Computational Intelligence SEW SS17
Homework 4

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Maximum Likelihood Estimation of Model Parameters

For scenario 2, find out which is the anchor with exponentially distributed measurements:

As seen in the plots below, 2, 3 and 4 show a gaussian distribution, whereas the plot from figure 1 shows an exponentially distributed plot. Also lambda for the first anchor has a very plausible value, instead of the ones from anchor 2 to 4.

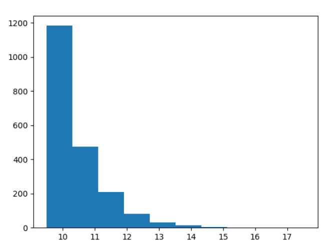

Figure 1: anchor r_0

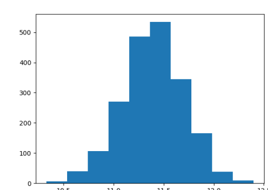

Figure 2: anchor r_1

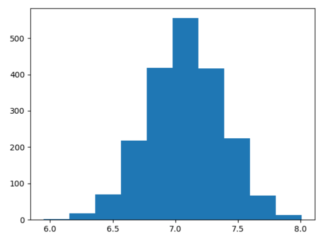

Figure 3: anchor r_2

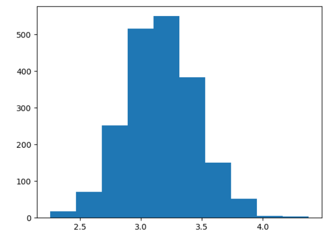

Figure 4: anchor r_3

Analytically derive the maximum likelihood solution for the exponential distribution:

Case II: $p(r_i, p) = \begin{cases} \lambda_i e^{-\lambda i(r_i - d_i(p))}, & r_i \geq d_i(p) \\ 0, & \text{else} \end{cases}$

$$\begin{aligned} L(r_i|p) &= ln(\lambda_i) - \lambda_i(r_i - d_i(p)) \\ \frac{\partial L(r_i|p)}{\partial \lambda_i} &= \frac{1}{\lambda_i} - (r_i - d_i(p)) \overset{!}{=} 0 \\ 1 - \lambda_i(r_i - d_i(p)) &= 0 \\ \lambda_i &= \frac{1}{r_i - d_i(p)} \end{aligned}$$

Estimate the parameters of the measurement models (2) and (3), i.e., estimate σ_i^2 and λ_i for all anchors in all 3 scenarios using the maximum likelihood method.

Table 1: HW4_1.data

	anchor r_0	**anchor** r_1	**anchor** r_2	**anchor** r_3
σ_i^2	0.091120787353436955	0.091704389499481176	0.096934463548551883	0.085391596621193597
λ_i	8.2643943490407317	8.0171346692797822	7.6841100110795288	8.6185831845268428

Table 2: HW4_2.data

	anchor r_0	**anchor** r_1	**anchor** r_2	**anchor** r_3
σ_i^2	1.5829503755101404	0.092789874308183223	0.089996782938954772	0.086182181972432853
λ_i	1.1123605389467395	8.482275948901135	8.0302966777533626	8.5787657875320349

Table 3: HW4_3.data

	anchor r_0	**anchor** r_1	**anchor** r_2	**anchor** r_3
σ_i^2	1.6446928698536829	1.6198260363284129	1.5693248239601287	1.6378092289139046
λ_i	1.1060260185511142	1.1157679060677703	1.1298612158821044	1.0963331410607486

1 Least-Squares Estimation of the Position

Scatter plots of the estimated positions. Do the estimated positions look Gaussian?

Yes, they do look gaussian. Their confidence intervals are different.

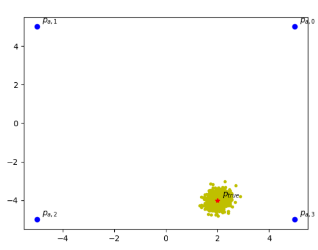

Figure 5: Scatter Scenario 1

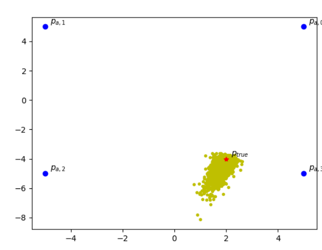

Figure 6: Scatter Scenario 2

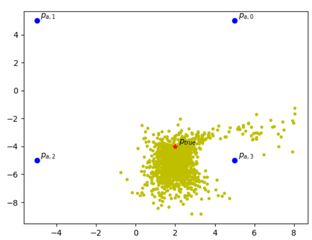

Figure 7: Scatter Scenario 3

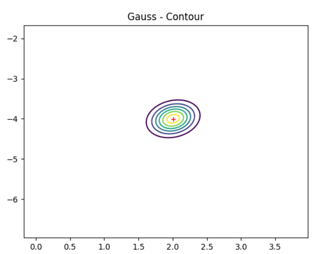

Figure 8: Gauss Contour Scenario 1

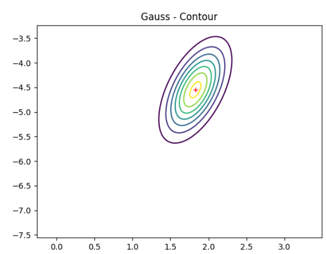

Figure 9: Gauss Contour Scenario 2

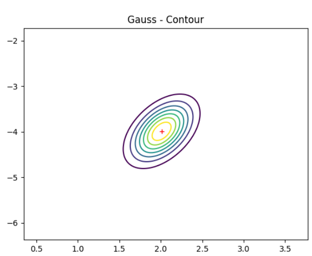

Figure 10: Gauss Contour Scenario 3

Compare the different scenarios by looking at the cumulative distribution function (CDF). What can you say about the probability of large estimation errors?

The probability of large estimation errors as you can see in image As you can see in figure 1 the probability of large estimation errors is very large, to be precise over 90 percent of all errors are below 0.6.

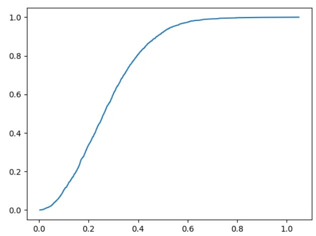

Figure 11: Error Scenario 1

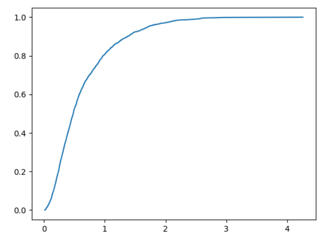

Figure 12: Error Scenario 2

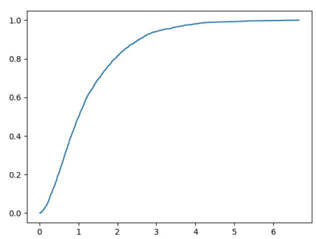

Figure 13: Error Scenario 3

Compare the performance of scenario 2 with the case that you do not use the anchor with the exponentially distributed measurements at all

As seen below in scenario 2 (figure 14) the covariance is higher and it is less gaussian distributed as in scenario 5 (figure 15). As seen below in figure 17

scenario 4 has less error then in scenario 2 (figure 16), because the dataset doesn't contain the first column which contained the exponential distribution.

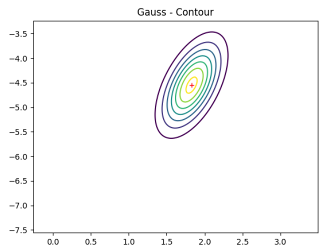

Figure 14: Gauss Contour Scenario 2

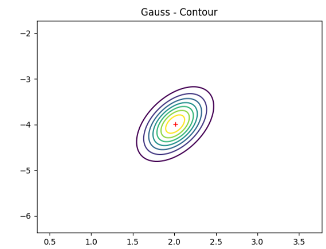

Figure 15: Gauss Contour Scenario 4

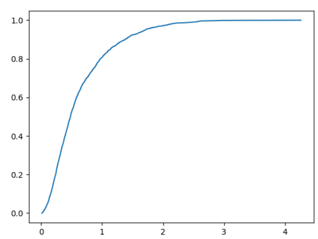

Figure 16: Error Scenario 2

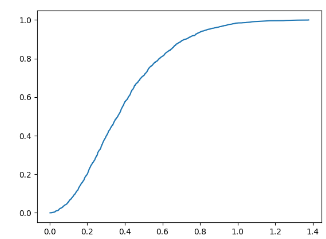

Figure 17: Error Scenario 4

Show analytically that for scenario 1 (joint likelihood for the distances is Gaussian), the least-squares estimator of the position is equivalent to the maximum likelihood estimator,i.e., (5) equals (6).

To show:

$$(5)\ \hat{P}ML = arg\ \underset{p}{max}\ p(r|p) = arg\ max \prod_{i=1}^{N_A} p(r_i|p)$$

$$(6)\ \overset{!}{=} \hat{P}LS = arg\ \underset{p}{min} \sum_{i=1}^{N_A} (r_i - d_i(p))^2 = ||\ r - d(p)||^2$$

$$(2)\ p(r_i|p) = \frac{1}{\sqrt{2\ \pi\ \sigma_i^2}} e^{\frac{(r_i - d_i(p))^2}{2\ \sigma_i^2}}$$

According to Wikipedia the standard form of the density function is to complex there is an alternative form shown below, which has the maximum on the same position as the standard form but is easier to work with.

$$l(q) = ln(\prod_{i=1}^{n} fx, (x_i; q)) = \sum_{i=1}^{n} lnfx_i(x_i; q)$$

therefore:

$$\sum_{i=1}^{N_A} ln(p(r_i|p)) \tag{1}$$

$$\sum_{i=1}^{N_A} ln(\frac{1}{\sqrt{2\ \pi\ \sigma_i^2}} e^{\frac{(r_i - d_i(p))^2}{2\ \sigma_i^2}}) \tag{2}$$

$$\sum_{i=0}^{M_A} ln(\frac{1}{\sqrt{2\ \pi\ \sigma_i^2}}) + ln(e^{\frac{(r_i - d_i(p))^2}{2\ \sigma_i^2}}) \tag{3}$$

$$\sum_{i=0}^{M_A} ln(\frac{1}{\sqrt{2\ \pi\ \sigma_i^2}}) - \frac{(r_i - d_i(p))^2}{2\ \sigma_i^2} \tag{4}$$

$$\sum_{i=0}^{M_A} ln(\frac{1}{\sqrt{2\ \pi\ \sigma_i^2}}) - \sum_{i=0}^{N_A} \frac{(r_i - d_i(p))^2}{2\ \sigma_i^2} \tag{5}$$