Linear Algebra

Inverted Pastry

Papyrus

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Preface

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Beijing, February 2024 *Inverted Pastry*

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Chapter 1

Vector Spaces

Definition 1. A *vector space* over a field F, whose elements are referred to as *scalars*, is a non-empty set V, whose elements are referred to as *vectors*, together with two binary operations. The first operation, called *vector addition* or simply *addition*, assigns to each pair $(\boldsymbol{u}, \boldsymbol{v})$ of vectors in V a vector $\boldsymbol{u} + \boldsymbol{v}$ in V. The second operation, called *scalar multiplication*, assigns to each pair $(a, \boldsymbol{v})$ in $F \times V$ a vector $a\boldsymbol{v}$ in V. Furthermore, if we let $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ be any vectors in V, and a, b be any scalars in F, the following properties must be satisfied.

1. Addition is *associative*, $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}$.
2. Addition is *commutative*, $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$.
3. There exists a vector $\mathbf{0}$ in V, called the *zero vector*, such that $\boldsymbol{u} + \mathbf{0} = \mathbf{0} + \boldsymbol{u} = \boldsymbol{u}$.
4. There exists a vector $-\boldsymbol{u}$ in V, called the *additive inverse* of $\boldsymbol{u}$, such that $\boldsymbol{u} + (-\boldsymbol{u}) = (-\boldsymbol{u}) + \boldsymbol{u} = \mathbf{0}$.
5. Scalar multiplication is *distributive with respect to vector addition*, $a(\boldsymbol{u} + \boldsymbol{v}) = a\boldsymbol{u} + a\boldsymbol{v}$.
6. Scalar multiplication is *distributive with respect to field addition*, $(a + b)\boldsymbol{u} = a\boldsymbol{u} + b\boldsymbol{u}$.
7. Scalar multiplication is *compatible with field multiplication*, $a(b\boldsymbol{u}) = (ab)\boldsymbol{u}$.
8. $1\boldsymbol{u} = \boldsymbol{u}$, where 1 denotes the multiplicative identity in F.

Such a vector space is also called an *F-vector space*.

Remark. Items 1 to 4 can be summarized by saying that $(V, +)$ is an *abelian group*.

Bibliography

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