Logistic Regression and Poisson Regression

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Logistic Regression: Horseshoe Crab Malting

Data Source: Brockmann, H. J. (1996). Satellite male groups in horseshoe crabs, Limulus polyphemus. *Ethology*, 102(1), 1-21.

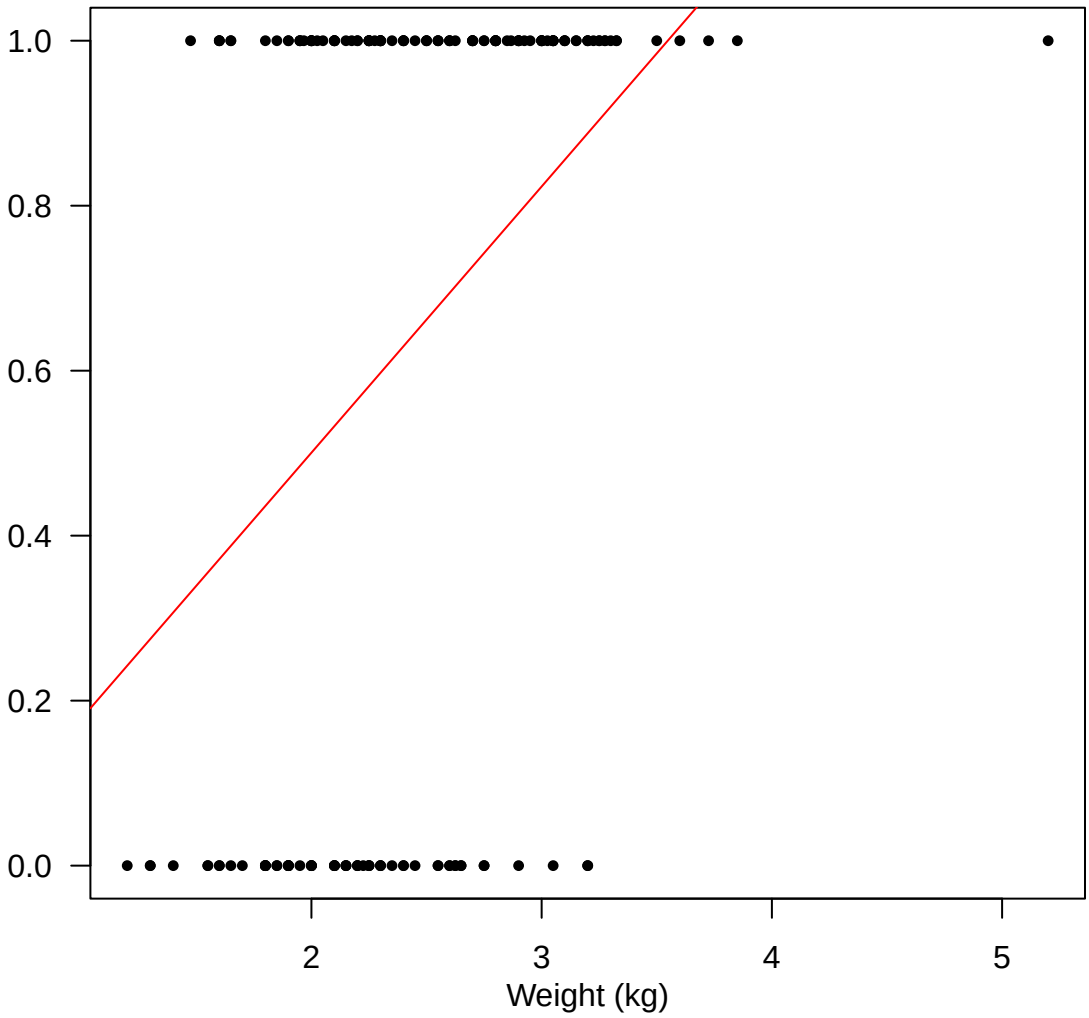
Load the Data

```
crab <- read.table("http://users.stat.ufl.edu/~aa/cda/data/Crabs.dat", header = T)
```

Fit a Linear Regression

Let's fit a simple linear regression using `weight` as the predictor.

```
lmFit <- lm(y ~ weight, data = crab)
par(mar = c(3.5, 3.5, 0.8, 0.6))
with(crab, plot(weight, y, pch = 16,
                cex = 0.75, las = 1, xlab = "", ylab = ""))
mtext("Weight (kg)", side = 1, line = 2)
abline(lmFit, col = "red")
```



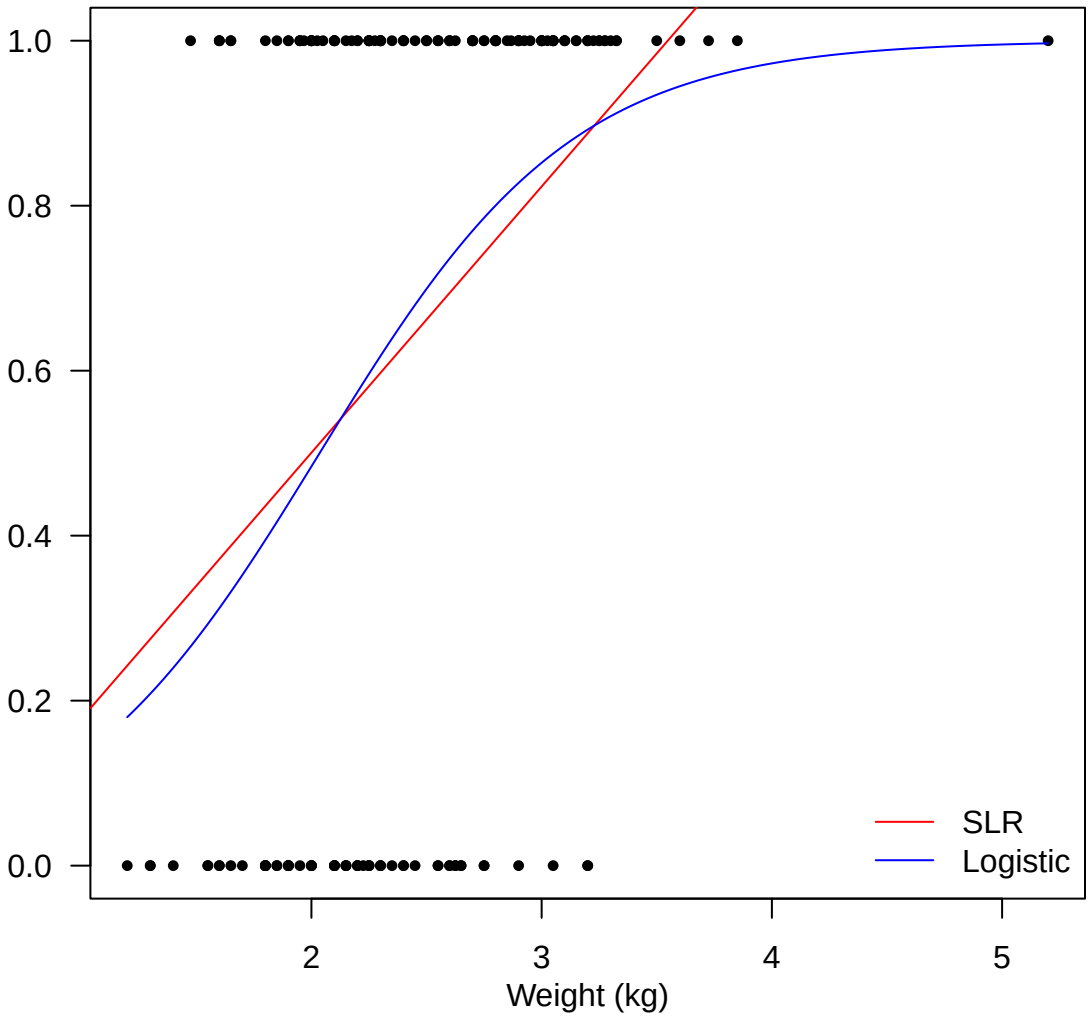
Fit a Logistic Regression

```
logitFit <- glm(y ~ weight, data = crab, family = "binomial")
summary(logitFit)
```

```
##
## Call:
## glm(formula = y ~ weight, family = "binomial", data = crab)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1108  -1.0749   0.5426   0.9122   1.6285
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -3.6947     0.8802  -4.198 2.70e-05 ***
## weight        1.8151     0.3767   4.819 1.45e-06 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 225.76  on 172  degrees of freedom
## Residual deviance: 195.74  on 171  degrees of freedom
## AIC: 199.74
##
## Number of Fisher Scoring iterations: 4
```

```r
# Plot the Fits
rg <- range(crab$weight)
xg <- seq(rg[1], rg[2], 0.01)
pred <- predict(logitFit, newdata = data.frame(weight = xg), type = "response")
par(mar = c(3.5, 3.5, 0.8, 0.6))
plot(crab$weight, crab$y, pch = 16, cex = 0.75, las = 1, xlab = "", ylab = "")
mtext("Weight (kg)", side = 1, line = 2)
abline(lmFit, col = "red")
lines(xg, pred, col = "blue")
legend("bottomright", legend = c("SLR", "Logistic"),
       col = c("red", "blue"), lty = 1, bty = "n")
```



Confidence Intervals

```
# Normal Approximation
est <- summary(logitFit)$coefficients
(CI_norm <- est[2, 1] + c(-1, 1) * qnorm(0.975) * est[2, 2])
```

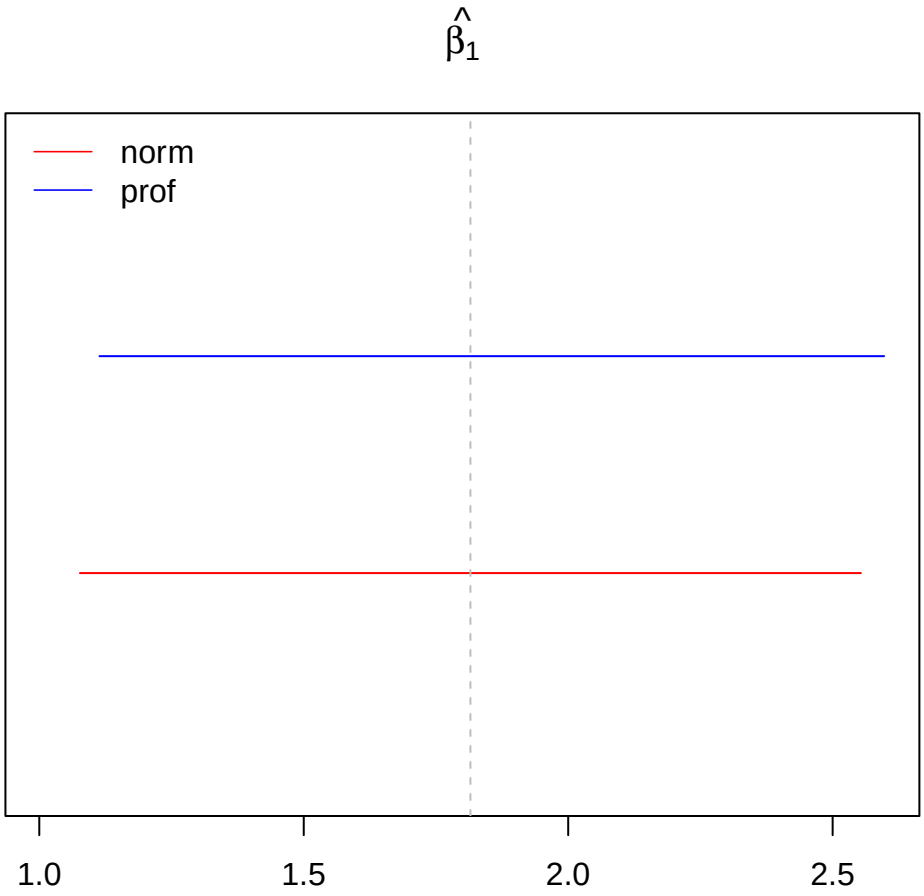
```
## [1] 1.076834 2.553455
```

```
# Profile Likelihood CI
library(MASS)
(CI_prof <- confint(logitFit)[2, ])
```

```
## Waiting for profiling to be done...
```

```
##    2.5 %   97.5 %
## 1.113790 2.597305
```

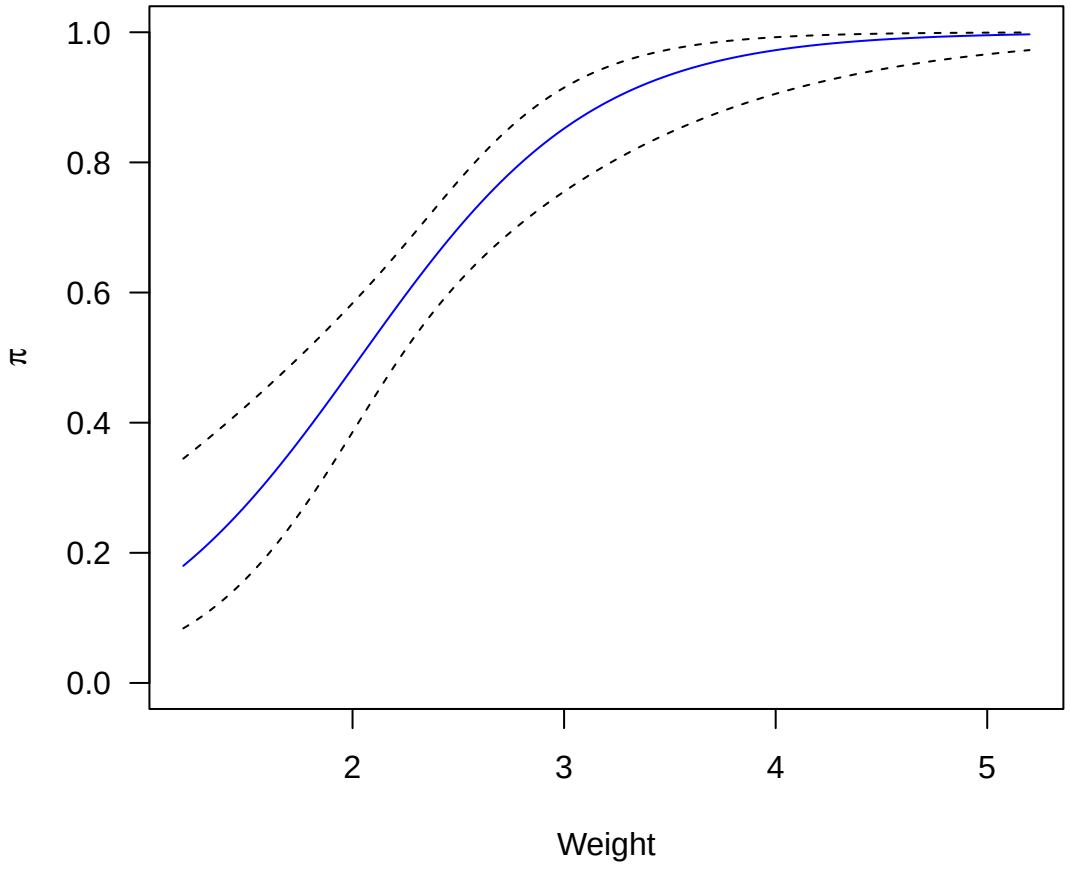
```r
plot(1, type = "n", xlab = "", ylab = "",
     xlim = c(1, 2.6), ylim = c(-0.05, 0.1),
     yaxt = "n", main = expression(hat(beta[1])))
segments(CI_norm[1], 0, CI_norm[2], col = "red")
segments(CI_prof[1], 0.05, CI_prof[2], col = "blue")
abline(v = est[2, 1], lty = 2, col = "gray")
legend("topleft", legend = c("norm", "prof"),
       col = c("red", "blue"), lty = 1,
       lwd = 0.8, bty = "n")
```



Prediction

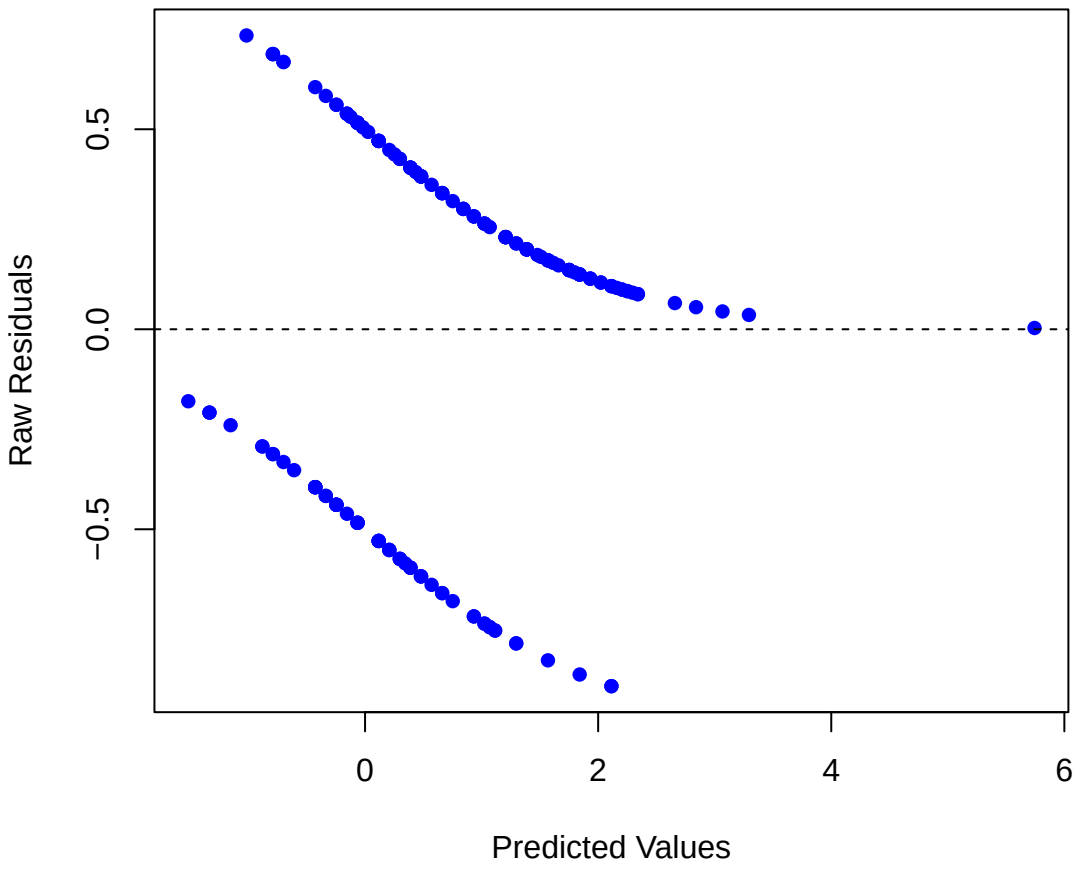
```r
rg <- range(crab$weight)
xg <- seq(rg[1], rg[2], 0.01)

pred <- predict(logitFit, newdata = data.frame(weight = xg), se.fit = TRUE)
```

```r
fit <- pred$fit; se <- pred$se.fit

plot(xg, exp(fit) / (1 + exp(fit)),
     type = "l", col = "blue", ylim = c(0, 1),
     las = 1, xlab = "Weight", ylab = expression(pi))
lines(xg, exp(fit + 1.96 * se) / (1 + exp(fit + 1.96 * se)), lty = 2)
lines(xg, exp(fit - 1.96 * se) / (1 + exp(fit - 1.96 * se)), lty = 2)
```



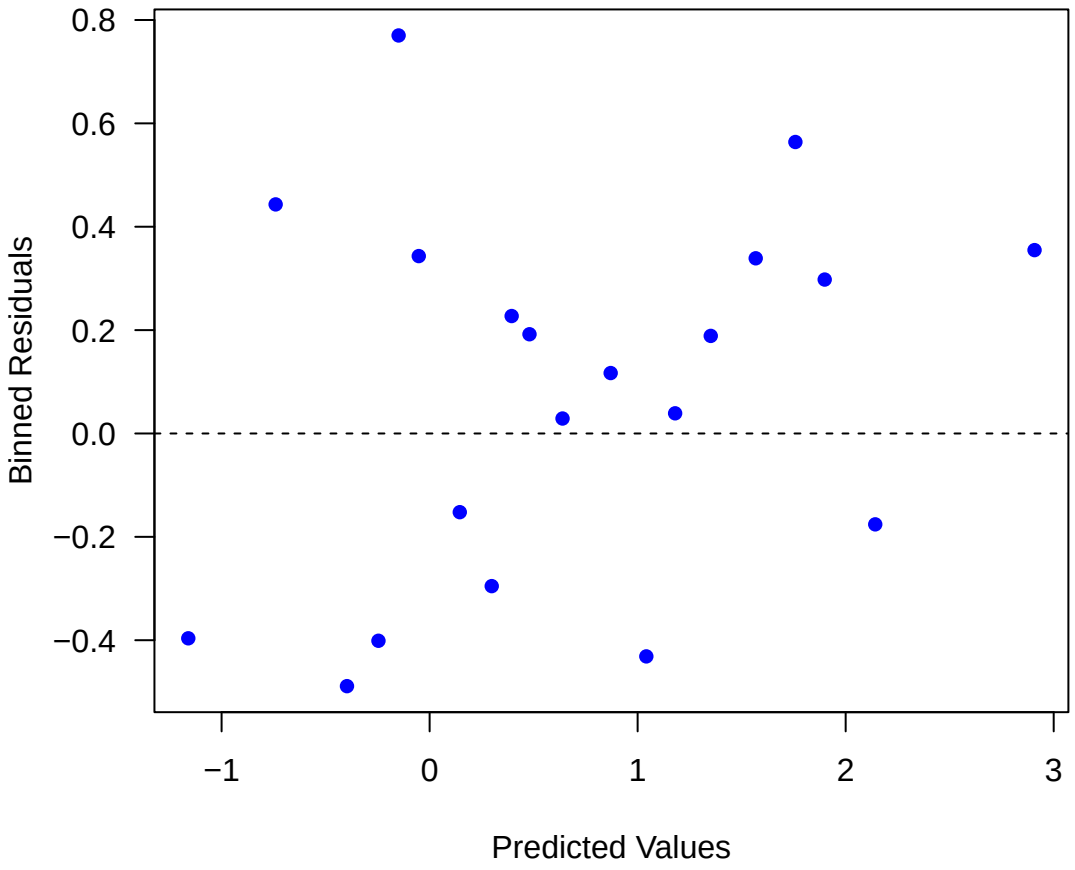
Raw Residual Plot

```r
res <- resid(logitFit, type = "response")
pred <- predict(logitFit)

plot(pred, res, col = "blue", pch = 16,
     xlab = "Predicted Values",
     ylab = "Raw Residuals")
abline(h = 0, lty = 2)
```



Binned Residuals

```
wt_bin <- findInterval(crab$weight, unique(quantile(crab$weight, seq(0, 1, length.out = 20 + 1))), righ
library(dplyr)
crab.res <- mutate(crab, residuals = residuals(logitFit), Linpred = predict(logitFit), bin = wt_bin)

res_bin <- tapply(crab.res$residuals, crab.res$bin, mean)
Lpred_bin <- tapply(crab.res$Linpred, crab.res$bin, mean)

plot(res_bin ~ Lpred_bin, xlab = "Predicted Values",
     ylab = "Binned Residuals", col = "blue", pch = 16, las = 1)
abline(h = 0, lty = 2)
```



Model Selection

```
logitFit2 <- glm(y ~ weight + width, data = crab, family = "binomial")
summary(logitFit2)
```

```
##
## Call:
## glm(formula = y ~ weight + width, family = "binomial", data = crab)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.1127  -1.0344   0.5304   0.9006   1.7207
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  -9.3547     3.5280  -2.652  0.00801 **
## weight        0.8338     0.6716   1.241  0.21445
## width         0.3068     0.1819   1.686  0.09177 .
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 225.76  on 172  degrees of freedom
## Residual deviance: 192.89  on 170  degrees of freedom
## AIC: 198.89
##
## Number of Fisher Scoring iterations: 4
```

```
step(logitFit2)
```

```
## Start:  AIC=198.89
## y ~ weight + width
##
##          Df Deviance    AIC
## - weight  1   194.45 198.45
## <none>        192.89 198.89
## - width   1   195.74 199.74
##
## Step:  AIC=198.45
## y ~ width
##
##         Df Deviance    AIC
## <none>       194.45 198.45
## - width  1   225.76 227.76

##
## Call:  glm(formula = y ~ width, family = "binomial", data = crab)
##
## Coefficients:
## (Intercept)        width
##    -12.3508       0.4972
##
## Degrees of Freedom: 172 Total (i.e. Null);  171 Residual
## Null Deviance:       225.8
## Residual Deviance: 194.5     AIC: 198.5
```

Poisson Regression

Flying-Bomb Hits on London During World War II [Clarke, 1946; Feller, 1950]

```
count <- c(229, 211, 93, 35, 7, 1)
grids <- 576
hits <- 537
lambda <- hits / grids
count_expected <- c(grids * dpois(0:4, lambda = lambda), grids * ppois(4, lambda = lambda, lower.tail =
round(count_expected, 1)
```

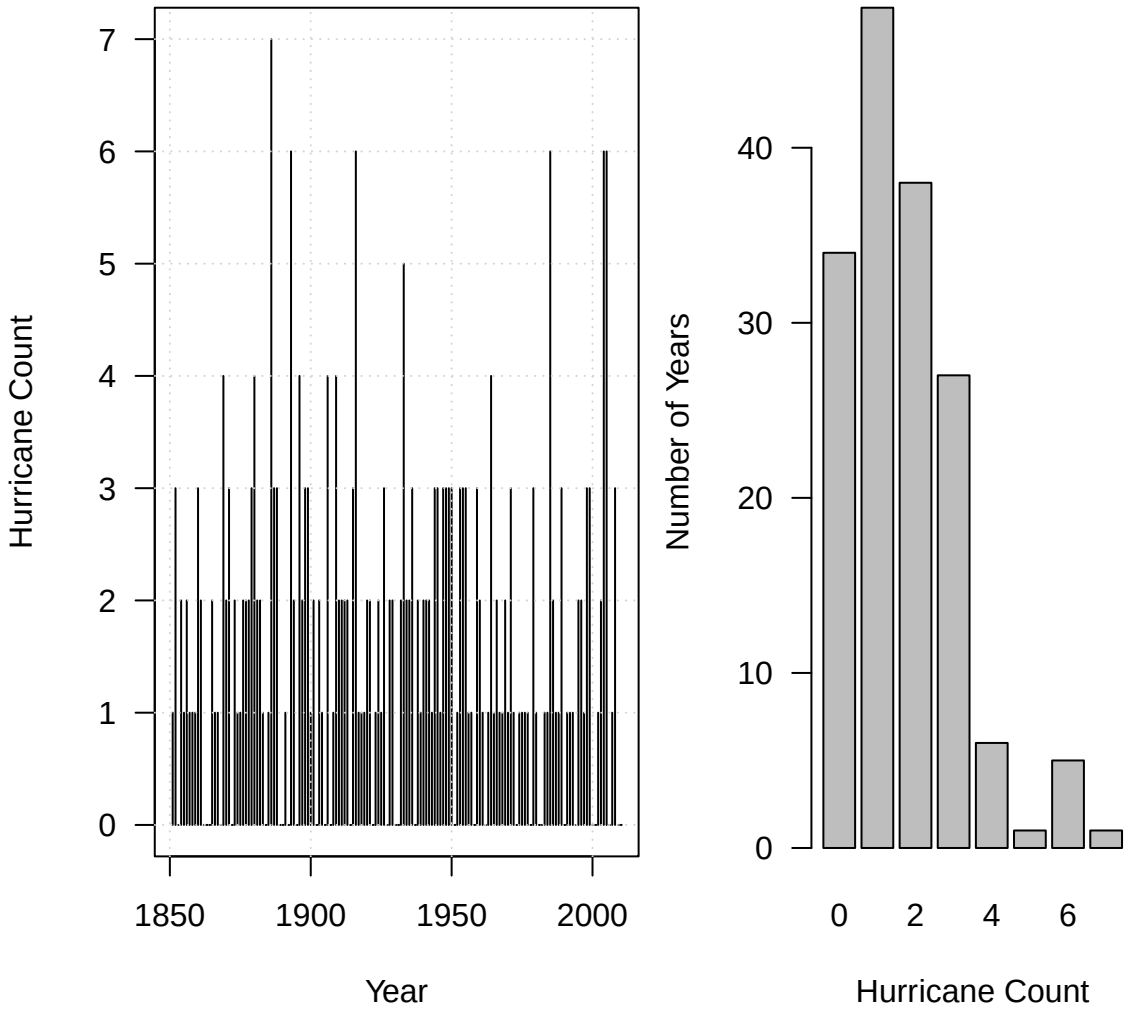
```
## [1] 226.7 211.4  98.5  30.6   7.1   1.6
```

US Landfalling Hurriances

```
# Load the Hurricane Count
con = "http://myweb.fsu.edu/jelsner/Book/Chap07/US.txt"
hurricanes = read.table(con, header = T)
head(hurricanes)
```

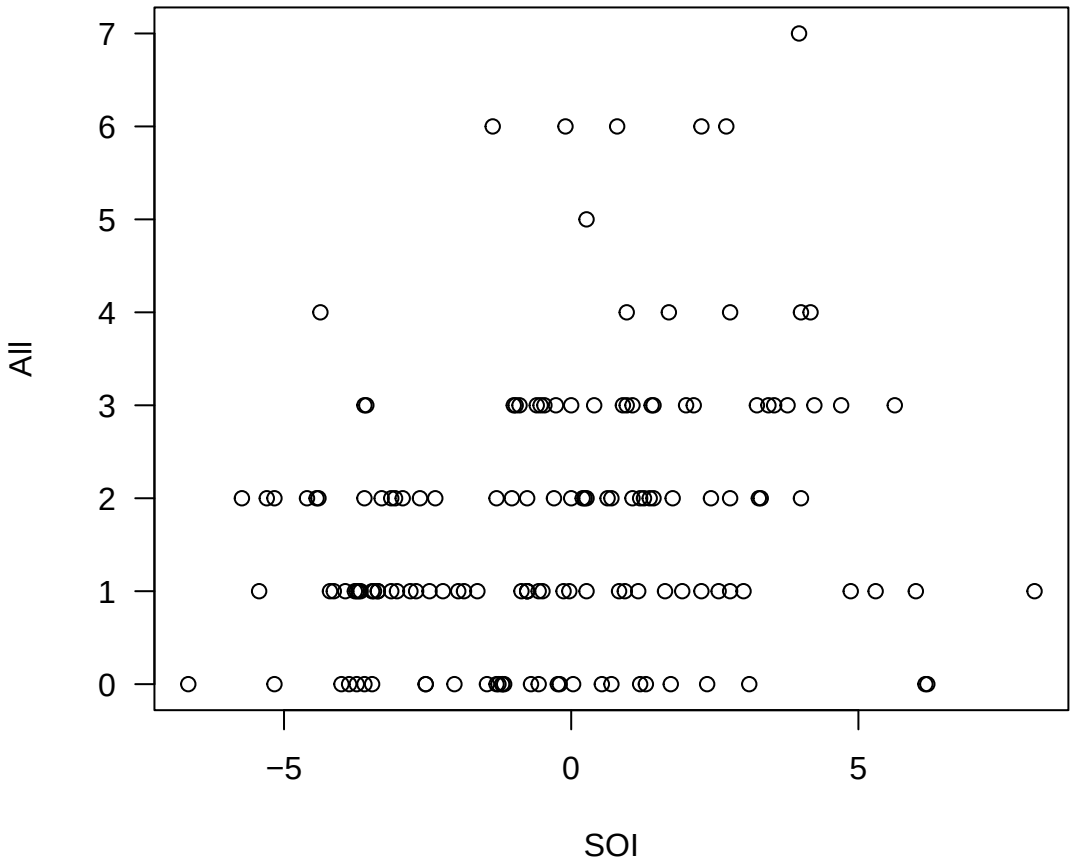
```
##   Year All MUS G FL E
## 1 1851   1   1 0  1 0
## 2 1852   3   1 1  2 0
## 3 1853   0   0 0  0 0
## 4 1854   2   1 1  0 1
## 5 1855   1   1 1  0 0
## 6 1856   2   1 1  1 0
```

```
par(las = 1, mar = c(4.6, 3.9, 0.8, 0.6))
layout(matrix(c(1, 2), 1, 2, byrow = TRUE), widths = c(0.57, 0.43))
plot(hurricanes$Year, hurricanes$All, type = "h", xlab = "Year", ylab = "Hurricane Count")
grid()
barplot(table(hurricanes$All), xlab = "Hurricane Count", ylab = "Number of Years", main = "")
```



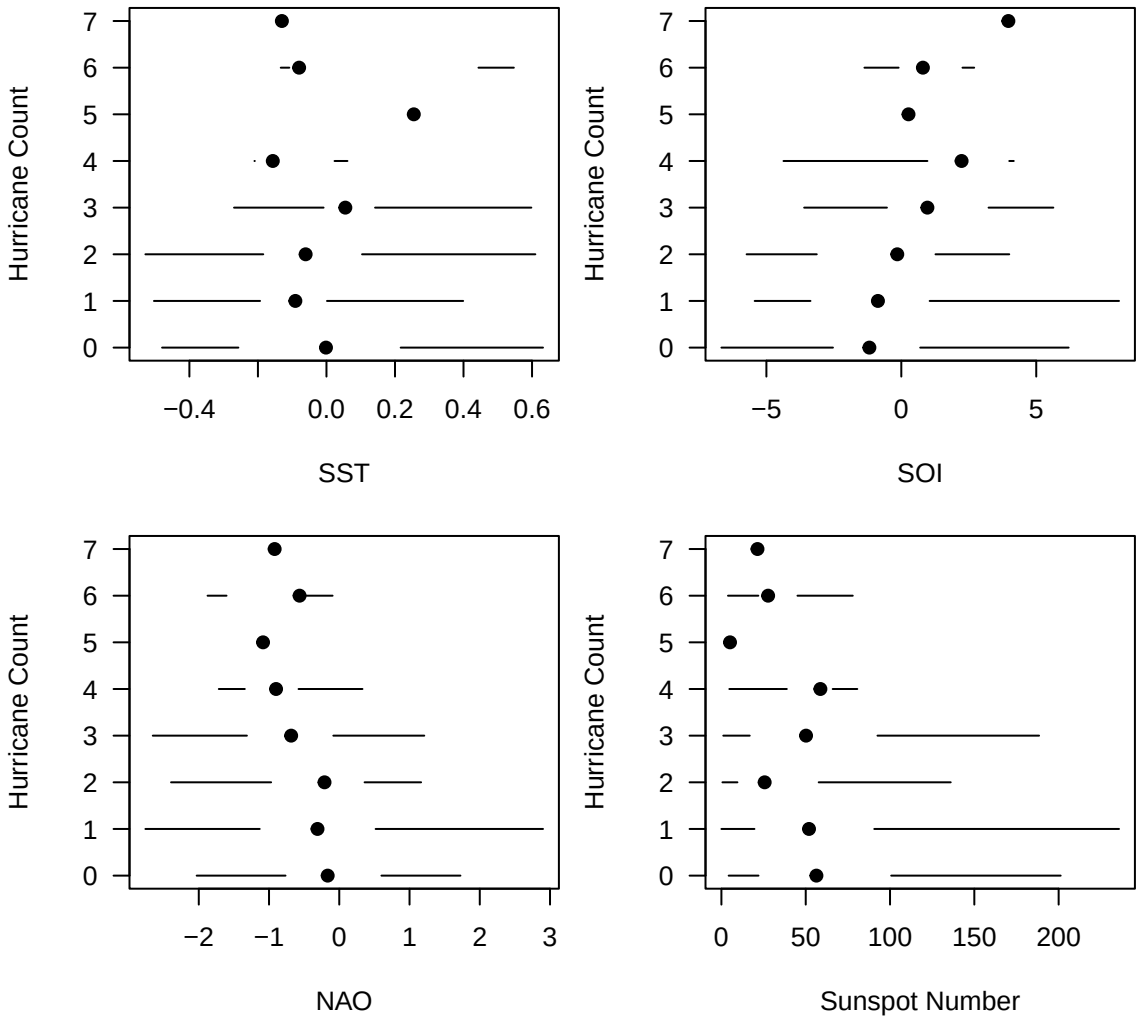
Load the Environmental Variables

```
load("annual.RData")
data <- data.frame(All = hurricanes$All, SOI = annual$soi,
                   NAO = annual$nao, SST = annual$sst,
                   SSN = annual$ssn)
data <- data[-(1:15), ]
with(data, plot(All ~ SOI, las = 1))
```




```
H <- hurricanes

par(mfrow = c(2, 2), mar = c(4.5, 4, 1, 0.6))
plot(range(annual$sst, na.rm = TRUE), c(0, 7), type = "n", ylab = "Hurricane Count", xlab = "SST",
     las = 1)
for(i in 0:7) {
  points(fivenum(annual$sst[H$All == i])[3], i, pch = 19)
  lines(c(fivenum(annual$sst[H$All == i])[1], fivenum(annual$sst[H$All == i])[2]), c(i, i))
  lines(c(fivenum(annual$sst[H$All == i])[4], fivenum(annual$sst[H$All == i])[5]), c(i, i))
}
plot(range(annual$soi, na.rm = TRUE), c(0, 7), type = "n", ylab = "Hurricane Count", xlab = "SOI",
     las = 1)

for(i in 0:7) {
  points(fivenum(annual$soi[H$All == i])[3], i, pch=19)
  lines(c(fivenum(annual$soi[H$All == i])[1], fivenum(annual$soi[H$All == i])[2]), c(i, i))
  lines(c(fivenum(annual$soi[H$All == i])[4], fivenum(annual$soi[H$All == i])[5]), c(i, i))
}
plot(range(annual$nao, na.rm = TRUE), c(0, 7), type = "n", ylab = "Hurricane Count", xlab = "NAO",
     las = 1)
```

```r
for(i in 0:7) {
  points(fivenum(annual$nao[H$All == i])[3], i, pch = 19)
  lines(c(fivenum(annual$nao[H$All == i])[1], fivenum(annual$nao[H$All == i])[2]), c(i, i))
  lines(c(fivenum(annual$nao[H$All == i])[4], fivenum(annual$nao[H$All == i])[5]), c(i, i))
}
plot(range(annual$ssn, na.rm = TRUE), c(0, 7), type = "n", ylab = "Hurricane Count",
     xlab = "Sunspot Number", las = 1)

for(i in 0:7) {
  points(fivenum(annual$ssn[H$All == i])[3], i, pch = 19)
  lines(c(fivenum(annual$ssn[H$All == i])[1], fivenum(annual$ssn[H$All == i])[2]), c(i, i))
  lines(c(fivenum(annual$ssn[H$All == i])[4], fivenum(annual$ssn[H$All == i])[5]), c(i, i))
}
```

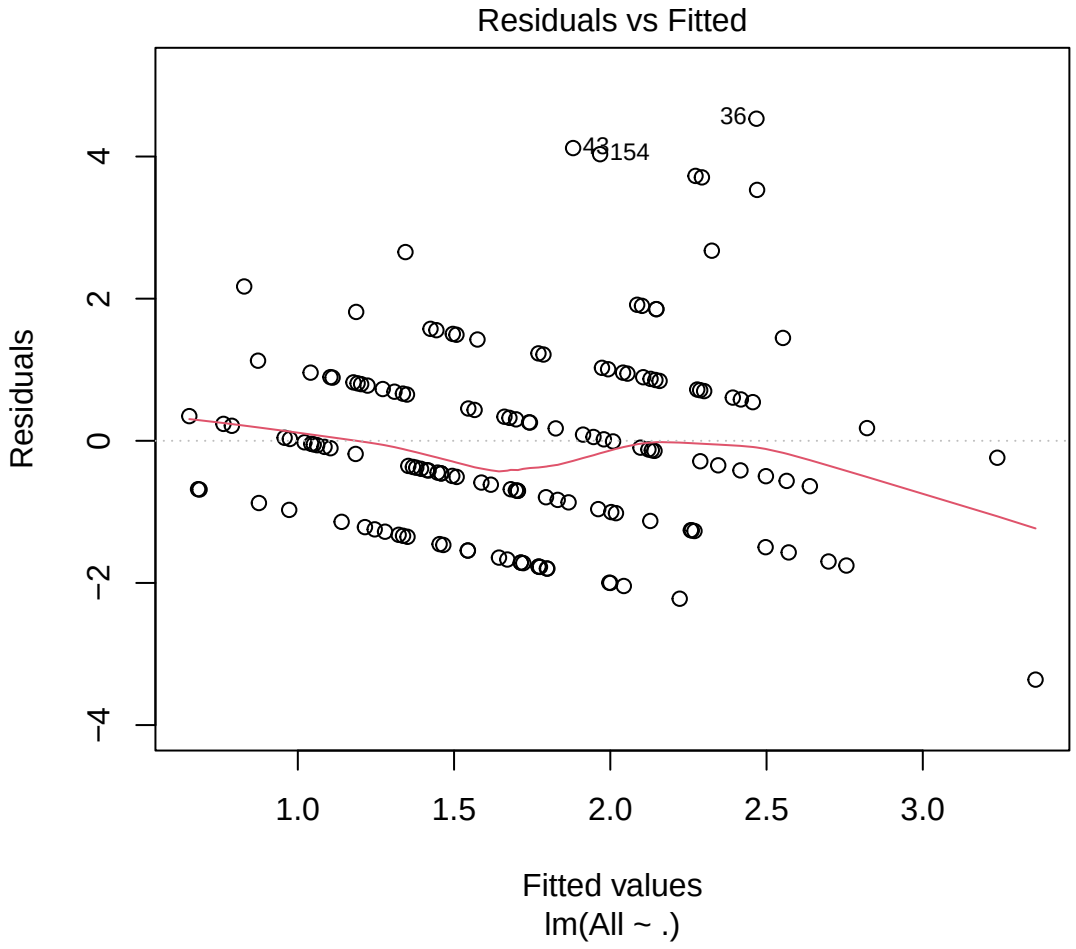


Linear Regression

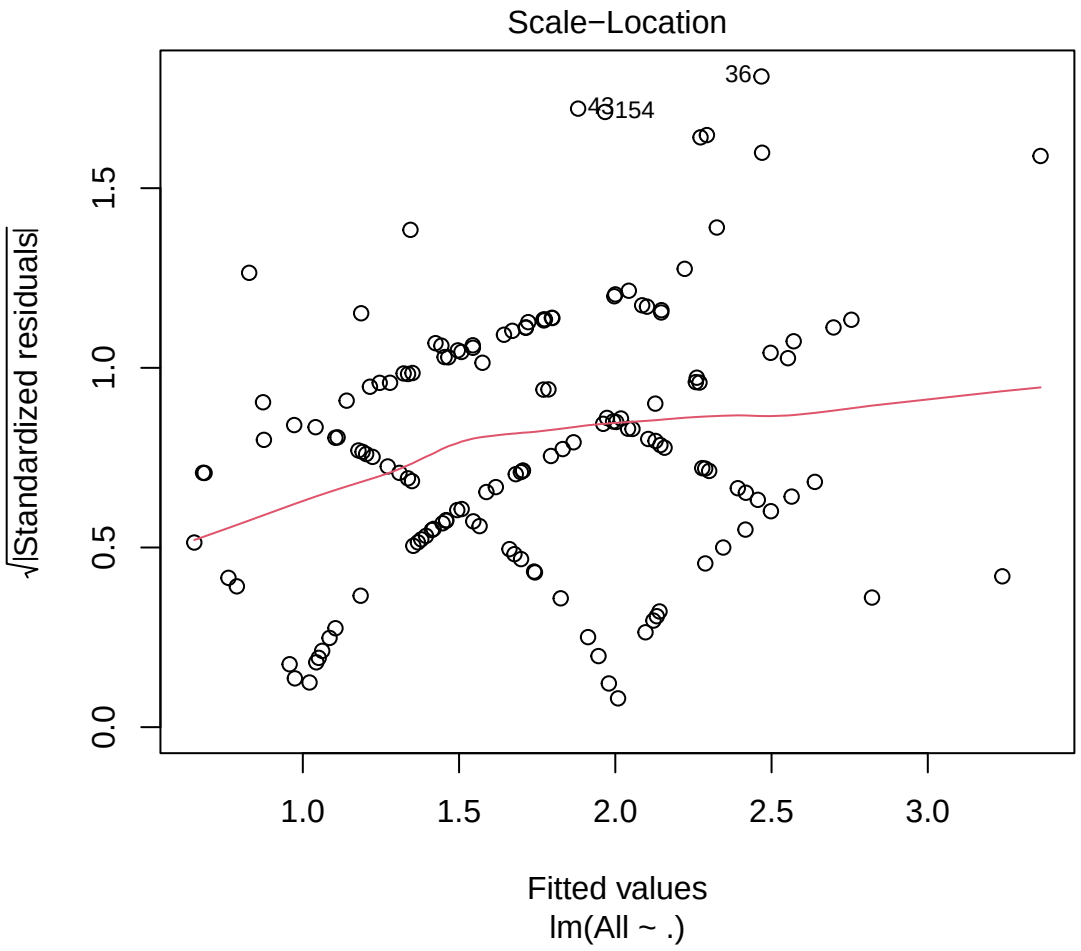
```
lmFull <- lm(All ~ ., data = data)
predict(lmFull, newdata = data.frame(SOI = -3, NAO = 3, SST = 0, SSN = 250))
```

```
##         1
## -0.318065
```

```
plot(lmFull, which = 1)
```




```
plot(lmFull, which = 3)
```



Poisson Regression

```
PoiFull <- glm(All ~ ., data = data, family = "poisson")
summary(PoiFull)
```

```
##
## Call:
## glm(formula = All ~ ., family = "poisson", data = data)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.8530  -0.8984  -0.1376   0.6027   2.4720
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.595288   0.103342   5.760 8.39e-09 ***
## SOI          0.061863   0.021319   2.902  0.00371 **
## NAO         -0.166595   0.064427  -2.586  0.00972 **
```

```
## SST          0.228972   0.255289   0.897  0.36977
## SSN         -0.002306   0.001372  -1.681  0.09284 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 197.89  on 144  degrees of freedom
## Residual deviance: 174.81  on 140  degrees of freedom
## AIC: 479.64
##
## Number of Fisher Scoring iterations: 5
```

```r
plot(data$SOI, hurricanes$All[-(1:15)], cex = 0.75, col = "gray",
     xlab = "", ylab = "", las = 1)
mtext("Hurricane Count", side = 2, line = 2)
mtext("Year", side = 1, line = 2)
points(data$SOI, predict(lmFull), col = "red",
       cex = 0.5, pch = 16)
points(data$SOI, predict(PoiFull, type = "response"), col = "blue", cex = 0.5, pch = 16)
```