Theorem 1. *Let $f : E \to X$ be a covering map. Let Y be a connected space and $g : Y \to X$ be a continuous map. The two lifts $H_1, H_2 : Y \to E$ of g with respect to f are equal if and only if there exists $y \in Y$ such that $H_1(y) = H_2(y)$.*

Alternatively, we can state the theorem as follows:

Theorem 2. *Let $f : E \to X$ be a covering map. Let Y be a topological space and $H_1, H_2 : Y \to E$ be two continuous maps such that*

$$f \circ H_1 = f \circ H_2.$$

Suppose for every connected component C of Y, there exists $y \in C$ such that $H_1(y) = H_2(y)$, then $H_1 = H_2$.

Proof sketch. Claim 1: If a closed and open set $S \subseteq Y$ intersects with all the connected components of Y, then $S = Y$.

Claim 2: A set $S \subseteq Y$ is closed and open in Y if and only if for every $y \in Y$ there is a neighborhood U_y of y such that $U_y \cap S$ is closed and open in U_y.

Claim 3: If $f, g : X \to Y$ are continuous and Y is a discrete topological space, then the set $\{x \in X \mid f(x) = g(x)\}$ is closed and open in X.

We now proceed with the proof of the theorem. Let

$$S = \{y \in Y \mid H_1(y) = H_2(y)\}.$$

We will show that S is closed and open in Y. Let $y \in Y$, since $f : E \to X$ is a covering space, we get open neighborhood $V_y \subseteq X$ of $(f \circ H_1)(y)$ such that

$$f^{-1}(V_y) \cong V_y \times D$$

for a discrete space D. Let

$$U_y = (f \circ H_1)^{-1}(V_y) = (f \circ H_2)^{-1}(V_y).$$

From claim 2, it is enough to show that $U_y \cap S$ is closed and open in U_y. Note that

$$U_y \cap S = \{u \in U_y \mid \mathrm{Proj}_D(H_1(u)) = \mathrm{Proj}_D(H_2(u)).\}$$

From claim 3, since D is a discrete set, we get that $U_y \cap S$ is closed and open in U_y. Hence, S is closed and open in Y. From assumption we get that S intersects with all the connected components of Y. From claim 1, we get that $S = Y$. Hence, $H_1 = H_2$. $\square$