Bottom-charmed hadrons from Lattice QCD

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Note to ourselves: To be written at the end of the paper preparation ...

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The physics of heavy hadrons with only one heavy flavor is affected by the presence of two widely separated scales : the Λ_{QCD} and the relatively large heavy quark mass. For this reason the chiral quark dynamics, that determine the low energy spectrum of light hadrons, could have a suppressed role in the heavy hadron excitations and hence heavy hadron physics plays important role in our knowledge about the quark confining nature of strong interactions. Phenomenological approaches based on empirical relations and simple minded calculations and abinitio non-perturbative calculations such as lattice QCD have been quite successful in interpreting and predicting their low lying spectrum []. Whereas, the predictions on heavy hadrons with two different flavors from such calculations are yet to be tested. Investigations of such systems (bottom-charm hadrons) are highly appealing, as these systems involve multiple scales : $1/m_b(\nu_b \sim 0.05)$, $1/m_b(\nu_c \sim 0.45)$ and Λ_{QCD} and hence can reveal numerous information on strongly interacting systems, which are hidden from heavy hadrons with single heavy flavor.

In the bottom-charm sector, only two B_c resonances have been observed experimentally : one at energy 6275(1) MeV interpreted as the $J^0 = 0^-$ B_c excitation based on quark model predictions and the other observed recently by ATLAS with energy 6842(6) MeV, interpreted as the former first radial excitation $B_c(2S)$. Multitude of phenomenological calculations exist in literature with predictions for bottom-charm mesons [] and baryons []. However, their predictions on the states vary widely in energy. e.g. hyperfine splittings predicted for B_c mesons spread over a range of 40-90 MeV []. The spread of the higher excited states are even more scattered in pattern. Lattice QCD methods provide a unique opportunity to study hadronic physics from frist principles and aid in complementing the phenomenological calculations and interpreting and predicting experimental observations. In light of this, multiple lattice computations of hadron spectrum using complementary lattice setups would be crucial to ensure the robustness of lattice predictions. However, not many lattice calculations on bottom-charm spectroscopy exist. Bottom-charm mesos have been studied by two collaborations [] and only few calculations exist for bottom-charm baryons [].

In this letter, we present the results from our lattice investigations of bottom-charm hadron spectrum. *Note to ourselves: May be we should write this para towards the end ...*

Three dynamical N_f =2+1+1 flavors lattice QCD ensembles generated by the MILC collaboration with one-loop, tadpole improved Symanzik gauge action and Highly Improved Staggered Quark (HISQ) fermion action [] and lattice extensions $24^3 \times 64$, $32^3 \times 96$ and $48^3 \times 144$ at gauge couplings $10/g^2 = 6.00$, 6.30 and 6.72 respectively are utilized for this calculation. Our estimates for the lattice spacings on these ensembles from the lattice value of Ω baryon mass (0.1192(14), 0.0877(10) and 0.0582(5)) are found to be consistent by indenpendent measurements by MILC collaboration (0.1207(11), 0.0888(8) and 0.0582(5)) using the $r1$ parameter. We tune the valence strange quark mass by setting the fictituous pseudoscalar $\bar{s}s$ mass to 685 MeV []. The quark masses for the strange and the charm sea are set at their physical values, whereas the ratio of quark masses m_l/m_s in the sea is fixed to 0.2 for all three ensembles. The details of these lattice QCD ensembles in use are summarized in Ref. [].

For the fermion measurements, we follow a mixed action approach by using an overlap action for quark masses up to charm, whereas the bottom quarks are realized using a Non-Relativistic QCD (NRQCD) formulation. Overlap fermion action has multiple desirable features that includes the exact chiral symmetry on the lattice and automatic $\mathcal{O}(a)$ improvement. By adopting such a mixed action approach, one get the advantage of having large set of configurations with small discretization errors as well as small taste breaking effects. One also gets the advantage of simulating the quark masses all the way up to charm quark with chiral fermions having no $\mathcal{O}(a)$ uncertainties. The numerical implementation of the overlap femion action follows the Ref. []. The NRQCD Hamiltonian and the Green's function we use in this calculation has been discussed in the Ref. []. We consider all the terms in the Hamiltonian up to $1/M_0^2$ and the leading term of the order of $1/M_0^3$, where M_0 is the bare bottom quark mass in lattice units. For the coarse and the fine lattice, we utilize the improvement co-efficients in the NRQCD Hamiltonian, as estimated non-perturbatively

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by HPQCD collaboration []. For the superfine lattice, we use the tree level co-efficients. The gauge configurations are fixed to Coulomb gauge and then smeared with a single level of HYP blocking. Fermion measurements are made on these HYP smeared configurations with wall sources and point sinks. Using such measurements for a large set of valence light quark masses on these ensembles, we are able to make systematic extrapolation towards the chiral and continuum limits.

pmn : The discussion on tuning the charm and the bottom quark masses, I would personally like to have a discussion similar to the one discussed in [1]. However, since I do not have the data or the results from the tuning using kinetic mass and the speed of light measurements, I keep the discussion short. But I give an outline of what should be discussed in my opinion. Once more info is written in this section, I can polish it to look better

Heavy hadron observables are severely affected by the discretization uncertainties on the lattice, owing to the large heavy quark mass. To quantify the discretization uncertainties in the charm and bottom sector, we investigate the energy momentum dispersion relation of their respective 1S onium states. Firstly we tune charm and the bottom quark masses by setting the spin averaged kinetic mass of their 1S ground states to their physical value. We then investigate the speed of light (c) as dictated by these kinetic masses and the continuum dispersion relation on the ensembles we use in our study. Closer the value of c to unity, smaller the discretization errors. A similar magnitude of cut-off errors, as quantified by c, are expected other observables with heavy quarks. In Table I, we present the speed of light measurements.

momentum($\|\mathbf{n}\|$)	Ensemble1	Ensemble2	Ensemble3
1	0.98(2)	0.96(2)	0.94(2)
2	0.98(2)	0.96(2)	0.94(2)
3	0.98(2)	0.96(2)	0.94(2)

TABLE I. Speed of light measurements. pmn : Number currently in Table I are temporary values.

To further demonstrate the quality of our heavy qaurk treatment, we present various hyperfine splittings involving heavy quarks. Hyperfine splittings are generally affected by discretization errors. In the Table II, we present the chiral extrapolated hyperfine splittings for a collection of heavy mesons on the ensembles we use in this study.

pmn : The section on heavy hadron chiral perturbation theory is a mere copy of the proceedings. I need more time to build some words for this. I keep this here to note the position of the discussions.

In order to reduce discretization errors, we perform chiral extrapolation on the ratios of baryon masses ($M_b a$) to the $1S$ spin-average mass ($M_{1S}a$), i.e., $M_b^r = M_b a/nM_{1S}a$, where $n = 1/2$ and 1 for singly-and doubly charmed baryons, respectively. The chiral extrapolations are made with a naive quadratic fit form in the quark

Meson	HFS	EXP
$\bar{c}u$	-	142
$\bar{c}s$	-	143.8(4)
$\bar{c}c$	-	113.5(5)
$\bar{b}u$	-	45.2(2)
$\bar{b}s$	-	49(2)
$\bar{b}b$	-	61(2)

TABLE II. Hyperfine splittings in heavy mesons pmn : Number currently in Table II are temporary values.

mass, $M_b^r = A + B.(m_\pi a)^2$, as well as with a chiral extrapolation form (equations below) using heavy baryon chiral perturbation theory (HBChPT), as described in Ref. [2].

$$\begin{aligned}\frac{m_{\Lambda_c}}{m_{spinav.}} =& \frac{m^0_{\Lambda_c}}{m^0_{spinav.}} + \frac{\sigma_{\Lambda_c}}{(4\pi f_\pi)m_{spinav.}} m_\pi^2 \\ &- \frac{6g_3^2}{(4\pi f_\pi)^2(m_{spinav.})}\Big(\frac{1}{3}\mathcal{F}(m_\pi, \Delta_{\Lambda_c\Sigma_c}, \mu) \\ &+ \frac{2}{3}\mathcal{F}(m_\pi, \Delta_{\Lambda_c\Sigma_c^*}, \mu)\Big) \end{aligned} \tag{1}$$

$$\begin{aligned}\frac{m_{\Xi_c}}{m_{spinav.}} =& \frac{m^0_{\Xi_c}}{m^0_{spinav.}} + \frac{\sigma_{\Xi_c}}{(4\pi f_\pi)m_{spinav.}} m_\pi^2 \\ &- \frac{3}{2}\frac{g_3^2}{(4\pi f_\pi)^2(m_{spinav.})}\Big(\frac{1}{3}\mathcal{F}(m_\pi, \Delta_{\Xi_c\Xi_c'}, \mu) \\ &+ \frac{2}{3}\mathcal{F}(m_\pi, \Delta_{\Xi_c\Xi_c^*}, \mu)\Big) \end{aligned} \tag{2}$$

for Λ_c and Ξ_c respectively. The chiral function $\mathcal{F}$ is defined as,

$$\begin{aligned}\mathcal{F}(m, \Delta, \mu) =& (\Delta^2 - m^2 + i\epsilon)^{3/2} \ln\left(\frac{\Delta + \sqrt{\Delta^2 - m^2 + i\epsilon}}{\Delta - \sqrt{\Delta^2 - m^2 + i\epsilon}}\right) \\ &- \frac{3}{2}\Delta m^2 \ln\left(\frac{m^2}{\mu^2}\right) - \Delta^3 \ln\left(\frac{4\Delta^2}{m^2}\right), \end{aligned} \tag{3}$$

with $\mathcal{F}(m, 0, \mu) = \pi m_\pi^3$. Splittings Δ used in the extrapolation formula are obtained by extrapolating the splittings between two baryons to the physical pion masses using trivial extrapolation form

$$\Delta_{ij} = \Delta^0_{ij} + A.(m_\pi a)^2, \tag{4}$$

where i and j are the baryons under consideration. For Λ_c and Ξ_c we could use HBChPT with $\chi^2/dof \sim 1$. We then perform continuum extrapolation of the chirally extrapolated ratios with a form up to $\mathcal{O}(a^2)$ terms. Finally to obtain the physical values we multiply the extrapolated values by $nM_{phy}(1S : c\bar{c})$.

Hadron spectroscopy on the lattice proceeds by computing two point correlation functions between hadronic observables, followed by non-linear fitting techniques to estimate the energy of the excitations. We follow the same conventional strategy. In what follows we describe the meson and baryon interpolating operators we use in this study. For mesons, we use the basic local interpolator type ($\bar{p}\Gamma\ q$), where p and q refer to the quark fields and Γ as given in the Table III for various quantum channels.

J^{PC}	0^{-+}	1^{--}	0^{++}	1^{++}	1^{+-}
Γ	$\gamma_5,\ \gamma_5\gamma_t$	γ_i	I, γ_t	$\gamma_i\gamma_5$	$\gamma_i\gamma_j$
info	pscalar	vector	scalar	pvector	tensor
for us	a4a4		scalar0		

TABLE III. Meson operators used. The subscripts i and j runs over x, y and z.

For the baryons, we use the following conventional three quarks operators

$$\mathcal{O}_5^{\gamma}(p,q,r) = \epsilon^{abc}\Big(p_a^{\alpha}(C\gamma_5)_{\alpha\beta}q_b^{\beta}\Big)r_c^{\gamma} \quad \text{and}$$
$$\mathcal{O}_j^{\gamma}(p,q,r) = \epsilon^{abc}\Big(p_a^{\alpha}(C\gamma_j)_{\alpha\beta}q_b^{\beta}\Big)r_c^{\gamma}, \tag{5}$$

where p, q and r refer to the quark fields, a, b and c refer to color indices, α, β and γ refer to the Dirac indices, ϵ^{abc} is the Levi-Civita tensor anti-symmetrizing the color indices, C is the charge conjugation matrix given by $\gamma_t\gamma_y$ and j runs over x, y and z. The positive parity and negative parity components are projected out using the projection operators

$$\mathcal{P}_{\pm} = \frac{1}{2}(1 \pm \gamma_t). \tag{6}$$

$\mathcal{O}_j^{\gamma}$ couples to $\frac{3}{2}$ as well as $\frac{1}{2}$ in general. To project out the individual spin components, we use the spin projection operators [3]

$$\mathcal{P}_{ij}^{3/2} = \delta_{ij} - \frac{1}{3}\gamma_i\gamma_j \quad \text{and} \quad \mathcal{P}_{ij}^{1/2} = \frac{1}{3}\gamma_i\gamma_j. \tag{7}$$

pmn : In Table IV, I list the collection operators implemented in our codes. We can finalize this table based on the analysis.

I. LITERATURE SURVEY

[4] : Karliner and Rosner, publication in 2014. Comments on the masses, production, decays and detection. Masses predicted using on empirical relations based on constituent quark masses.

[5] : 1896 Fermilab report by Bjorken. Simple-minded calculation based on observed ground state light and strange baryon spectra.

Baryon	J^P	Operator	Internal name
Ξ_{bc}	$\frac{1}{2}^+$	$\mathcal{O}_5^{\gamma}([b,c,u])$	1o2XI
Ξ'_{bc}	$\frac{1}{2}^+$	$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}([b,c,u])$	1o2XIprime
Ξ^*_{bc}	$\frac{3}{2}^+$	$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}([b,c,u])$	3o2XI
Ω_{bc}	$\frac{1}{2}^+$	$\mathcal{O}_5^{\gamma}([b,c,s])$	1o2XI
Ω'_{bc}	$\frac{1}{2}^+$	$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}([b,c,s])$	1o2XIprime
Ω^*_{bc}	$\frac{3}{2}^+$	$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}([b,c,s])$	3o2XI
Ω_{bcc}	$\frac{1}{2}^+$	$\mathcal{O}_5^{\gamma}(c,b,c)$	1o2OMEGA
		$\mathcal{O}_5^{\gamma}(c,c,b)$	1o2DIAG_DIQ
Ω'_{bcc}	$\frac{1}{2}^+$	$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}(c,b,c)$	1o2prime
		$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}(c,c,b)$	1o2DIAG_DIQprime
Ω^*_{bcc}	$\frac{3}{2}^+$	$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}(c,b,c)$	3o2OMEGA
		$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}(c,c,b)$	3o2DIAG_DIQ
Ω_{cbb}	$\frac{1}{2}^+$	$\mathcal{O}_5^{\gamma}(b,c,b)$	1o2OMEGA
		$\mathcal{O}_5^{\gamma}(b,b,c)$	1o2DIAG_DIQ
Ω'_{cbb}	$\frac{1}{2}^+$	$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}(b,c,b)$	1o2prime
		$\mathcal{P}_{ij}^{1/2}\mathcal{O}_j^{\gamma}(b,b,c)$	1o2DIAG_DIQprime
Ω^*_{cbb}	$\frac{3}{2}^+$	$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}(b,c,b)$	3o2OMEGA
		$\mathcal{P}_{ij}^{3/2}\mathcal{O}_j^{\gamma}(b,b,c)$	3o2DIAG_DIQ

TABLE IV. Baryon operators used. The subscripts i and j runs over x, y and z. $[b,c,u]$ implies three such operators are build by forming cyclic rotation of the flavors. Some of baryons mentioned in the first column are fictituous.

[6] : Tevatron B-Physics report 2001. Masses based on relations proposed by Bjorken [53] and DGG [54].

[7] : 1994 Potential model for masses by Bagan, Richad et al. decay constants, decay modes are discussed with QCDSR framework.

[8] : 1995 (Roncaglia, Lichtenberg, Predazzi) Predictions based on Feynmann-Hellman Theorem (FHT) and semi-empirical mass relations.

[9] : 1996 (Lichtenberg, Roncaglia, Predazzi) Prediction based on CQM, FHT, empirical mass formulaes.

[10] : 1996 Ebert et al. Relativistic quasipotential quark models.

[11] : 1996 Silvestre-Brac; Interquark potentials plus Faddeev equations for three body systems.

[12] : 2001 Kiselve and Likhoded; Potential approach and QCD sum rules.

[13] : 2002 Narodetskii and Trusov; Non-perturbative string approach

[14] : 2002, Ebert et al. Relativistic quark model within light quark heavy diquark picture.

[15] : 2004, He et al. Doubly heavy baryons based on MIT bag model

Reference	Ξ_{bc}	Ξ'_{bc}	Ξ^*_{bc}	Ω_{bc}	Ω^*_{bc}
[4]	6914(13)	6933(12)	-	-	-
[5]	6916(139)	6976(99)	7013(84)	7073(130)	7160(84)
[6]	6938	6971	7000	7095	7128
[7]	6930(50)	-	-	7000(50)	-
[8]	6990(90)	7040(90)	7060(90)	7060(90)	7120(90)
[9]	7029	7053	7083	7126	7165
[10]	6950	7000	7020	7050	7110
[11]	6916	-	6991	7005	7073
[12]	6820	6850	6900	6910	6990
[13]	6960	-	-	7130	-
[14]	6933	6963	6980	7088	7130
[15]	6838	7028	6986	6941	7077
[16]	$6919(^{+17}_{-7})$	$6948(^{+17}_{-6})$	$6986(^{+14}_{-5})$	$6986(^{+27}_{-17})$	$7046(^{+11}_{-9})$
[17]	7011	7047	7074	7136	7187
[18]	6840	-	-	6945	-
[19]	6750(50)	6950(80)	8000(260)	7020(80)	7540(80)
[20, 21]	6920	-	6986	7136	7187
[22, 23]	6720(200)	6790(200)	7250(200)	6750(300)	7300(200)
[24]	7014	-	7064	-	-
[25]	6904	6920	6936	6994	7017
[26]	6846	6891	6919	6999	7063
[27]	6820	6850	-	-	-
[28]	6650	6610	6690	6750	6770
[29]	6792	6825	6827	6999	7024
[30]	6842	-	6919	6988	7054
[31]	7037(50)	-	7114(31)	7164(61)	7242(42)
[32]	$6493(^{33}_{28})$	$6959(^{36}_{28})$	$6985(^{36}_{28})$	$6998(^{27}_{20})$	$7059(^{28}_{21})$
				$7032(^{28}_{20})$	

TABLE V.

[16] : 2006 Albertus et al. Variational method plus heavy quark spin symmetry constraints.

[17] : 2008 Pervin and Roberts, Constituent Quark model

[18] : 2010, Weng et al, Covariant instantaneous approximation plus Bethe-Salpeter equations. Interesting naming to divert the reader? Covariant instantaneous approximation? Does it not simply mean an instantaneous potential?

[19, 33] : 2008 Zhang and Huang, QCD sum rules.

[20, 21] : 2016-7 Zalak Shah, CQM within hyper Coulomb potential.

[22, 23] 2012 Aliev et al. QCD sum rules.

[24] : 2012 Eakins and Roberts, CQM plus superflavor symmetry (frozen or relatively high diquark excitations) Reconsidering reading this reference.

Reference	Ω_{bcc}	Ω^*_{bcc}	Ω_{bbc}	Ω^*_{bbc}
[5]	8250(107)	8254(105)	11549(149)	11553(147)
[6]	8198	8202	11476	11481
[17]	8245	8265	11535	11554
[25]	7832	7839	11108	11115
[33]	7410(130)	7450(160)	10300(100)	10540(110)
[27]	-	-	11120	11180
[29]	8018	8025	11280	11287
[34]	8230(130)	8230(130)	11500(110)	11490(110)
[35]	8018	8046	11214	11245
[36]	7980(70)	-	11480(120)	-
[37]	-	8030	-	11200
[30]	-	8039	-	11152
[38]	7984	8005	11139	11163
[31]	8274(84)	8353(63)	11640(98)	11725(76)
[32]	$8007(^{9}_{20})$	$8037(^{9}_{20})$	$11195(^{8}_{20})$	$11229(^{8}_{20})$

TABLE VI.

Reference	0^-	1^-	0^+	1^+
[7]	6255(20)	6330(20)	-	-
[30]	6347	6388	-	-
[38]	6304	6342	-	-
[39]	6270	6340	-	-
	6850	6890	-	-
[40]	6277	-	-	-
[41]	6260	6340	6680	6730/40
[42]	6270	6332	6699	6734/49
[43]	6247	6308	6688	6738/57
[44]	6253	6317	6683	6717/29
[45]	6264	6337	6700	6730/36
[46]	6286	6341	6701	6737/60
[47]	6275	6314	6672	6766/828
[48]	$6304(4)(11)(^{+18}_{0})$	-	-	-
[49, 50]	6280(4)	6300(7)(2)(6)	-	-
[51]	-	6333(3)	6711(2)	6752(2)
	6843(6)	6878(6)	-	6764(2)
[52]	6278(4)(8)	6332(5)(8)	6707(14)(8)	6742(14)(8)
	6894(19)(8)	6922(19)(8)	-	-

TABLE VII. B_c

[25] : 2009, Giannuzzi. Semi-relativistic quark model, with potential model inspired from AdS/QCD correspondence.

[26, 38] : 2008 Bernotas and Simonis, MIT bag model.

[27] : 2000 Gershtein, Kiselev, et al.

[28] : 2011 Tang et al. QCD sum rules plus diquark model.

[29] : 2008 Matrynenko. Relativistic quark model with hyperspherical expansion.

[37] : 1980 Peter Hasenfratz, Bag model

[34] : 2011 Wang. Triply heavy baryons from QCD sum rules.

[35] : 2011 Flynn et al. HQSS.

[36] : 2006 Jia, Weakly coupled Coulomb bound states.

[32] : Brown et al. Lattice calculation. u, d and s domain wall; c RHQA and NRQCD for b. SU(4—2) heavy-hadron chiral perturbation theory including $1/m_Q$ and finite volume effects. Continuum extrapolation also performed.

[30] : 1978 Ponce, Bag model.

[55] : 1985 Tsuge, Morii and Morishita. Potential model with relativisitic corrections. Paper not accessible to me.

[56] : 2011 Llanes-Estrada, NNLO pNRQCD and three body variational approach.

[31] : 2014, Ghalenovi et al. NR CQM with hypercentral approach.

[57] : 2002 Nilmani quenched calculation. Look into the table V. Nilmani will take care of these numbers.

[58] : 2016 Garcilazo. Potential model studies with inputs/constraints from lattice QCD results. No numbers, but plots. One has to pixel read them.

[48] : 2005 PRL. HPQCD, Fermilab, UKQCD study of Bc meson in 3 flavor QCD.

[39] : i1985 Godfrey and Isgur. Relativised CQM.

[40] : 2004 Vijande et al. CQM.

[41] : 1994 Zeng, Roberts, Van Orden. Spectator equation. Relation to Bethe-Salpeter equations.

[42] : 2002 Ebert et al. Relativistic quark model

[43] : 1995 Gupta and Johnson. Potential model.

[44] : 1995 Kiselev et al. QCD sum rules and potential model.

[45] : 1994 Eichten and Quigg, empirical determination based on observed states in charmonia and bottomonia.

[46] : 2004 Godfrey. Relativized quark model

[59] : 1998 Fulcher, Potential model.

[47] : 2016 Antony Prakash et al. RQM with coupled channel effects.
[49, 50] : 2010 Lattice calculation HPQCD

[51] : 2015 Lewis, free form smearing, lattice calculation

[52] : 2012 HPQCD Dowdall lattice calculation

[60] : PDG

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[6] K. Anikeev *et al.*, in *Workshop on B Physics at the Tevatron: Run II and Beyond Batavia, Illinois, September 23-25, 1999* (2001) arXiv:hep-ph/0201071 [hep-ph].
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