

# Homework-Romberg Numerical integration

PB18010496 杨乐园

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## 1 Introduction

编写计算定义在任意区间 $[a, b]$ 上的函数 $f$ 的Romberg算法, 可指定计算到Romberg阵列的任意行数, 并且输出整个阵列与计算误差, 并用如上程序计算积分值验证算法正确性。

$$I_1(f) = \int_0^1 \frac{\sin x}{x} dx \quad I_2(f) = \int_{-1}^1 \frac{\cos x - e^x}{\sin x} dx \quad I_3(f) = \int_1^\infty \frac{1}{xe^x} dx$$

其中相关函数值时, 用 $f(x_0) = \lim_{x \rightarrow x_0} f(x)$ 代替一些无法直接定义的函数值。而对于第三个积分计算, 采取变量替换 $x = \frac{1}{t}$ 即可。

计算到Romberg阵列中的第七行, 输出误差。

## 2 Method

Romberg算法计算公式如下:

$$\begin{cases} R(0, 0) = \frac{1}{2}(b-a)[f(a) + f(b)] \\ R(n, 0) = \frac{1}{2}R(n-1, 0) + h_n \sum_{i=1}^{2^{n-1}} f(a + (2i-1)h_n) & n \geq 1 \\ R(n, m) = R(n, m-1) + \frac{1}{4^m-1}[R(n, m-1) - R(n-1, m-1)] & n \geq 1, m \geq 1 \end{cases}$$

其中

$$h_0 = b - a \quad h_n = \frac{h_{n-1}}{2} \quad (n \geq 1)$$

故只需根据算法直接编写计算过程即可, Romberg算法如下:

**romberg** (f,a,b,M)

$h = (b - a)$

$R(0, 0) = \frac{1}{2}(b - a)[f(a) + f(b)]$

**for**  $n = 1$  **to** M

$h = h/2$

$R(n, 0) = \frac{1}{2}R(n-1, 0) + h \sum_{i=1}^{2^{n-1}} f(a + (2i-1)h)$

**for**  $m = 1$  **to**  $n$

$R(n, m) = R(n, m-1) + \frac{1}{4^m-1}[R(n, m-1) - R(n-1, m-1)]$

### 3 Results

输出结果如下：

图 1:  $I_1(f) = \int_0^1 \frac{\sin x}{x}$  的Romberg阵列与误差

```
0.9207354924039482533
0.93979328480617712694 0.9461458822735867515
0.94451352166538954864 0.9460869339517936892 0.9460830040636741517
0.94569086358270127850 0.9460833108884718551 0.9460830693509170662 0.9460830703872225093
0.94598502993438603340 0.9460830853849476184 0.9460830703513793359 0.9460830703672596894 0.9460830703671814038
0.94608585096276806718 0.9460830713055620784 0.9460830703669363758 0.9460830703671833129 0.9460830703671830134 0.9460830703671830150
0.94607694306006308775 0.9460830704258280946 0.9460830703671791623 0.9460830703671830161 0.9460830703671830149 0.9460830703671830149 0.9460830703671830149
```

积分误差为:  $0. \times 10^{-20}$

图 2:  $I_2(f) = \int_{-1}^1 \frac{\cos x - e^x}{\sin x}$  的Romberg阵列与误差

```
-3.5883002051320011102
-2.7941501025660005551 -2.5294334017106670367
-2.4839910862753402814 -2.3806047475117868569 -2.3706828372318615115
-2.3557125631257739417 -2.3129530554092518285 -2.3084429426024161600 -2.3074550077670281385
-2.2987235976443990650 -2.2797272758172741061 -2.2775122238444755913 -2.27702126009546670108 -2.2769019120244224574
-2.2720483405253772079 -2.2631565881523699222 -2.2620518756413763099 -2.2618064732889461626 -2.2617468074584923554 -2.2617319930845276143
-2.2591675634261543011 -2.2548739710597466655 -2.2543217965869051151 -2.2541990969193738263 -2.2541692640708656995 -2.2541618568925786744 -2.2541600862634719714
```

积分误差为:  $0.5851506926406663590$

图 3:  $I_3(f) = \int_1^\infty \frac{1}{xe^x}$  的Romberg阵列与误差

```
0.18393972058572116080
0.22730514352947327229 0.24176028451072397612
0.21983392335871307313 0.21734351663512634008 0.21571573211008649768
0.21935095793150070119 0.21918996945576324387 0.21931306631047237079 0.21937016685333563862
0.21938357975286012172 0.21939445369331326190 0.21940808597581659643 0.21940959422447285398 0.21940974884161456855
0.21938393240575668160 0.21938404095672220156 0.21938335637428279754 0.21938296384092511819 0.21938285940804846041 0.21938283312316716411
0.21938393427337269840 0.21938393489591137067 0.21938392722519064861 0.21938393628631617006 0.21938394009982750752 0.21938394115622220844 0.21938394142680415107
```

积分误差为:  $7.03128387739 \times 10^{-9}$

### 4 Discussion

通过对数据的观察我们发现：

Romberg算法较好的给出积分的数值计算值，误差除了第二个积分计算有0.585较大，其他均十分小，结果良好。

### 5 Computer Code

代码部分请参见附件!(Homework9.0429.nb)。