mod1)

## 
## Call:
## lm(formula = TIME ~ colorf)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.8750 -2.2500  0.0687  2.0531  6.2250
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   18.337      1.162  15.778 1.83e-15 ***
## colorf2        4.237      1.644   2.578   0.0155 *
## colorf3        3.538      1.644   2.152   0.0401 *
## colorf4       -0.150      1.644  -0.091   0.9279
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.287 on 28 degrees of freedom
## Multiple R-squared:  0.2967, Adjusted R-squared:  0.2214
## F-statistic: 3.938 on 3 and 28 DF,  p-value: 0.01836
```

```
anova(mod1)
```

```
## Analysis of Variance Table
##
## Response: TIME
##           Df Sum Sq Mean Sq F value  Pr(>F)
## colorf     3 127.66  42.554  3.9379 0.01836 *
## Residuals 28 302.58  10.806
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Residuals
r <- residuals(mod1)
s <- rstandard(mod1)
var(s)
```

```
## [1] 1.032258
```

```
t <- rstudent(mod1)
```

Assess Equal Variance

```
# Levene’s Test for Equal Variance
# install.packages("lawstat")
library(lawstat)
levene.test(TIME, colorf, location = "mean")
```

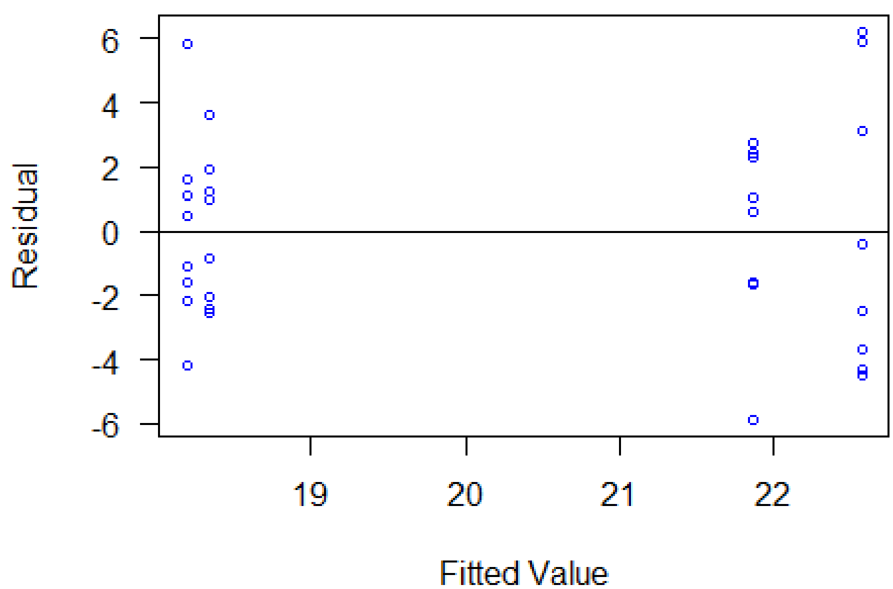
```
##
##  Classical Levene's test based on the absolute deviations from the mean
##  ( none not applied because the location is not set to median )
##
## data:  TIME
## Test Statistic = 2.1682, p-value = 0.1141
```

```r
# Brown-Forsythe test
levene.test(TIME, colorf, location = "median")
```

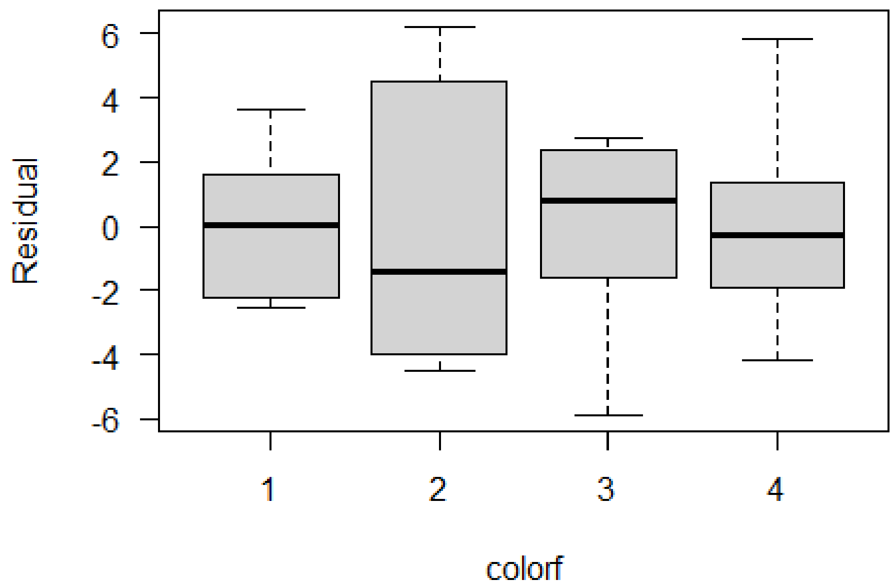
```
##
##  Modified robust Brown-Forsythe Levene-type test based on the absolute
##  deviations from the median
##
## data:  TIME
## Test Statistic = 1.3975, p-value = 0.2642
```

Plot r_{ij} vs. $\hat{y}_{i.}$ and Treatments

```r
plot(mod1$fitted, mod1$resid, las = 1, xlab = "Fitted Value", ylab =
"Residual", cex = 0.75, col = "blue")
abline(h = 0)
```

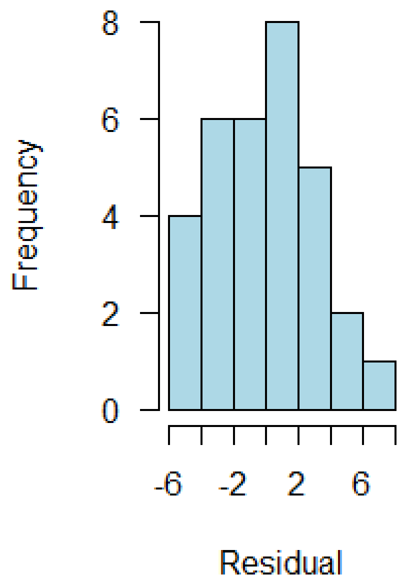


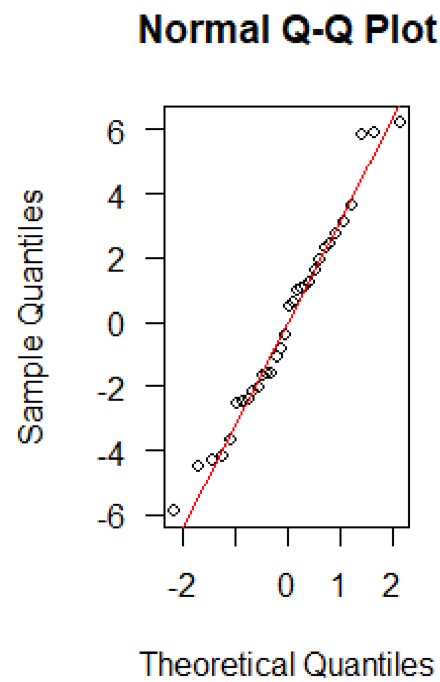

```r
plot(mod1$resid ~ colorf, ylab = "Residual", las = 1)
```



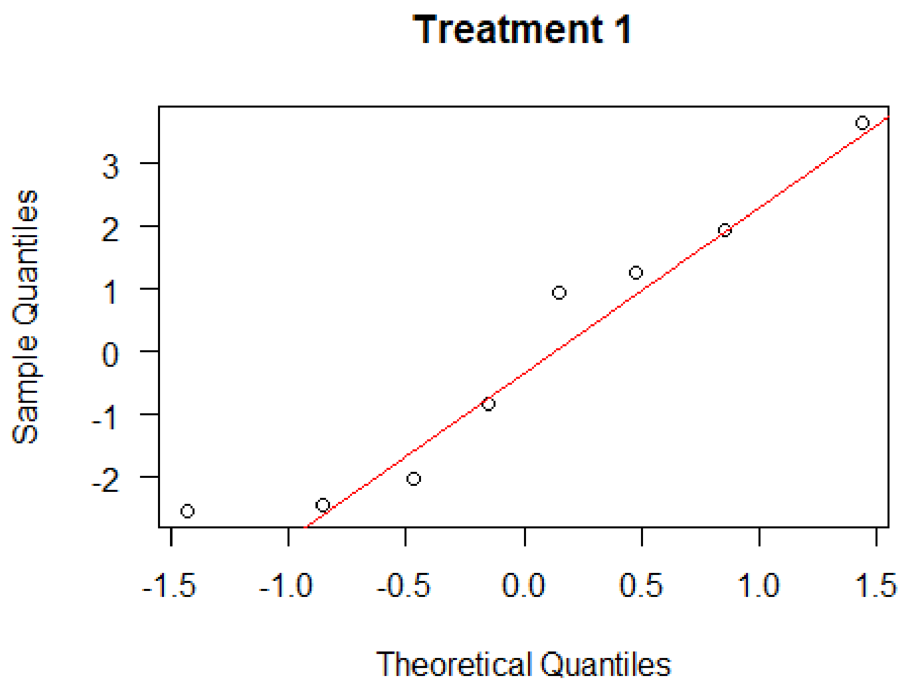
Assess Normality

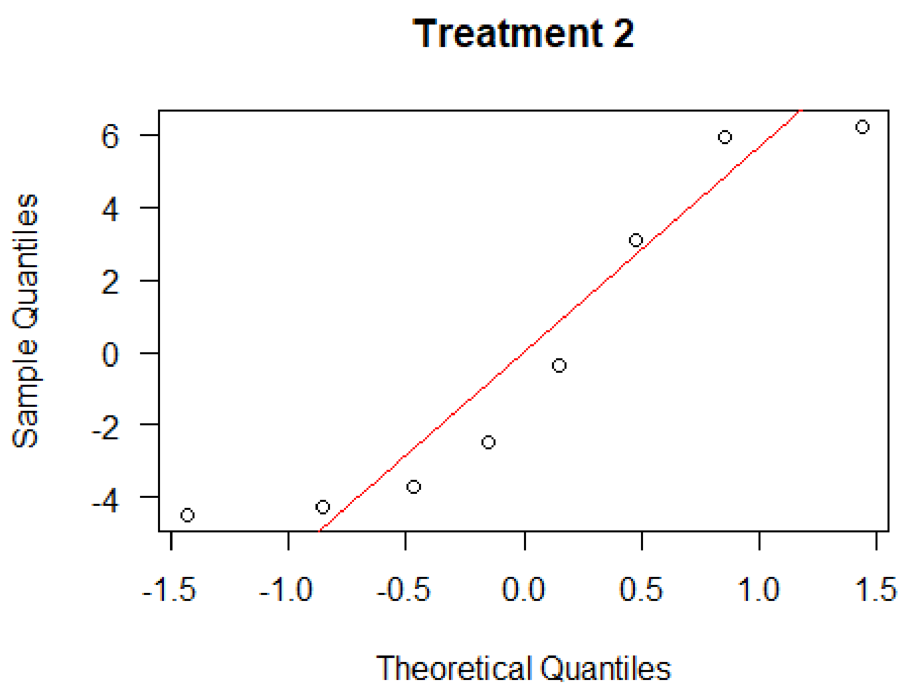
```
par(mfrow = c(1, 2), las = 1)
hist(mod1$resid, 8, main = "", xlab = "Residual", col = "lightblue")
qqnorm(mod1$resid, cex = 0.8)
qqline(mod1$resid, col = "red", lwd = 1.5)
```

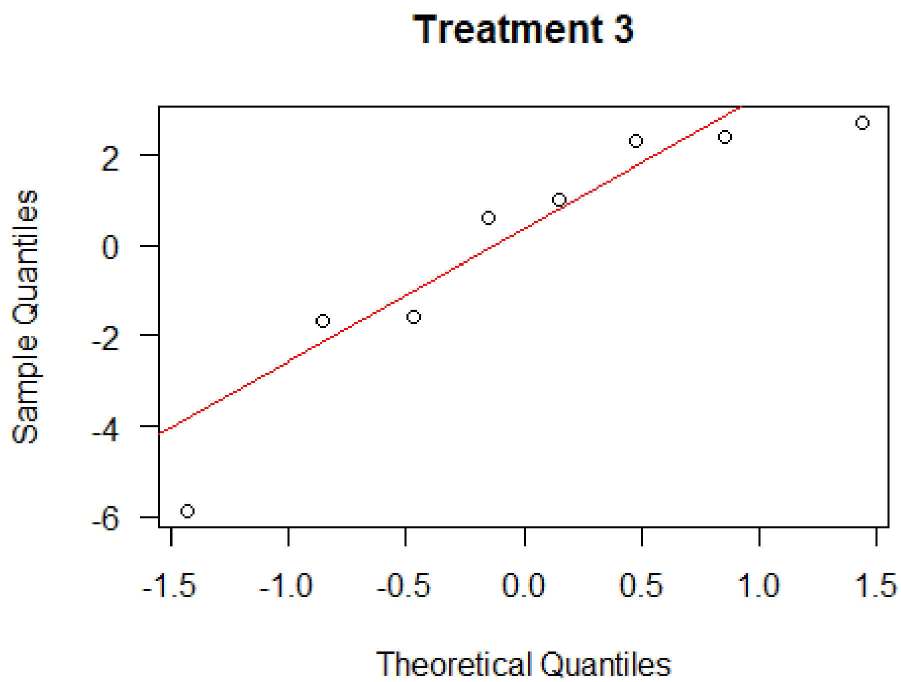


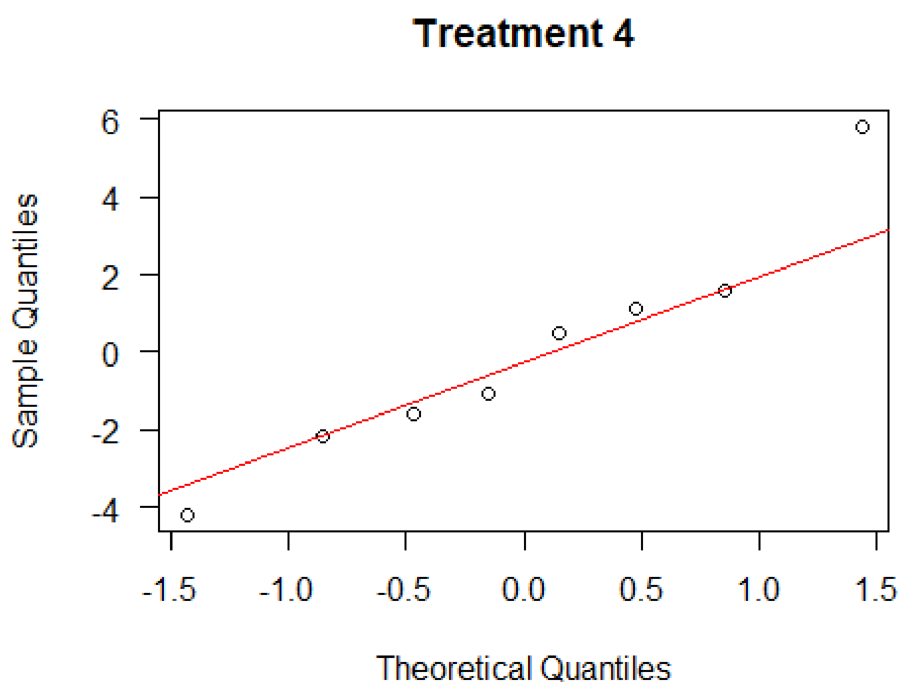



```
new <- data.frame(colorf, mod1$resid)
trt <- paste("Treatment", 1:4)
for (i in 1:4) {
  newc1 <- new[colorf == i, ]
  qqnorm(newc1$mod1.resid, las = 1,
         main = trt[i])
  qqline(newc1$mod1.resid,
         col = "red", lwd = 1.5)
}
```

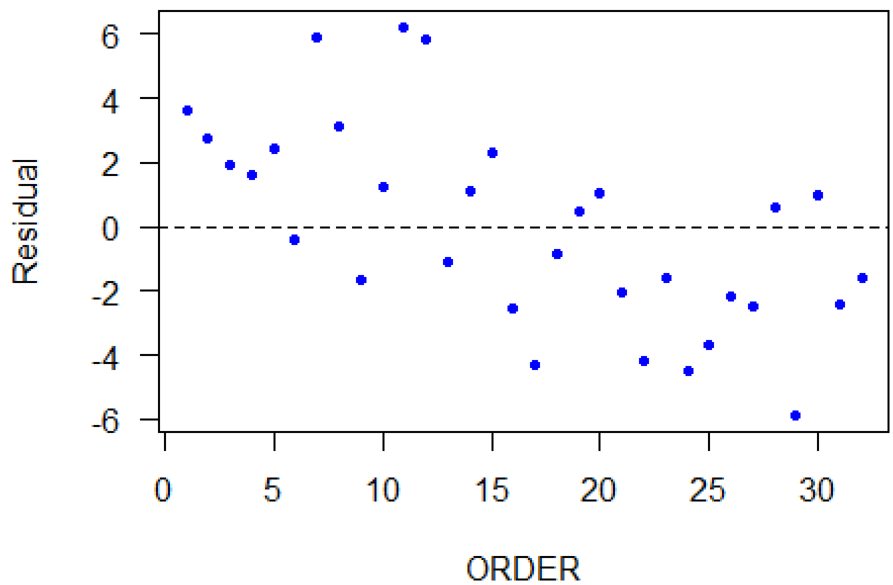

Treatment 1
Sample Quantiles
3
2
1
0
-1
-2
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Theoretical Quantiles

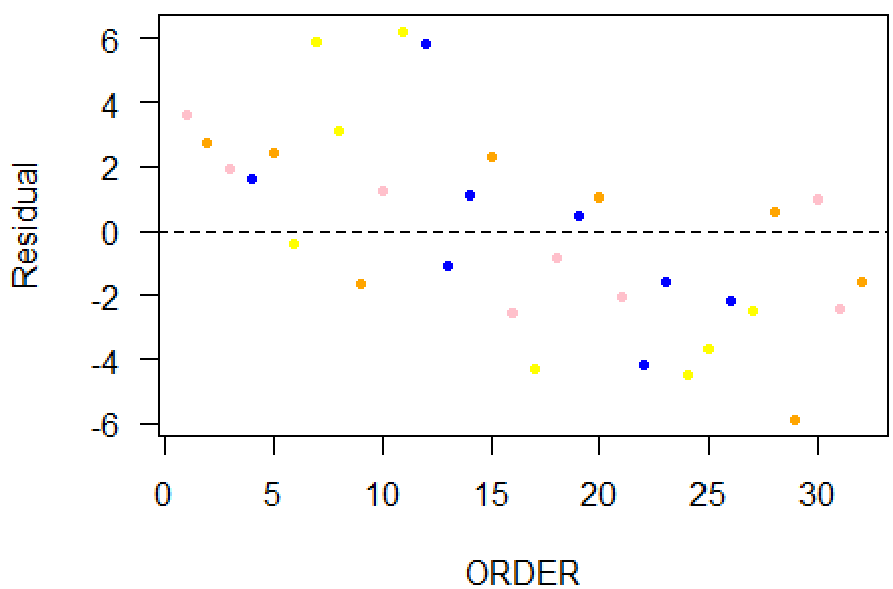

Treatment 2
Sample Quantiles
6
4
2
0
-2
-4
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Theoretical Quantiles


Treatment 3
Sample Quantiles
Theoretical Quantiles
2
0
-2
-4
-6
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5


Treatment 4
Sample Quantiles
Theoretical Quantiles
6
4
2
0
-2
-4
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5

Assess Indepdence





Durbin-Watson Test

```r
# install.packages("lmtest")
library(lmtest)
dwtest(TIME ~ colorf, data = balloon)
```

```
##
##  Durbin-Watson test
##
## data:  TIME ~ colorf
## DW = 1.1617, p-value = 0.006005
## alternative hypothesis: true autocorrelation is greater than 0
```

Fit a Model with Correlated AR(1) Error

```r
# install.packages("nlme")
library(nlme)
mod2 <- gls(TIME ~ colorf, correlation = corARMA(p = 1, q = 0))
mod2
```

```
## Generalized least squares fit by REML
##   Model: TIME ~ colorf
##   Data: NULL
##   Log-restricted-likelihood: -74.42885
##
## Coefficients:
## (Intercept)     colorf2     colorf3     colorf4
##  18.5860865   3.7248742   3.4233901  -0.3578644
##
## Correlation Structure: AR(1)
##  Formula: ~1
##  Parameter estimate(s):
##       Phi
## 0.4285025
## Degrees of freedom: 32 total; 28 residual
## Residual standard error: 3.321057
```