**Theorem 1.** *Let $f : E \to X$ be a covering map. Let $Y$ be a connected space and $g : Y \to X$ be a continuous map. The two lifts $H_1, H_2 : Y \to E$ of $g$ with respect to $f$ are equal if and only if there exists $y \in Y$ such that $\tilde{g}_1(y) = \tilde{g}_2(y)$.*

Alternatively, we can state the theorem as follows:

**Theorem 2.** *Let $f : E \to X$ be a covering map. Let $Y$ be a topological space and $H_1, H_2 : Y \to E$ be two continuous maps such that*

$$f \circ H_1 = f \circ H_2.$$

*Suppose for every connected component $C$ of $Y$, there exists $y \in C$ such that $H_1(y) = H_2(y)$, then $H_1 = H_2$.*

*Proof sketch.* Claim 1: If a closed and open set $S \subseteq Y$ intersects with all the connected components of $Y$, then $S = Y$.

Claim 2: A set $S \subseteq Y$ is closed and open in $Y$ if and only if for every $y \in Y$ there is a neighborhood $U_y$ of $y$ such that $U_y \cap S$ is closed and open in $U_y$.

Claim 3: If $f, g : X \to Y$ are continuous and $Y$ is a discrete topological space, then the set $\{x \in X \mid f(x) = g(x)\}$ is closed and open in $X$.

We now proceed with the proof of the theorem. Let

$$S = \{y \in Y \mid H_1(y) = H_2(y)\}.$$

We will show that $S$ is closed and open in $Y$. Let $y \in Y$, since $f : E \to X$ is a covering space, we get open neighborhood $V_y \subseteq X$ of $(f \circ H_1)(y)$ such that

$$f^{-1}(V_y) \cong V_y \times D$$

for a discrete space $D$. Let

$$U_y = (f \circ H_1)^{-1}(V_y) = (f \circ H_2)^{-1}(V_y).$$

From claim 2, it is enough to show that $U_y \cap S$ is closed and open in $U_y$. Note that

$$U_y \cap S = \{u \in U_y \mid \mathrm{Proj}_D(H_1(u)) = \mathrm{Proj}_D(H_2(u)).\}$$

From claim 3, since $D$ is a discrete set, we get that $U_y \cap S$ is closed and open in $U_y$. Hence, $S$ is closed and open in $Y$. From assumption we get that $S$ intersects with all the connected components of $Y$. From claim 1, we get that $S = Y$. Hence, $H_1 = H_2$. $\square$