Statistical Inference Course Project

Part 1: Simulation Exercise

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Overview

We investigate the exponential distribution and compare it to the Central Limit Theorem, which essentially states that a distribution of sample means (calculated from different samples of the same population) tends toward a normal distribution as the number of sample means collected tends to infinity. What is remarkable about this theorem is that it holds even when the observations sampled are *not* normally distributed themselves.

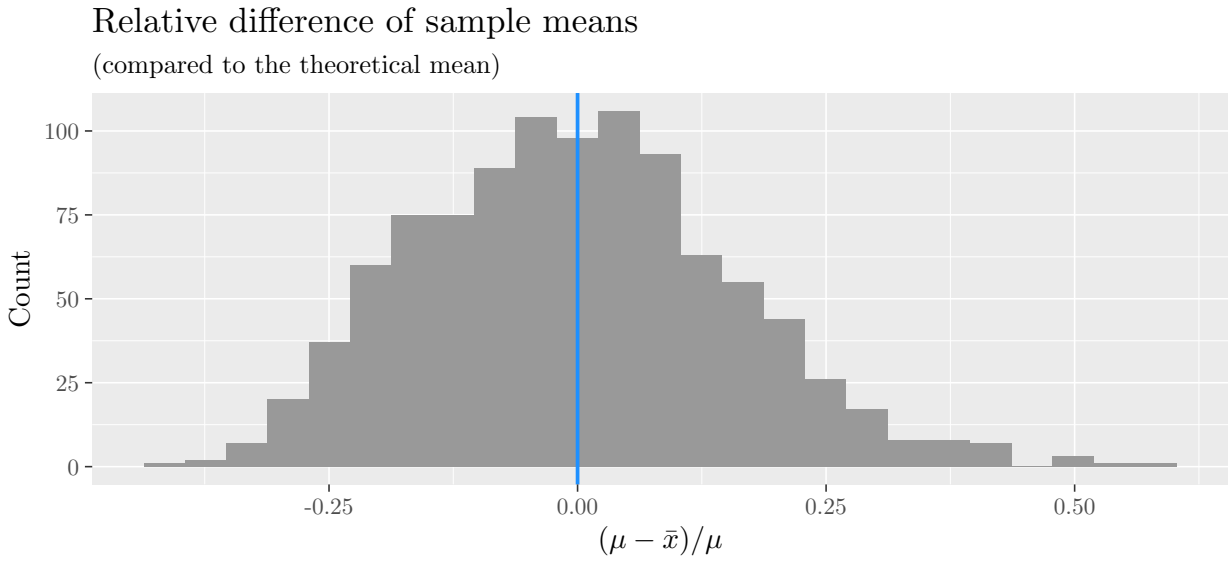
Simulating draws from the exponential distribution

We illustrate the Central Limit Theorem via simulation. We sample 40 observations from an *exponential* distribution with $\lambda = 0.2$ and then take the sample mean. We repeat this process 1000 times and plot the resulting distribution of the sample means.

Sample means vs. theoretical mean

The theoretical mean of this exponential distribution is $\mu = 1/\lambda = 1/0.2 = 5$, so we expect our simulated sample means to be close to this value.

We calculate and plot a histogram of the relative difference between our theoretical mean and our 1000 sample means ($(\mu - \bar{X}_i)/\mu$ for $i = 1, 2, ..., 1000$).

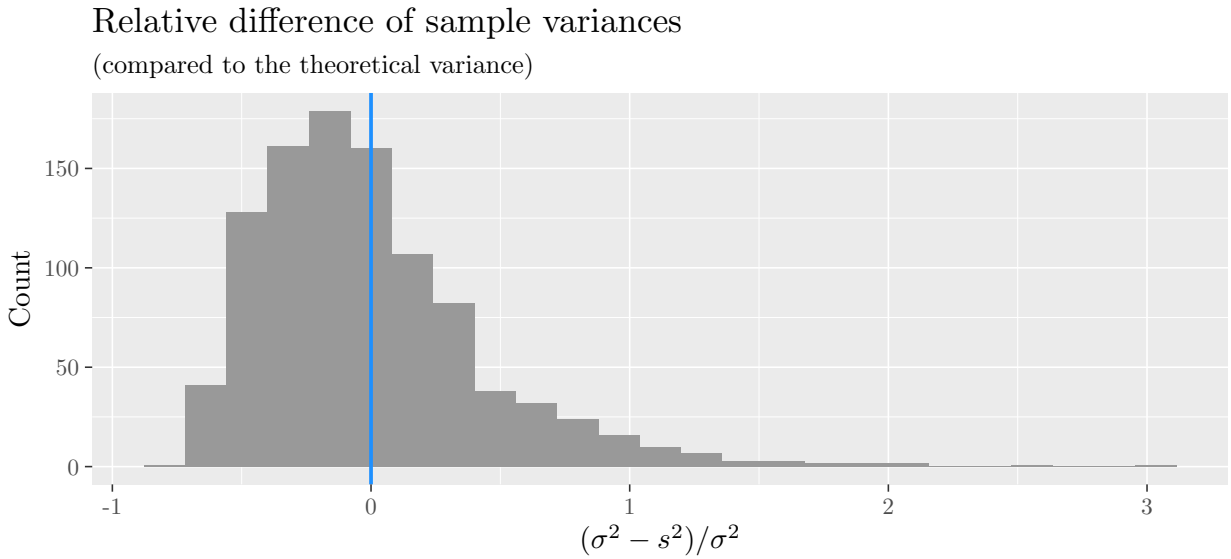


We can see that the relative difference between the sample means and the theoretical mean accumulate around zero, *i.e.* the sample means accumulate around the theoretical mean. Moreover, the majority of sample means are within 25% of the theoretical mean.

Sample variances vs. theoretical variance

The theoretical variance this exponential distribution is also $\sigma^2 = 1/\lambda = 1/0.2 = 5$, so we expect our simulated sample variances to be close to this value.

We calculate and plot a histogram of the relative difference between our theoretical variance and our 1000 sample variances ($(\sigma^2 - s_i^2)/\sigma^2$ for $i = 1, 2, ..., 1000$).



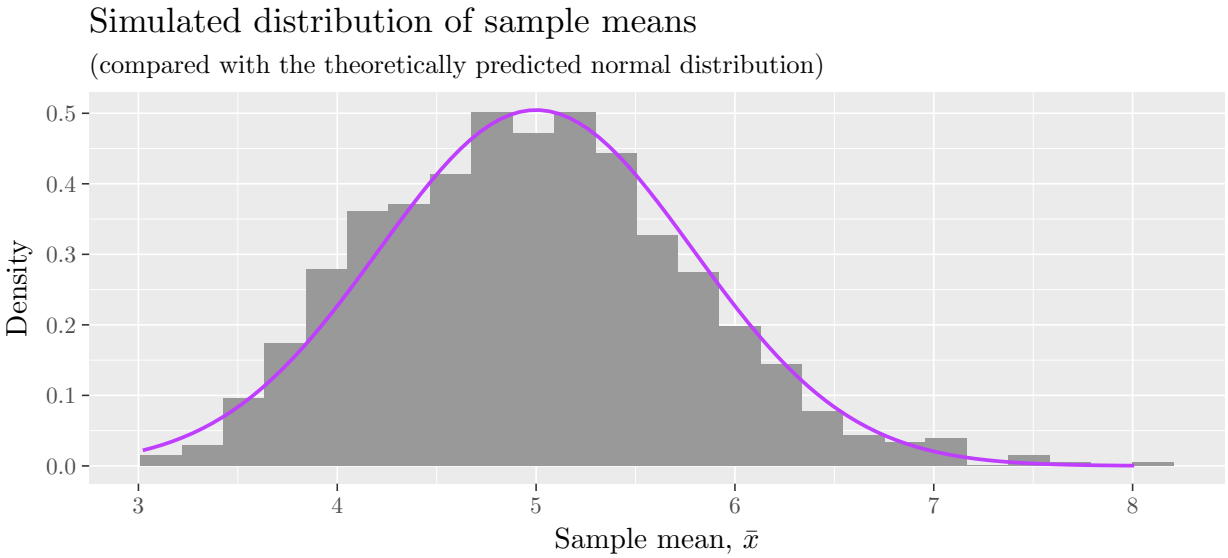
Again, the relative difference between the sample variances and the theoretical variance accumulate around zero, *i.e.* the sample variances accumulate around the theoretical variance.

However, the sample variances vary more from their associated theoretical value than the sample means; the majority of sample variances are within 100% of the theoretical variance.

Distribution of sample means

According to the Central Limit Theorem, the distribution of sample means should be normal (with mean μ and variance σ^2/n) as the number of samples, n, goes to infinity. Thus the distribution of sample means for our 40 draws should be *approximately* normal with these parameters.

To illustrate this point, we plot this histogram of sample means and overlay a normal distribution with mean μ and variance $\sigma^2/40$.



We see that the theorically predicted normal distribution matches reasonably well with the simulated distribution, and so we conclude that the simulated distribution is approximately normal.

Appendix

The following is all the code used to generate the figures in this report.

```
## Set default knitr chunk options
knitr::opts_chunk$set(dev = "tikz", echo = FALSE,
    fig.height = 3)

## Load necessary libraries
library(purrr)
library(ggplot2)

## Set seed for reproducible results
set.seed(15)

## Set parameters
lambda <- 0.2
n_draw <- 40
n_sim <- 1000

## Simulate draws and calculate sample
## means and variances
x <- replicate(n_sim, rexp(n_draw, lambda),
    simplify = FALSE)
xbar <- map_dbl(x, mean)
s2 <- map_dbl(x, var)

## Set theoretical mean
mu <- 1/lambda

## Calculate relative differences in means
## from mu (compared to mu)
diff_means <- data.frame(diffs = (xbar -
    mu)/mu)

## Set histogram parameters
n_bins <- 25
fill <- "grey60"

## Plot histogram of differences
p <- ggplot(data = diff_means, mapping = aes(x = diffs)) +
    geom_histogram(bins = n_bins, fill = fill) +
    labs(x = "$(\\mu - \\bar{x})/\\mu$",
        y = "Count", title = "Relative difference of sample means",
        subtitle = "(compared to the theoretical mean)") +
```

```
    geom_vline(xintercept = 0, colour = "dodgerblue",
        size = 1.2)

print(p)

## Set theoretical mean
sigma2 <- 1/lambda^2

## Calculate relative differences in means
## from mu (compared to mu)
diff_vars <- data.frame(diffs = (s2 - sigma2)/sigma2)

## Plot histogram of differences
p <- ggplot(data = diff_vars, mapping = aes(x = diffs)) +
    geom_histogram(bins = n_bins, fill = fill) +
    labs(x = "$(\\sigma^2 - s^2)/\\sigma^2$",
        y = "Count", title = "Relative difference of sample variances",
        subtitle = "(compared to the theoretical variance)") +
    geom_vline(xintercept = 0, colour = "dodgerblue",
        size = 1.2)

print(p)

## Convert sample mean data to a data
## frame
data <- data.frame(xbar = xbar)

## Plot histogram of xbar with CLT
## predicted normal distribution overtop
p <- ggplot(data = data, mapping = aes(x = xbar)) +
    geom_histogram(aes(y = ..density..),
        bins = n_bins, fill = fill) + labs(x = "Sample mean, $\\bar{x}$",
    y = "Density", title = "Simulated distribution of sample means",
    subtitle = "(compared with the theoretically predicted normal distribution)") +
    stat_function(fun = dnorm, args = list(mean = mu,
        sd = sqrt(sigma2/n_draw)), colour = "darkorchid1",
        size = 1.2) + NULL

print(p)
```