$Experiment3:\ FTCS$近似求解

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问题描述

针对下述偏微分方程初值问题：

$$\begin{cases} u_t = u_x, & -\infty < x < +\infty, t > 0 \\ u(x,0) = \sin 2\pi x, & -\infty < x < +\infty \\ \text{Periodic boundary condition}, & T = 1 \end{cases}$$

该方程的精确解为$u(x,t) = sin(2\pi(x+t))$，对时空区域均匀剖分，其中$x_j = j \cdot \Delta x, j = 0,1,2,\ldots,J$，空间步长$\Delta x = \frac{1}{J}$，令$\lambda = \frac{\Delta t}{\Delta x}$。取$\lambda = 0.5, J = 80$，分别取终止时间$T = 0.1, 0.4, 0.8, 1.0$。用$FTCS$方法计算其数值解，绘制出最大误差随时间变化图，并给出相应评论。

数值方法

记$v_j^n \approx u(x_j, t_n)$，有导数近似$u_t \approx \frac{u(x,t+\Delta t)-u(x,t)}{\Delta t}$，$u_x \approx \frac{u(x+\Delta x,t)-u(x-\Delta x,t)}{\Delta x}$，从而由偏微分方程得到相应的离散方程如下：

$$v_j^{n+1} = v_j^n + \frac{\Delta t}{2\Delta x}(v_{j+1}^n - v_{j-1}^n)$$

其中定解条件为：初始条件：$v_j^0 = sin2\pi x_j$，边界条件：$v_j^n = v_{j+J}^n$。

数值结果

1. $t = 0.1$

{0, 0. 594268}, {0. 0125, 0. 656742}, {0. 025, 0. 715167}, {0. 0375, 0. 769182}, {0. 05, 0. 818455}, {0. 0625, 0. 862682}, {0. 075, 0. 90159}, {0. 0875, 0. 93494}, {0. 1, 0. 962526}, {0. 1125, 0. 984177}, {0. 125, 0. 99976}, {0. 1375, 1. 00918}, {0. 15, 1. 01238}, {0. 1625, 1. 00933}, {0. 175, 1. 00007}, {0. 1875, 0. 984634}, {0. 2, 0. 963131}, {0. 2125, 0. 93569}, {0. 225, 0. 90248}, {0. 2375, 0. 863706}, {0. 25, 0. 819606}, {0. 2625, 0. 770454}, {0. 275, 0. 716552}, {0. 2875, 0. 658231}, {0. 3, 0. 595853}, {0. 3125, 0. 529801}, {0. 325, 0. 460483}, {0. 3375, 0. 388325}, {0. 35, 0. 313773}, {0. 3625, 0. 237287}, {0. 375, 0. 159338}, {0. 3875, 0. 0804067}, {0. 4, 0. 00097943}, {0. 4125, −0. 0784538}, {0. 425, −0. 157403}, {0. 4375, −0. 235383}, {0. 45, −0. 31191}, {0. 4625, −0. 386515}, {0. 475, −0. 458737}, {0. 4875, −0. 528131}, {0. 5, −0. 594268}, {0. 5125, −0. 656742}, {0. 525, −0. 715167}, {0. 5375, −0. 769182}, {0. 55, −0. 818455}, {0. 5625, −0. 862682}, {0. 575, −0. 90159}, {0. 5875, −0. 93494}, {0. 6, −0. 962526}, {0. 6125, −0. 984177}, {0. 625, −0. 99976}, {0. 6375, −1. 00918}, {0. 65, −1. 01238}, {0. 6625, −1. 00933}, {0. 675, −1. 00007}, {0. 6875, −0. 984634}, {0. 7, −0. 963131}, {0. 7125, −0. 93569}, {0. 725, −0. 90248}, {0. 7375, −0. 863706}, {0. 75, −0. 819606}, {0. 7625, −0. 770454}, {0. 775, −0. 716552}, {0. 7875, −0. 658231}, {0. 8, −0. 595853}, {0. 8125, −0. 529801}, {0. 825, −0. 460483}, {0. 8375, −0. 388325}, {0. 85, −0. 313773}, {0. 8625, −0. 237287}, {0. 875, −0. 159338}, {0. 8875, −0. 0804067}, {0. 9, −0. 00097943}, {0. 9125, 0. 0784538}, {0. 925, 0. 157403}, {0. 9375, 0. 235383}, {0. 95, 0. 31191}, {0. 9625, 0. 386515}, {0. 975, 0. 458737}, {0. 9875, 0. 528131}, {1, 0. 594268},

2. $t = 0.4$

{0, 0. 620717}, {0. 0125, 0. 552315}, {0. 025, 0. 480508}, {0. 0375, 0. 405738}, {0. 05, 0. 328467}, {0. 0625, 0. 249171}, {0. 075, 0. 168339}, {0. 0875, 0. 0864684}, {0. 1, 0. 004065}, {0. 1125, −0. 0783635}, {0. 125, −0. 160309}, {0. 1375, −0. 241266}, {0. 15, −0. 320735}, {0. 1625, −0. 398227}, {0. 175, −0. 473264}, {0. 1875, −0. 545383}, {0. 2, −0. 61414}, {0. 2125, −0. 67911}, {0. 225, −0. 739893}, {0. 2375, −0. 796115}, {0. 25, −0. 847428}, {0. 2625, −0. 893516}, {0. 275, −0. 934096}, {0. 2875, −0. 968917}, {0. 3, −0. 997764}, {0. 3125, −1. 02046}, {0. 325, −1. 03686}, {0. 3375, −1. 04687}, {0. 35, −1. 05043}, {0. 3625, −1. 04751}, {0. 375, −1. 03814}, {0. 3875, −1. 02236}, {0. 4, −1. 00028}, {0. 4125, −0. 972028}, {0. 425, −0. 937787}, {0. 4375, −0. 897764}, {0. 45, −0. 852207}, {0. 4625, −0. 801395}, {0. 475, −0. 745642}, {0. 4875, −0. 685292}, {0. 5, −0. 620717}, {0. 5125, −0. 552315}, {0. 525, −0. 480508}, {0. 5375, −0. 405738}, {0. 55, −0. 328467}, {0. 5625, −0. 249171}, {0. 575, −0. 168339}, {0. 5875, −0. 0864684}, {0. 6, −0. 004065}, {0. 6125, 0. 0783635}, {0. 625, 0. 160309}, {0. 6375, 0. 241266}, {0. 65, 0. 320735}, {0. 6625, 0. 398227}, {0. 675, 0. 473264}, {0. 6875, 0. 545383}, {0. 7, 0. 61414}, {0. 7125, 0. 67911}, {0. 725, 0. 739893}, {0. 7375, 0. 796115}, {0. 75, 0. 847428}, {0. 7625, 0. 893516}, {0. 775, 0. 934096}, {0. 7875, 0. 968917}, {0. 8, 0. 997764}, {0. 8125, 1. 02046}, {0. 825, 1. 03686}, {0. 8375, 1. 04687}, {0. 85, 1. 05043}, {0. 8625, 1. 04751}, {0. 875, 1. 03814}, {0. 8875, 1. 02236}, {0. 9, 1. 00028}, {0. 9125, 0. 972028}, {0. 925, 0. 937787}, {0. 9375, 0. 897764}, {0. 95, 0. 852207}, {0. 9625, 0. 801395}, {0. 975, 0. 745642}, {0. 9875, 0. 685292}, {1, 0. 620717}

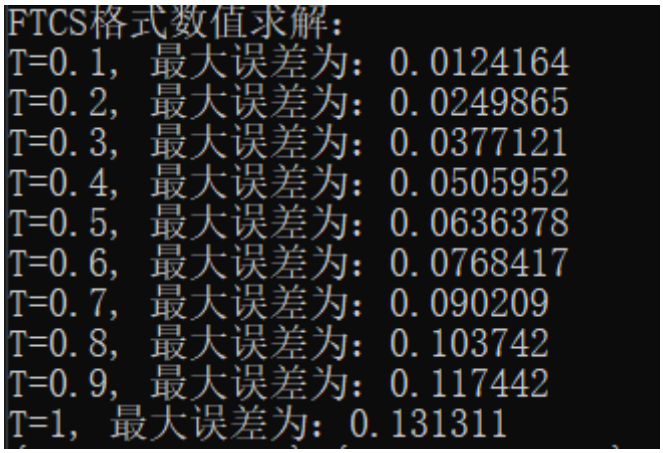
3. $t = 0.8$

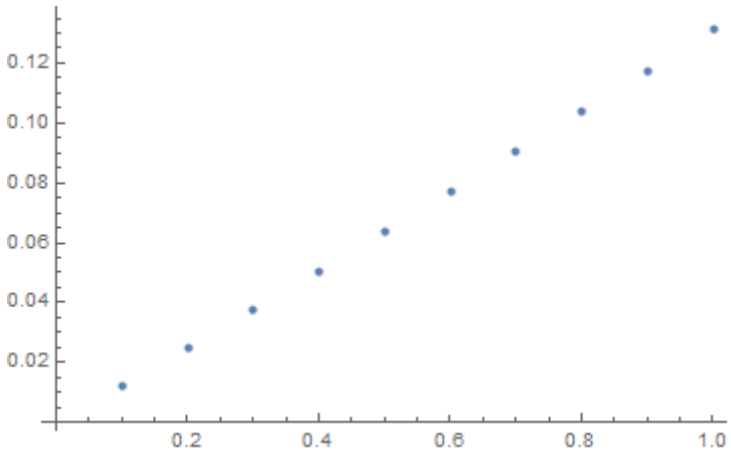
{0, -1. 05203}, {0. 0125, -1. 02267}, {0. 025, -0. 987005}, {0. 0375, -0. 945257}, {0. 05, -0. 897681}, {0. 0625, -0. 844571}, {0. 075, -0. 786254}, {0. 0875, -0. 723089}, {0. 1, -0. 655466}, {0. 1125, -0. 583801}, {0. 125, -0. 508538}, {0. 1375, -0. 430139}, {0. 15, -0. 349088}, {0. 1625, -0. 265885}, {0. 175, -0. 181043}, {0. 1875, -0. 0950847}, {0. 2, -0. 00854002}, {0. 2125, 0. 0780573}, {0. 225, 0. 164173}, {0. 2375, 0. 249277}, {0. 25, 0. 332844}, {0. 2625, 0. 414359}, {0. 275, 0. 49332}, {0. 2875, 0. 569238}, {0. 3, 0. 641648}, {0. 3125, 0. 710101}, {0. 325, 0. 774176}, {0. 3375, 0. 833478}, {0. 35, 0. 887642}, {0. 3625, 0. 936333}, {0. 375, 0. 979251}, {0. 3875, 1. 01613}, {0. 4, 1. 04675}, {0. 4125, 1. 07091}, {0. 425, 1. 08847}, {0. 4375, 1. 09932}, {0. 45, 1. 10339}, {0. 4625, 1. 10066}, {0. 475, 1. 09114}, {0. 4875, 1. 0749}, {0. 5, 1. 05203}, {0. 5125, 1. 02267}, {0. 525, 0. 987005}, {0. 5375, 0. 945257}, {0. 55, 0. 897681}, {0. 5625, 0. 844571}, {0. 575, 0. 786254}, {0. 5875, 0. 723089}, {0. 6, 0. 655466}, {0. 6125, 0. 583801}, {0. 625, 0. 508538}, {0. 6375, 0. 430139}, {0. 65, 0. 349088}, {0. 6625, 0. 265885}, {0. 675, 0. 181043}, {0. 6875, 0. 0950847}, {0. 7, 0. 00854002}, {0. 7125, -0. 0780573}, {0. 725, -0. 164173}, {0. 7375, -0. 249277}, {0. 75, -0. 332844}, {0. 7625, -0. 414359}, {0. 775, -0. 49332}, {0. 7875, -0. 569238}, {0. 8, -0. 641648}, {0. 8125, -0. 710101}, {0. 825, -0. 774176}, {0. 8375, -0. 833478}, {0. 85, -0. 887642}, {0. 8625, -0. 936333}, {0. 875, -0. 979251}, {0. 8875, -1. 01613}, {0. 9, -1. 04675}, {0. 9125, -1. 07091}, {0. 925, -1. 08847}, {0. 9375, -1. 09932}, {0. 95, -1. 10339}, {0. 9625, -1. 10066}, {0. 975, -1. 09114}, {0. 9875, -1. 0749}, {1, -1. 05203},

4. $t = 1.0$

{0, -0. 0109409}, {0. 0125, 0. 0778188}, {0. 025, 0. 166099}, {0. 0375, 0. 253355}, {0. 05, 0. 339048}, {0. 0625, 0. 422652}, {0. 075, 0. 50365}, {0. 0875, 0. 581542}, {0. 1, 0. 655849}, {0. 1125, 0. 726113}, {0. 125, 0. 7919}, {0. 1375, 0. 852804}, {0. 15, 0. 908451}, {0. 1625, 0. 958497}, {0. 175, 1. 00263}, {0. 1875, 1. 04059}, {0. 2, 1. 07213}, {0. 2125, 1. 09706}, {0. 225, 1. 11522}, {0. 2375, 1. 12651}, {0. 25, 1. 13086}, {0. 2625, 1. 12823}, {0. 275, 1. 11865}, {0. 2875, 1. 10216}, {0. 3, 1. 07889}, {0. 3125, 1. 04896}, {0. 325, 1. 01257}, {0. 3375, 0. 96993}, {0. 35, 0. 921313}, {0. 3625, 0. 867015}, {0. 375, 0. 807373}, {0. 3875, 0. 742752}, {0. 4, 0. 673552}, {0. 4125, 0. 600199}, {0. 425, 0. 523146}, {0. 4375, 0. 442868}, {0. 45, 0. 359859}, {0. 4625, 0. 274632}, {0. 475, 0. 187711}, {0. 4875, 0. 0996331}, {0. 5, 0. 0109409}, {0. 5125, -0. 0778188}, {0. 525, -0. 166099}, {0. 5375, -0. 253355}, {0. 55, -0. 339048}, {0. 5625, -0. 422652}, {0. 575, -0. 50365}, {0. 5875, -0. 581542}, {0. 6, -0. 655849}, {0. 6125, -0. 726113}, {0. 625, -0. 7919}, {0. 6375, -0. 852804}, {0. 65, -0. 908451}, {0. 6625, -0. 958497}, {0. 675, -1. 00263}, {0. 6875, -1. 04059}, {0. 7, -1. 07213}, {0. 7125, -1. 09706}, {0. 725, -1. 11522}, {0. 7375, -1. 12651}, {0. 75, -1. 13086}, {0. 7625, -1. 12823}, {0. 775, -1. 11865}, {0. 7875, -1. 10216}, {0. 8, -1. 07889}, {0. 8125, -1. 04896}, {0. 825, -1. 01257}, {0. 8375, -0. 96993}, {0. 85, -0. 921313}, {0. 8625, -0. 867015}, {0. 875, -0. 807373}, {0. 8875, -0. 742752}, {0. 9, -0. 673552}, {0. 9125, -0. 600199}, {0. 925, -0. 523146}, {0. 9375, -0. 442868}, {0. 95, -0. 359859}, {0. 9625, -0. 274632}, {0. 975, -0. 187711}, {0. 9875, -0. 0996331}, {1, -0. 0109409},

5. 误差输出以及随时间变化图





讨论

通过观察误差随时间变化图可以发现：最大误差随时间增长逐渐变大。