Randomized Complete Block Designs, Factorial Designs, and Split-Plot Designs

Blake Pappas

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RCBD: Randomized Complete Block Design

Create the Data Set

```
x <- c(52, 47, 44, 51, 42, 60, 55, 49, 52, 43, 56, 48, 45, 44, 38)
trt <- rep(c("A", "B", "C"), each = 5)
blk <- rep(1:5, 3)
dat <- data.frame(x = x, trt = trt, blk = as.factor(blk))
```

Two-Way ANOVA

```
lm <- lm(x ~ trt + blk, data = dat)
anova(lm)
```

```
## Analysis of Variance Table
##
## Response: x
##           Df Sum Sq Mean Sq F value    Pr(>F)
## trt        2   89.2   44.60  7.6239 0.0140226 *
## blk        4  363.6   90.90 15.5385 0.0007684 ***
## Residuals  8   46.8    5.85
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

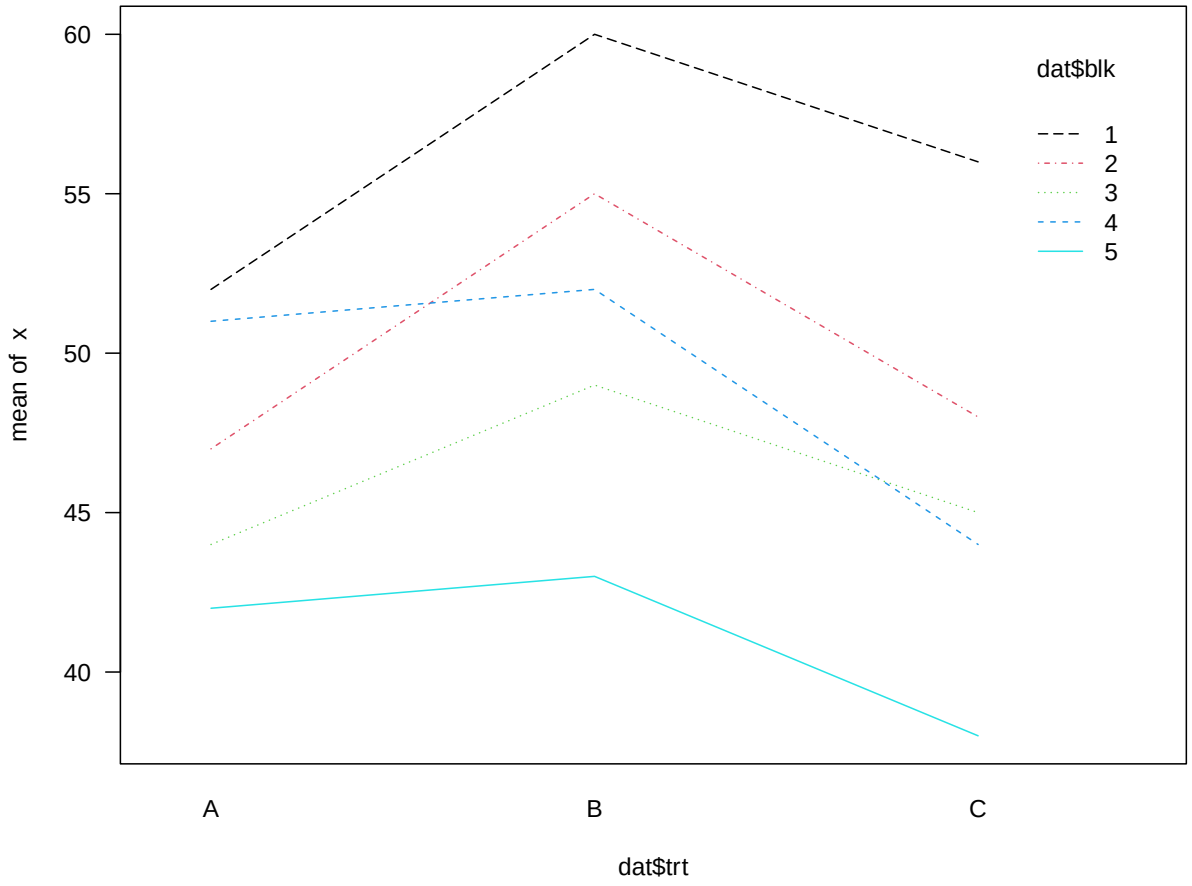
One-Way ANOVA

```
lm2 <- lm(x ~ trt, data = dat)
anova(lm2)
```

```
## Analysis of Variance Table
##
## Response: x
##           Df Sum Sq Mean Sq F value Pr(>F)
## trt        2   89.2    44.6  1.3041 0.3073
## Residuals 12  410.4    34.2
```

Interaction Plot: Assessing the Additivity Assumption

```
interaction.plot(dat$trt, dat$blk, x, las = 1, col = 1:5)
```



Factorial Design

Create the Data Set

```
y <- c(130, 74, 150, 159, 138, 168, 155, 180, 188, 126, 110, 160,
       34, 80, 136, 106, 174, 150, 40, 75, 122, 115, 120, 139,
       20, 82, 25, 58, 96, 82, 70, 58, 70, 45, 104, 60)
temp <- c(rep(15, 12), rep(70, 12), rep(125, 12))
material <- rep(c(1, 1, 2, 2, 3, 3), 6)
dat <- data.frame(cbind(y, temp, material))
dat$temp <- as.factor(dat$temp); dat$material <- as.factor(dat$material)

(meanA <- tapply(dat$y, dat$temp, mean))
```

```
##        15        70       125
## 144.83333 107.58333  64.16667
```

```
(meanB <- tapply(dat$y, dat$material, mean))
```

```
##         1         2         3
##  83.16667 108.33333 125.08333
```

```
(meanAB <- tapply(dat$y, list(dat$temp, dat$material), mean))
```

```
##          1      2      3
## 15  134.75 155.75 144.00
## 70   57.25 119.75 145.75
## 125  57.50  49.50  85.50
```

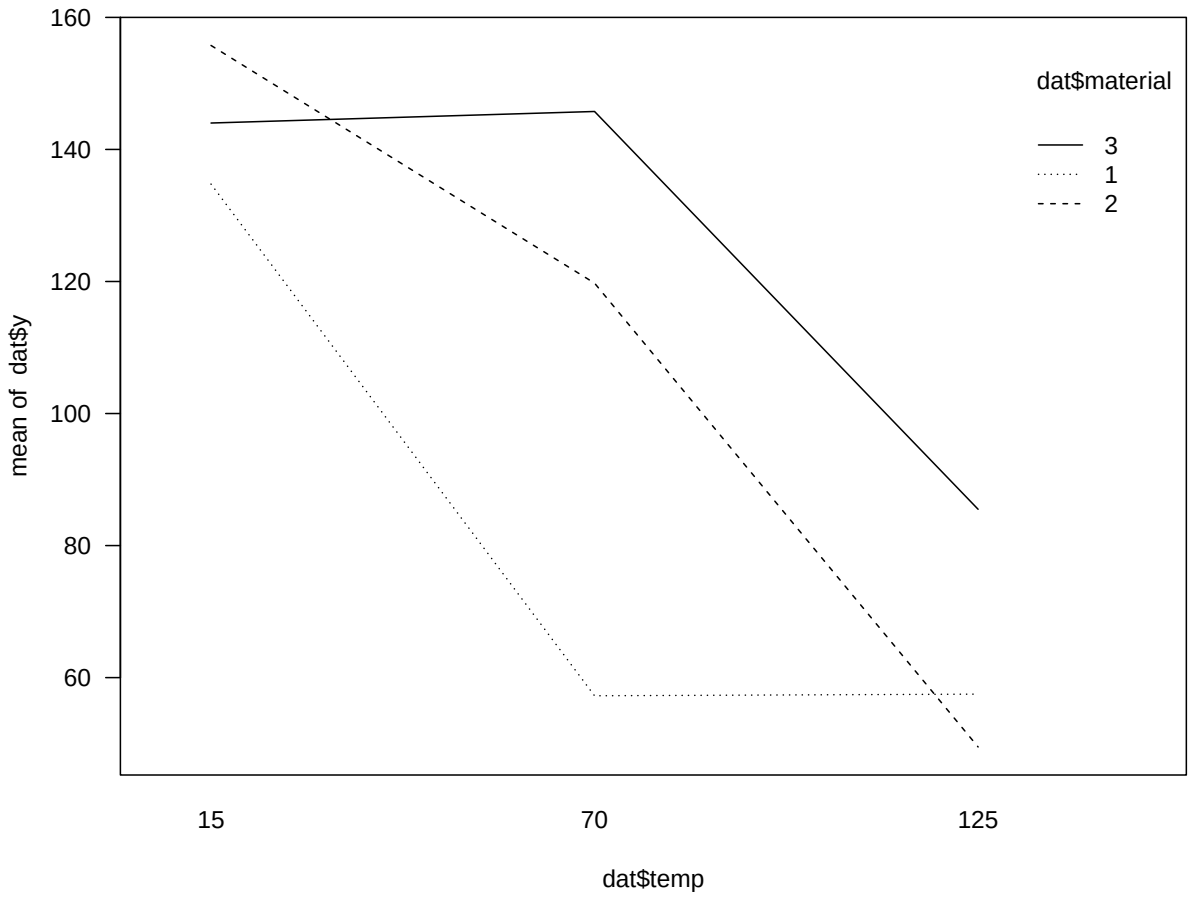
Two-Way ANOVA

```
lm <- lm(y ~ temp * material, data = dat)
anova(lm)
```

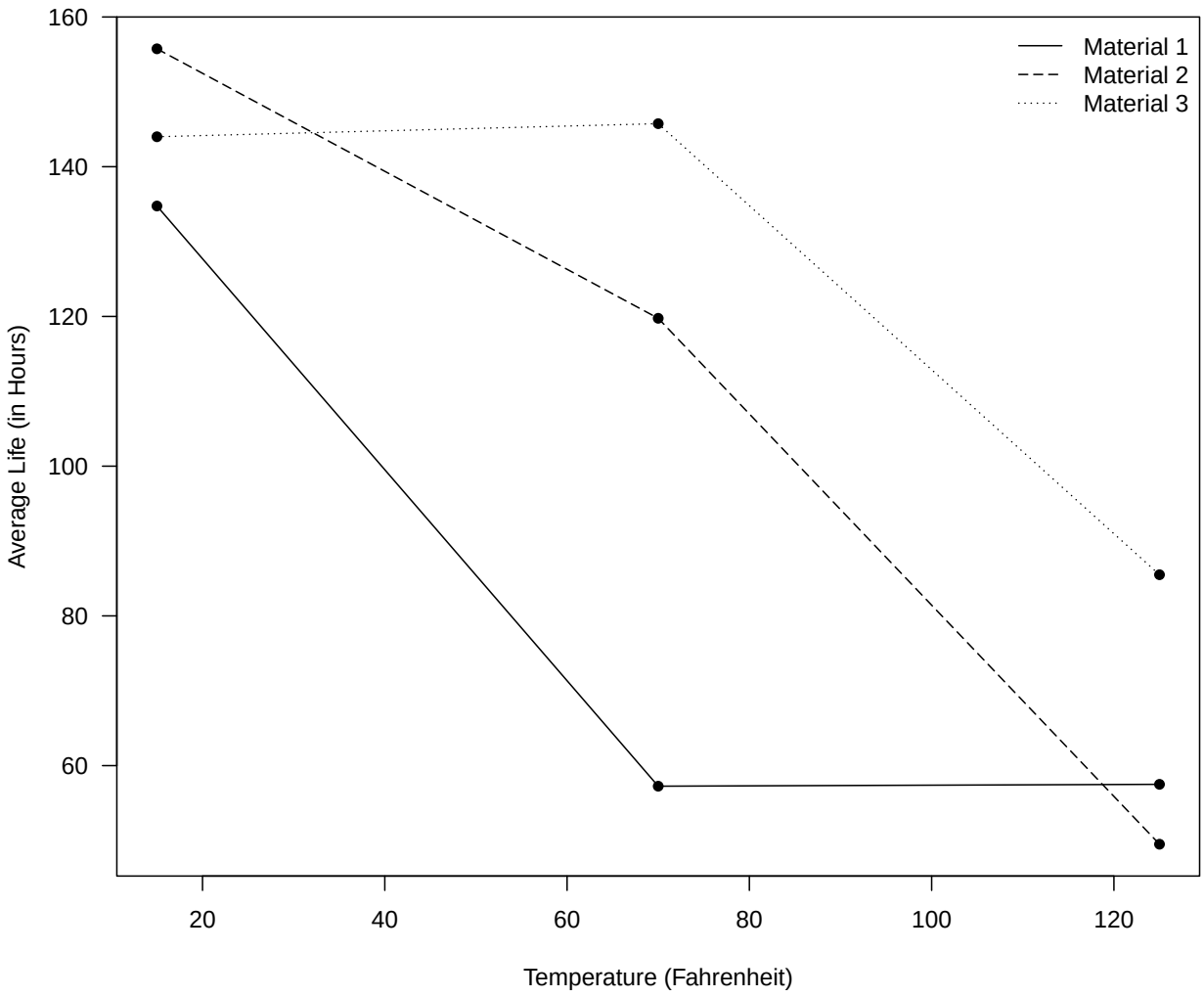
```
## Analysis of Variance Table
##
## Response: y
##               Df Sum Sq Mean Sq F value    Pr(>F)
## temp           2  39119 19559.4 28.9677 1.909e-07 ***
## material       2  10684  5341.9  7.9114  0.001976 **
## temp:material  4   9614  2403.4  3.5595  0.018611 *
## Residuals     27  18231   675.2
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interaction Plot

```
interaction.plot(dat$temp, dat$material, dat$y, las = 1)
```




```
par(mar = c(4.1, 4, 1, 0.8))
plot(rep(c(15, 70, 125), 3), c(meanAB), pch = 16, las = 1,
     xlab = "Temperature (Fahrenheit)", ylab = "Average Life (in Hours)")
lines(c(15, 70, 125), meanAB[, 1])
lines(c(15, 70, 125), meanAB[, 2], lty = 5)
lines(c(15, 70, 125), meanAB[, 3], lty = 3)
legend("topright", legend = paste("Material", 1:3), bty = "n",
lty = c(1, 5, 3))
```



Split-Plot Design

This example is taken from Lukas Meier's ANOVA using R [Link]

Farmer John has eight plots of land. He randomly assign two fertilization "schemes" ("control" and "new") to the eight plots. In addition, each plot (the "whole-plot") is divided into four subplots ("split-plots"). In each subplot, four different strawberry varieties are randomized to the subplots. John is interested in the effect of fertilization scheme and strawberry variety on fruit mass.

Read the Data

```
dat <- read.table("http://stat.ethz.ch/~meier/teaching/data/john.dat", header = TRUE)
dat[, "plot"] <- factor(dat[, "plot"])
str(dat)
```

```
## 'data.frame':    32 obs. of  4 variables:
##  $ plot      : Factor w/ 8 levels "1","2","3","4",..: 7 7 7 7 5 5 5 5 6 6 ...
##  $ fertilizer: chr  "control" "control" "control" "control" ...
```

```
##  $ variety   : chr  "A" "B" "C" "D" ...
##  $ mass      : num  11.6 7.7 12 14 8.9 9.5 11.7 15 10.8 11 ...
```

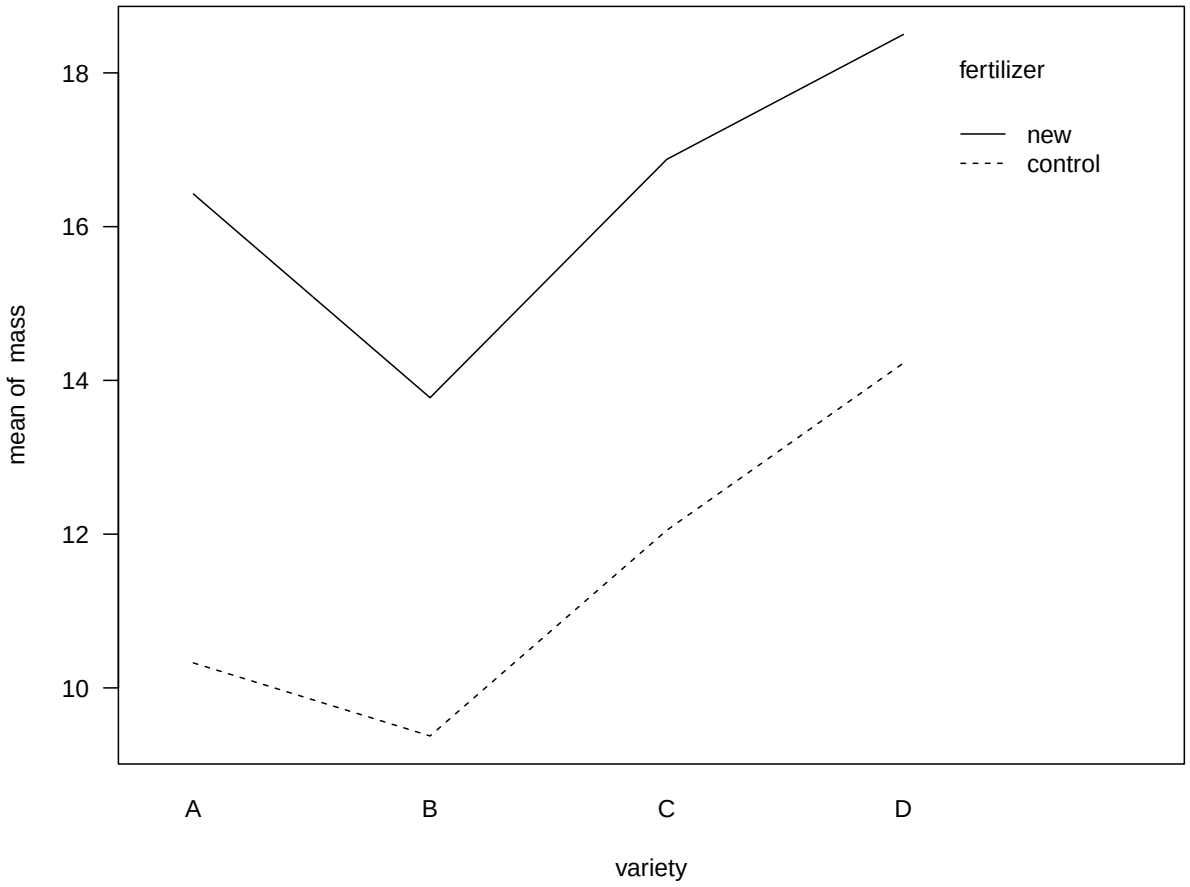
ANOVA

```
# install.packages("lmerTest")
library(lmerTest)
fit <- lmer(mass ~ fertilizer + variety + (1 | plot), data = dat)
anova(fit)
```

```
## Type III Analysis of Variance Table with Satterthwaite's method
##             Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
## fertilizer 131.341 131.341     1     6  68.240 0.0001702 ***
## variety     96.431  32.144     3    21  16.701 8.879e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Interaction Plot

```
with(dat, interaction.plot(x.factor = variety, trace.factor = fertilizer, response = mass, las = 1))
```


18
16
14
12
10
mean of mass
A
B
C
D
variety
fertilizer
new
control