# Introduction to Machine Learning

## Lecture 15: Neural Networks

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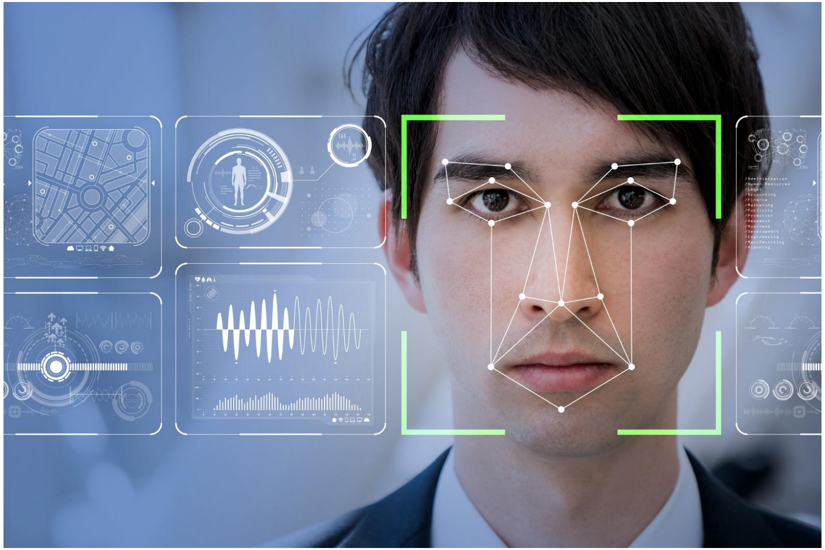
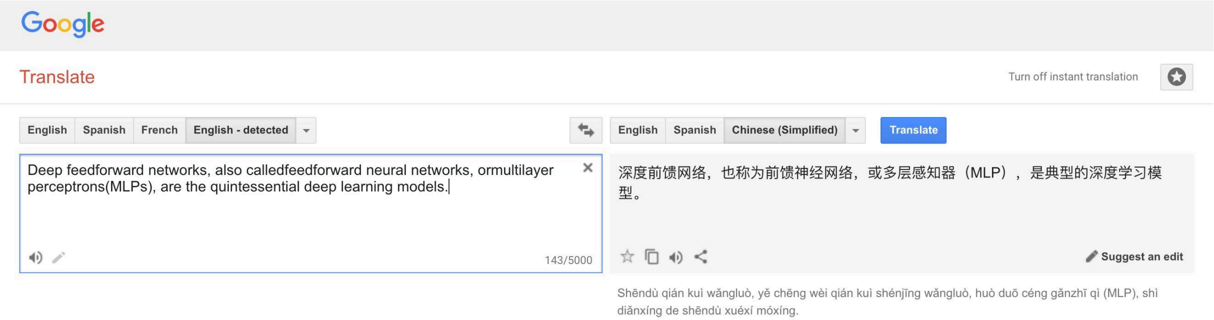
# Breakthroughs by Deep Learning

## Face recognition



# Breakthroughs by Deep Learning

## Machine translation

# Breakthroughs by Deep Learning

## Speech recognition

### Microsoft AI Beats Humans at Speech Recognition

*By Richard Adhikari*
*Oct 20, 2016 11:40 AM PT*

*Print*
*Email*

# Breakthroughs by Deep Learning

## Self-driving

# Breakthroughs by Deep Learning

## Machine reading comprehension

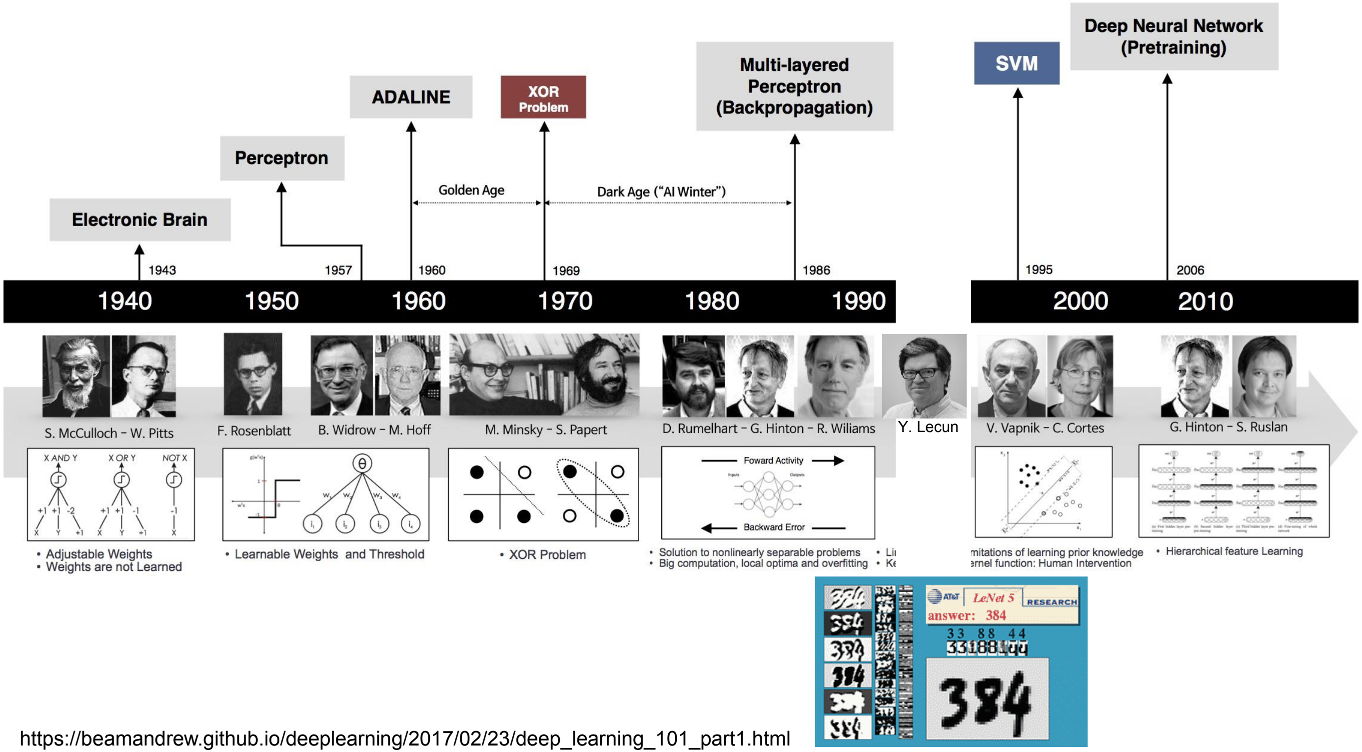
SQuAD Home

### SQuAD1.1 Leaderboard

Since the release of SQuAD1.0, the community has made rapid progress, with the best models now rivaling human performance on the task. Here are the ExactMatch (EM) and F1 scores evaluated on the test set of v1.1.

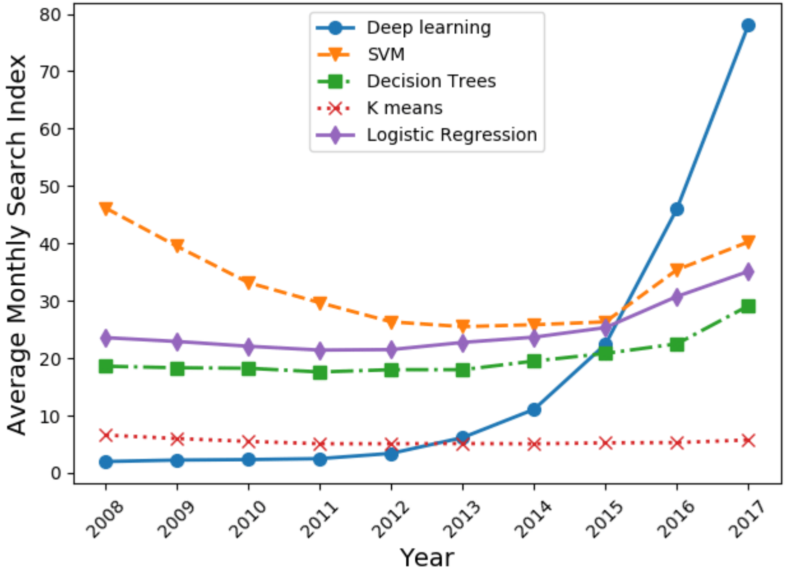
| Rank | Model | EM | F1 |
|---|---|---|---|
| | Human Performance<br>*Stanford University*<br>(Rajpurkar et al. '16) | 82.304 | 91.221 |
| 1<br>Sep 09, 2018 | nlnet (ensemble)<br>*Microsoft Research Asia* | **85.356** | **91.202** |
| 2<br>Jul 11, 2018 | QANet (ensemble)<br>*Google Brain & CMU* | 84.454 | 90.490 |
| 3<br>Jul 08, 2018 | r-net (ensemble)<br>*Microsoft Research Asia* | 84.003 | 90.147 |

# Milestones of Deep Learning




https://beamandrew.github.io/deeplearning/2017/02/23/deep_learning_101_part1.html

# Google Trend of Deep Learning




Mehdi Mohammadi, at.al. Deep Learning for IoT Big Data and Streaming Analytics: A Survey. IEEE Communications Surveys and Tutorials Journal, 2018.

# Motivation of Neural Networks

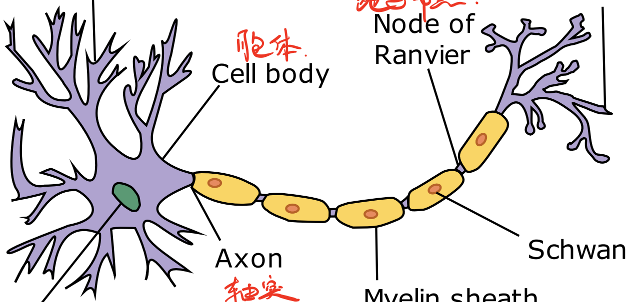
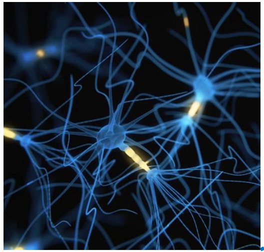



Diagram of neuron

https://simple.wikipedia.org/wiki/Neuron

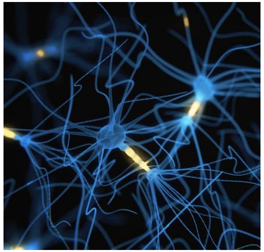
# Motivation of Neural Networks

This is a car

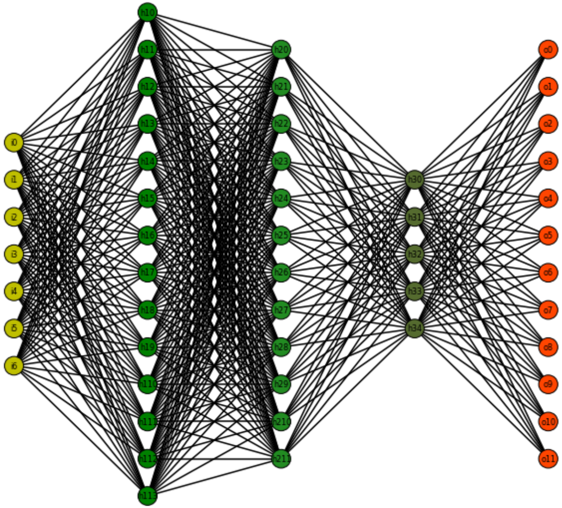


http://news.mit.edu/2015/how-brain-recognizes-objects-1005

# What is Neural Network?



Biological Neural Network



Artifical Neural Network

感知器

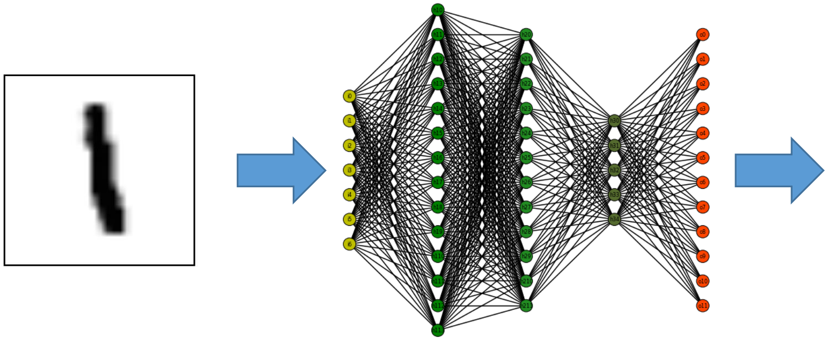
# Multi-Layer Perceptron

# Hand-written Digits Recognition

The MNIST dataset

By Josef Steppan - Own work, CC BY-SA 4.0,
https://commons.wikimedia.org/w/index.php?curid=64810040

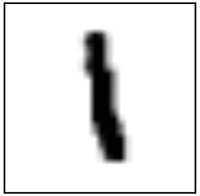
# Hand-written Digits Recognition



1 or not?

# Vector representation

$x$: image



Input domain

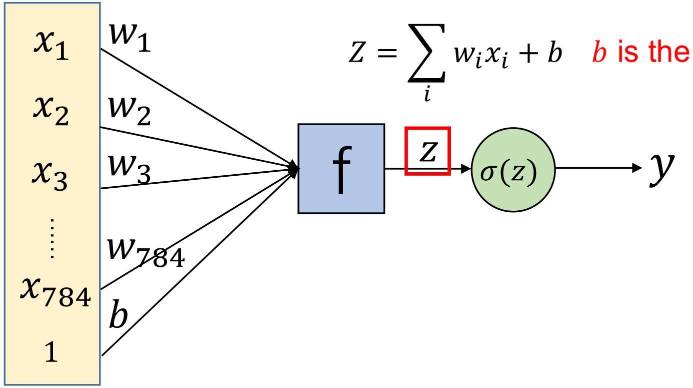
$28 \times 28$ pixels

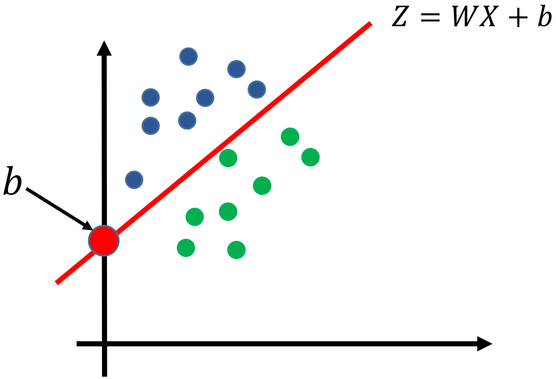
$$\begin{bmatrix} 0 \\ 1 \\ \dots \\ \dots \\ 0 \end{bmatrix} \in R^{784}$$

1: for ink
0: otherwise

# Single Neuron 单个神经元.

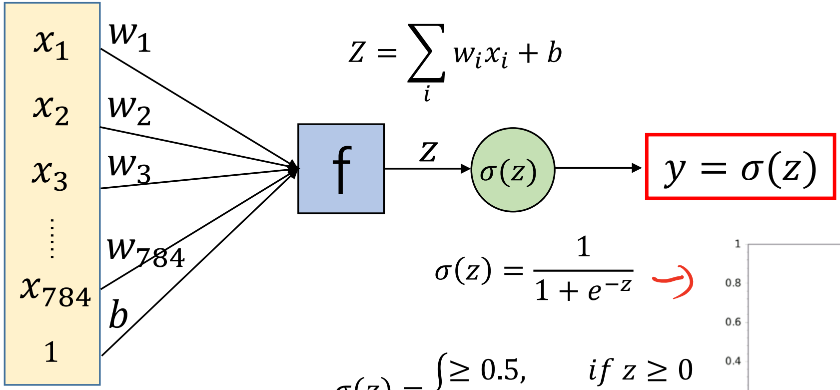
$$Z = \sum_i w_i x_i + b$$ $b$ is the bias




**Why do we need a bias *b*?**

# Single Neuron

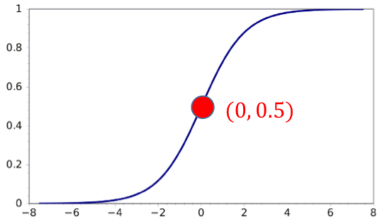



$$Z = \sum_i w_i x_i + b$$

$$prediction = \begin{cases} 1 & if\ y \geq 0.5 \\ -1 & if\ y < 0.5 \end{cases}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

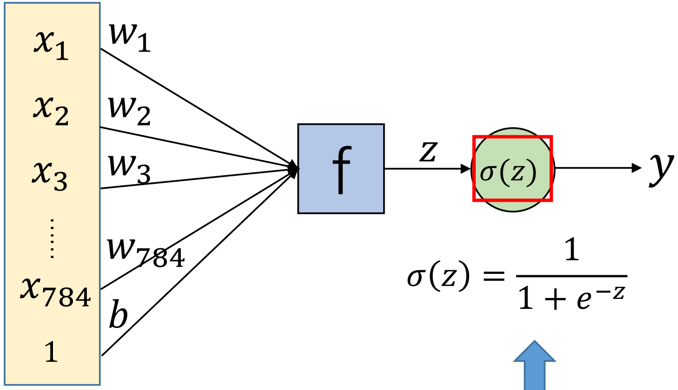
$$\sigma(z) = \begin{cases} \geq 0.5, & if\ z \geq 0 \\ < 0.5, & if\ z < 0 \end{cases}$$



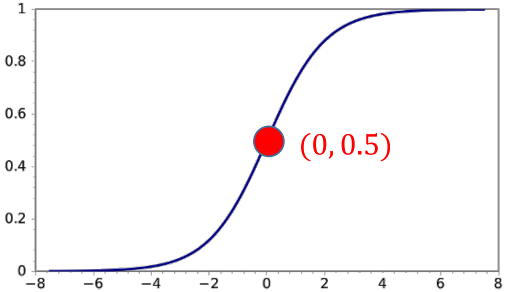

Sigmoid Function

This is a linear classifier.

# Activation Function




$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

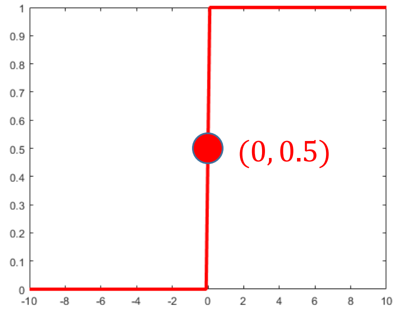



Activation function: The function that acts on the weighted combination of inputs.

We also have other activation function.

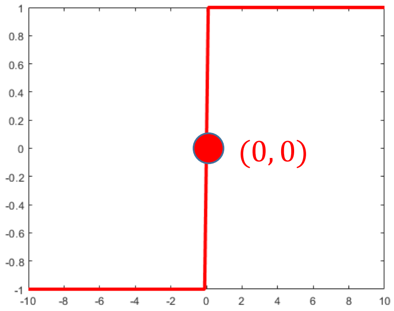
# Activation Function

Boolean




$$\sigma(z) = \begin{cases} 1 & z > 0 \\ 0.5 & z = 0 \\ 0 & z < 0 \end{cases}$$

Unit step function

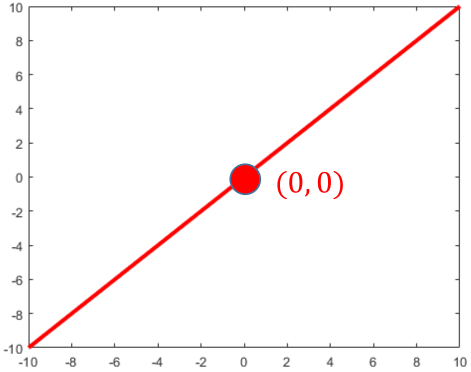



$$\sigma(z) = \begin{cases} 1 & z > 0 \\ 0 & z = 0 \\ -1 & z < 0 \end{cases}$$

Sign function

# Activation Function

Linear

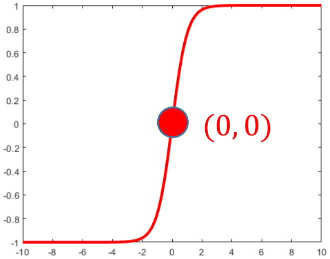



$$\sigma(z) = z$$

Linear function

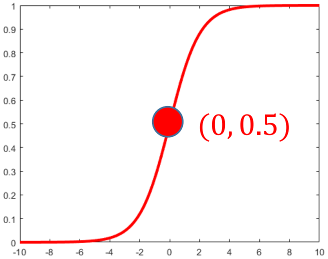
# Activation Function

Non-linear



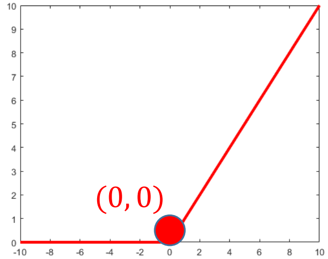

$$\sigma(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Tanh function




$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid function




$$\sigma(z) = \max(0, z)$$

ReLU function

Non-linear activation functions are frequently used in neural networks.

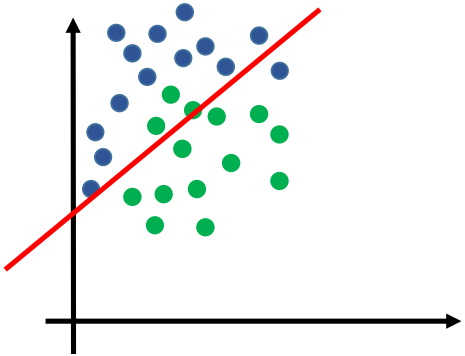
Why?

# Why Non-Linearity?

## Without non-linearity

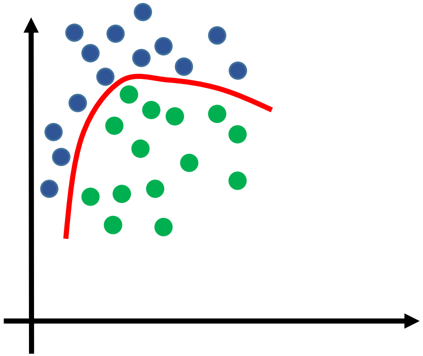
➢ Deep neural networks are equivalent to linear transforms.

$$W_1\big(W_2(W_3 \cdot x)\big) = Wx$$



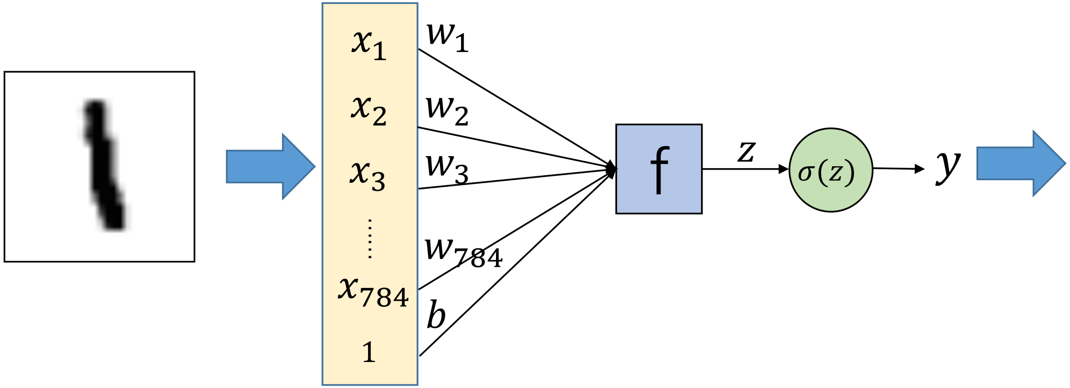
## With non-linearity

➢ The neural networks can approximate complicated functions.



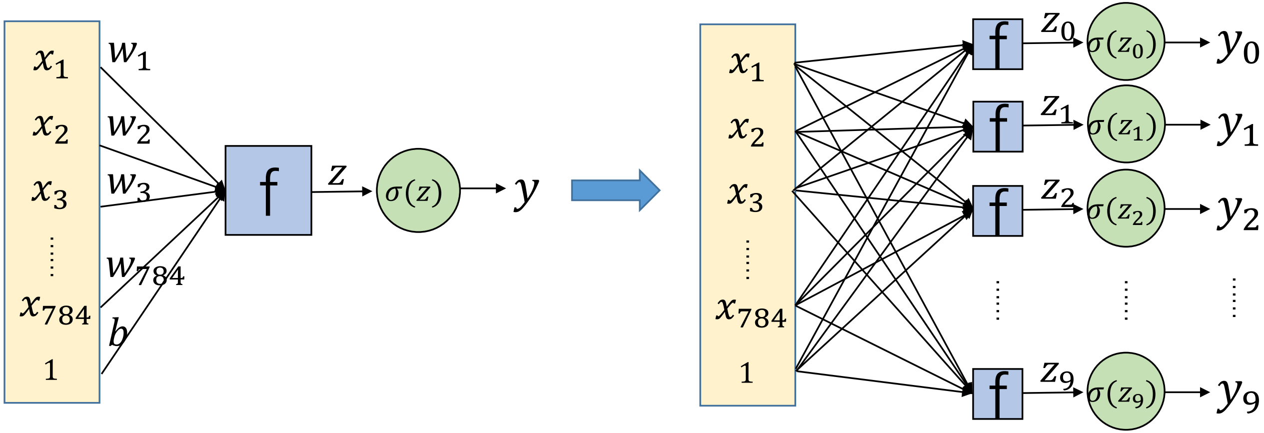
http://cs224d.stanford.edu/lectures/CS224d-Lecture4.pdf

# A More Complicated Task




What is the number in the image?

# Multiple Outputs

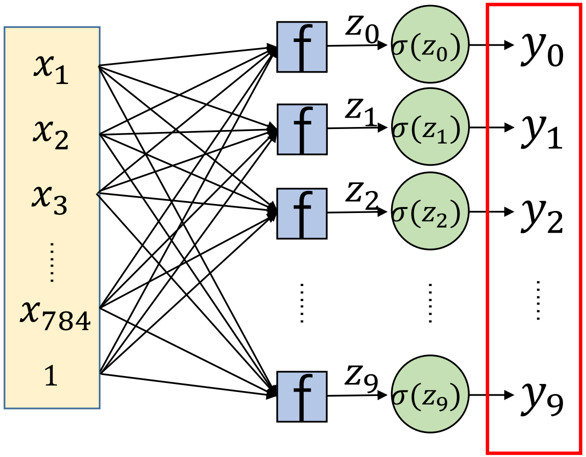



$W \in R^{\cancel{784\times10}}$ 10×784

$b \in R^{1\times10}$

# Multiple Outputs

$$\sigma(z) = \frac{1}{1+e^{-z}}$$




$W \in R^{\cancel{784\times 10}}$ $10\times 784$

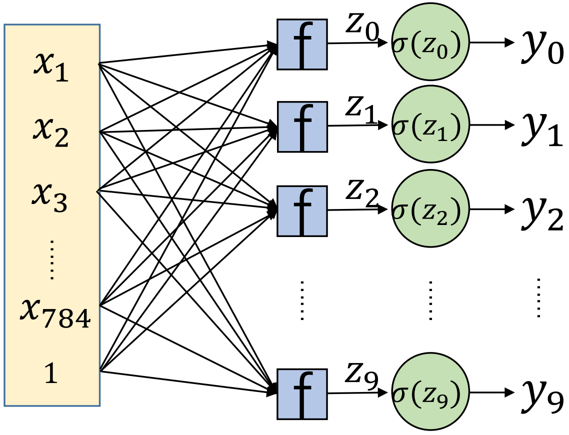
$b \in R^{1\times 10}$

We choose label corresponding to the maximum value of $y_i$.

Question:

How do we evaluate the performance of the model?

# Loss Function




$W \in R^{\cancel{784\times 10}}$ 10x784

$b \in R^{1\times 10}$

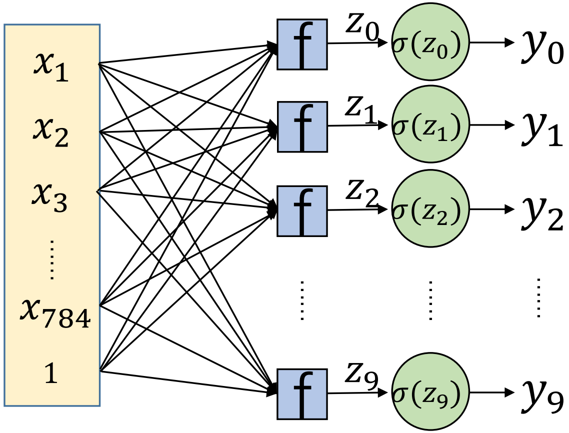
Ground truth: $Q = \begin{bmatrix} 0 \\ 1 \\ \dots \\ \dots \\ 0 \end{bmatrix} \in R^{10}$ One hot vector The component corresponding to the true label is “1”.

$$p_i = softmax(y_i) = \frac{e^{y_i}}{\sum_i e^{y_i}}$$

$$Loss = cross\ entropy = -\sum_i q_i \log(p_i)$$

The goal is to minimize the loss!

# Model Parameters




$W \in R^{\cancel{784\times10}}$ $10\times784$

$b \in R^{1\times10}$

$$y = f(x) = \sigma(\boxed{W}x + \boxed{b})$$

Model parameter set $\theta = \{W, b\}$

Minimize the loss = Pick the best $\theta$

# Optimization

**Any idea to pick the optimal parameter values ?**

↓

**(Stochastic) Gradient Descent**

↓

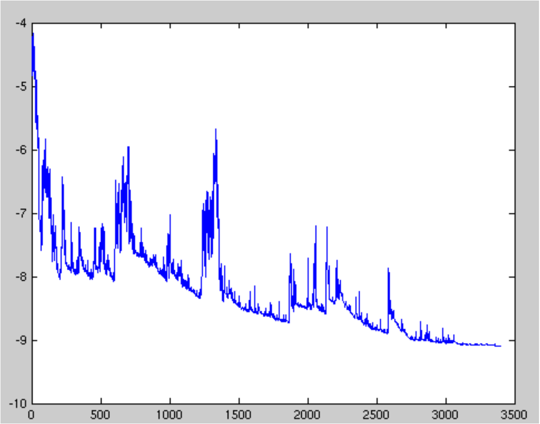
**Backpropagation**

# Stochastic Gradient Descent

$$\min_x F(x) = \sum_{i=1}^{n} f_i(x)$$

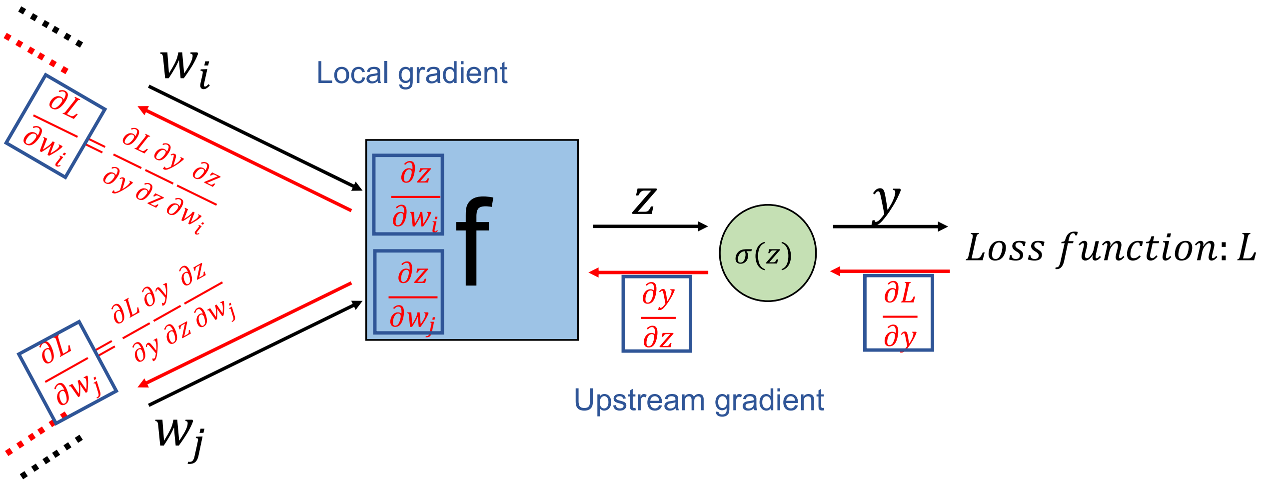
- Initialize the parameter x and learning rate $\eta$
- Repeat until the termination condition is met
  - Randomly shuffle examples in the training set
  - For $i = 1, \ldots, n$

$$x_{k+1} \leftarrow x_k - \eta \nabla f_i(x_k)$$



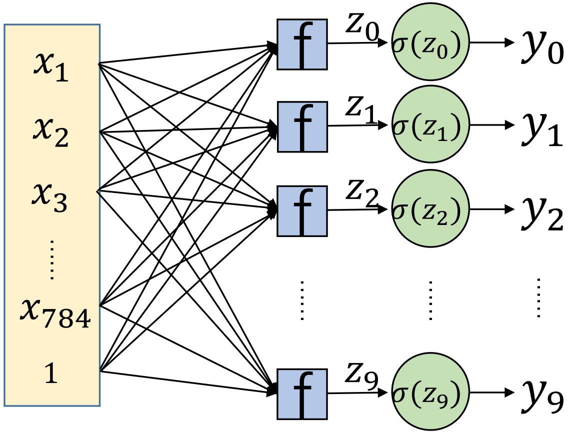

**Descent is in the sense of expectation.**

# Backpropagation 反向传播.




Upstream gradient * Local gradient

# Backpropagation




$W \in R^{784 \times 10}$

$b \in R^{1 \times 10}$

Ground truth: $Q = \begin{bmatrix} 0 \\ 1 \\ \dots \\ \dots \\ 0 \end{bmatrix} \in R^{10}$

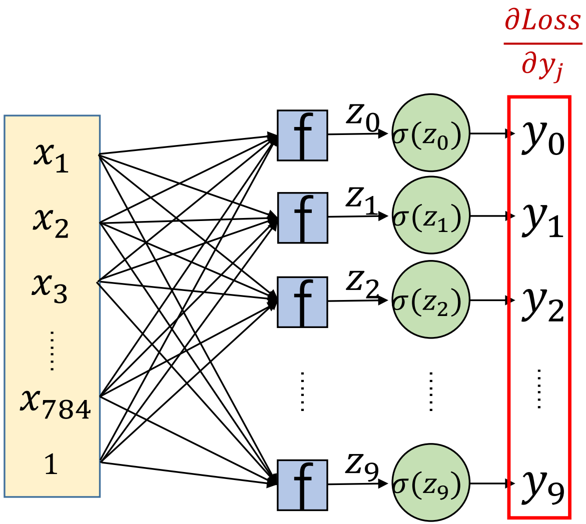
One hot vector: the component corresponding to the true label is "1".

$$p_i = softmax(y_i) = \frac{e^{y_i}}{\sum_i e^{y_i}}$$

Suppose that, the true label of a given data instance is $i$. Then

$$Loss = cross\ entropy = -\sum_i q_i \log(p_i) = -\log(p_i)$$

# Backpropagation




$W \in R^{784\times 10}$

$b \in R^{1\times 10}$

$$Loss = cross\ entropy = -\log(p_i)$$

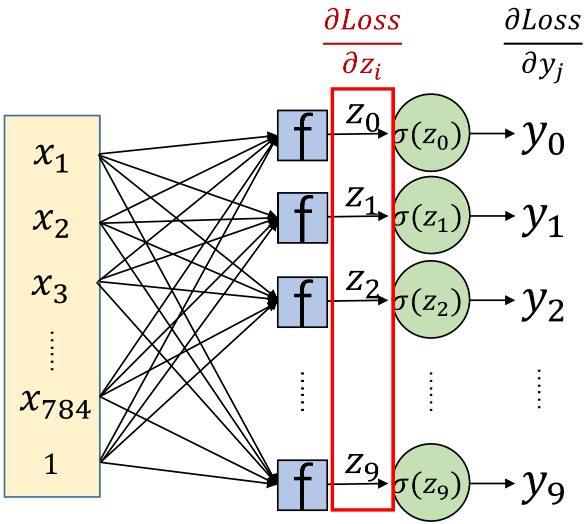
$$p_i = softmax(y_i) = \frac{e^{y_i}}{\sum_i e^{y_i}}$$

$$\frac{\partial Loss}{\partial y_j} = \frac{\partial Loss}{\partial p_i}\frac{\partial p_i}{\partial y_j}$$

$$\frac{\partial Loss}{\partial p_i} = -\frac{1}{p_i} \qquad \frac{\partial p_i}{\partial y_j} = \begin{cases} p_i(1-p_i) & i = j \\ -p_i p_j & i \neq j \end{cases}$$

$$\frac{\partial Loss}{\partial y_j} = \frac{\partial Loss}{\partial p_i}\frac{\partial p_i}{\partial y_j} = \begin{cases} p_i - 1 & i = j \\ p_j & i \neq j \end{cases}$$

# Backpropagation




$W \in R^{784\times10}$

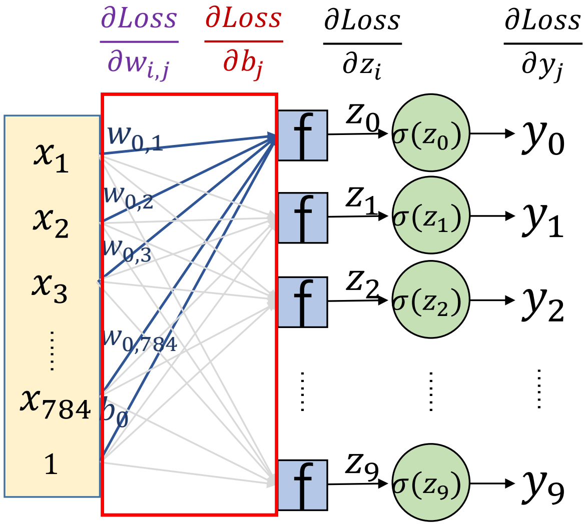
$b \in R^{1\times10}$

$$y_i = \frac{1}{1+e^{-z_i}}$$

$$\frac{\partial Loss}{\partial z_i} = \frac{\partial Loss}{\partial y_i}\frac{\partial y_i}{\partial z_i}$$

$$\frac{\partial y_i}{\partial z_i} = y_i(1-y_i)$$

# Backpropagation




$W \in R^{784\times10}$

$b \in R^{1\times10}$

$$z_i = w_{i,1}x_1 + w_{i,2}x_2 + \cdots + w_{i,784}x_{784} + b_i$$

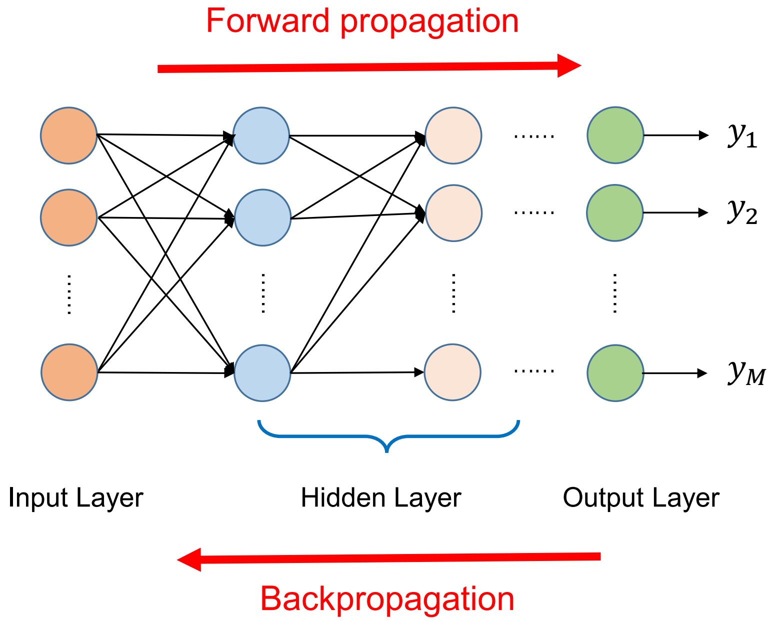
$$\frac{\partial Loss}{\partial w_{i,j}} = \frac{\partial Loss}{\partial z_i}\frac{\partial z_i}{\partial w_{i,j}} \qquad \frac{\partial z_i}{\partial w_{i,j}} = x_j$$

$$\frac{\partial Loss}{\partial b_i} = \frac{\partial Loss}{\partial z_i}\frac{\partial z_i}{\partial b_i} \qquad \frac{\partial z_i}{\partial b_i} = 1$$

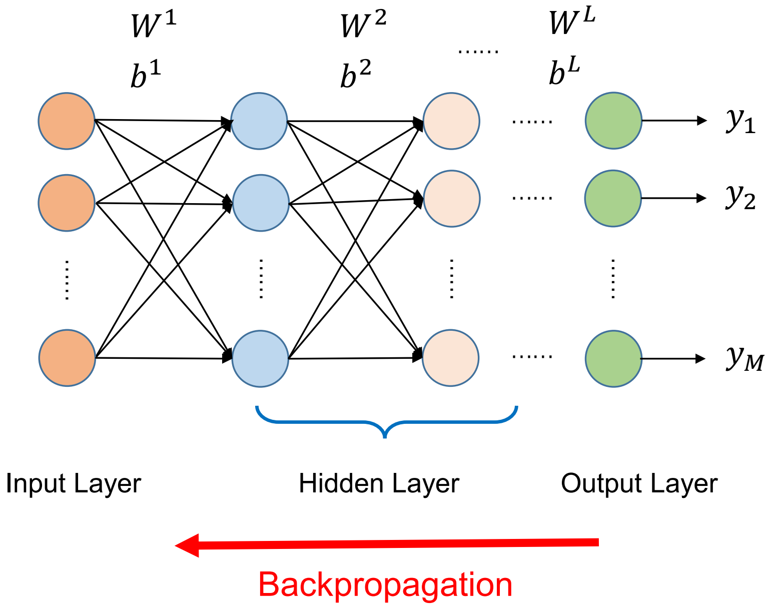
$$W = W - \eta\frac{\partial Loss}{\partial W}$$

$$b = b - \eta\frac{\partial Loss}{\partial b}$$

# Backpropagation：Multi-Layer Perceptron



# Backpropagation：Multi-Layer Perceptron




$$\theta = \{W^1, b^1, W^2, b^2, \dots, W^L, b^L\}$$

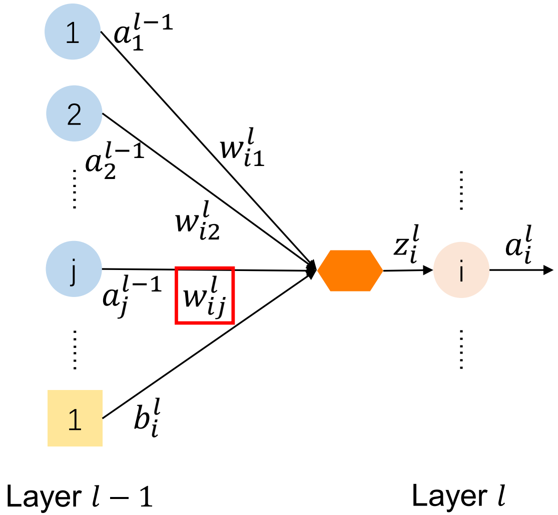
$$W^l = \begin{bmatrix} w_{11}^l & w_{12}^l & \cdots \\ w_{21}^l & w_{22}^l & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \qquad b^l = \begin{bmatrix} \vdots \\ b_i^l \\ \vdots \end{bmatrix}$$

$$\frac{\partial Loss(\theta)}{\partial W^l} = \begin{bmatrix} \frac{\partial Loss(\theta)}{\partial W_{11}^l} & \frac{\partial Loss(\theta)}{\partial W_{12}^l} & \cdots \\ \frac{\partial Loss(\theta)}{\partial W_{21}^l} & \frac{\partial Loss(\theta)}{\partial W_{22}^l} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$\frac{\partial Loss(\theta)}{\partial b^l} = \begin{bmatrix} \vdots \\ \frac{\partial Loss(\theta)}{\partial b_i^l} \\ \vdots \end{bmatrix}$$

$$W = W - \eta \frac{\partial Loss}{\partial W} \qquad b = b - \eta \frac{\partial Loss}{\partial b}$$

# Backpropagation：Multi-Layer Perceptron




$\boldsymbol{a_i^l}$ : output of a neuron

$\boldsymbol{w_{ij}^l}$ : a weight of layer $\boldsymbol{l}$

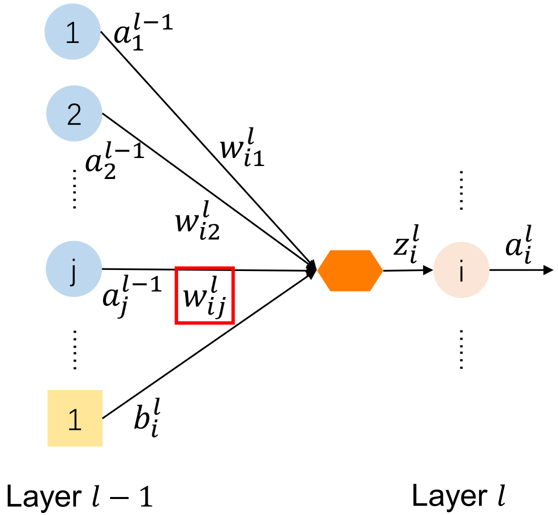
$\boldsymbol{b_i^l}$ : a bias of layer $\boldsymbol{l}$

$\boldsymbol{z_i^l}$ : input of an activation function

$$z^l = W^l a^{l-1} + b^l$$

$$a^l = \sigma(z^l)$$

# Backpropagation: Multi-Layer Perception




$$\frac{\partial Loss(\theta)}{\partial w_{ij}^l} = \frac{\partial Loss(\theta)}{\partial z_i^l} \boxed{\frac{\partial z_i^l}{\partial w_{ij}^l}}$$

If $l$ >1:

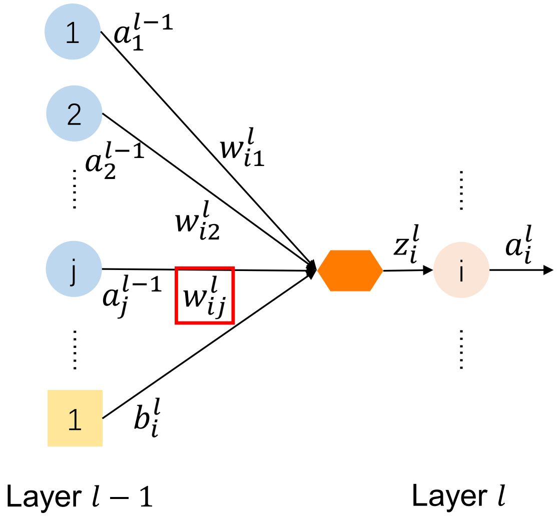
$$z_i^l = \sum_j w_{ij}^l a_j^{l-1} + b_i^l$$

$$\frac{\partial z_i^l}{\partial w_{ij}^l} = a_j^{l-1}$$

If $l$=1:

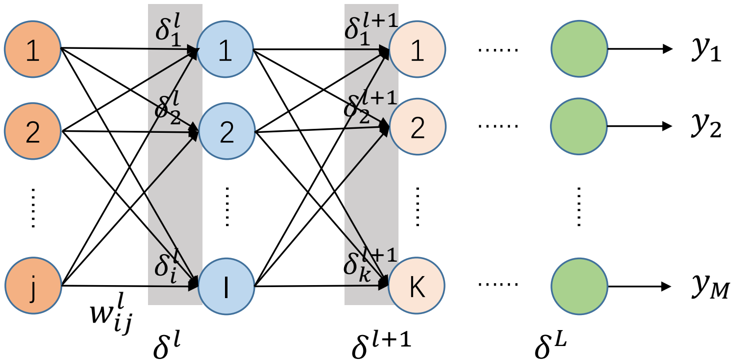
$$\frac{\partial z_i^l}{\partial w_{ij}^l} = x_j$$

# Backpropagation：Multi-Layer Perception

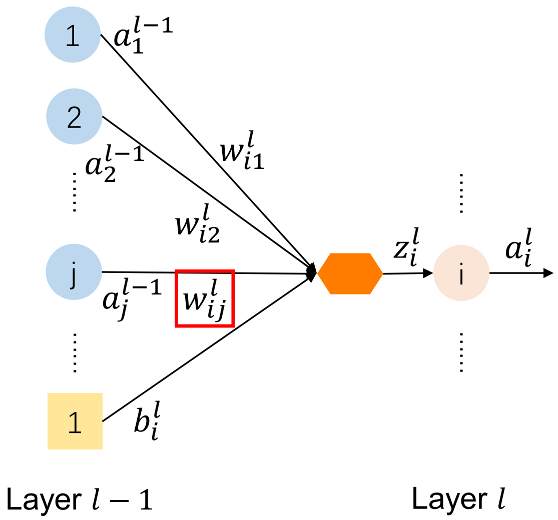


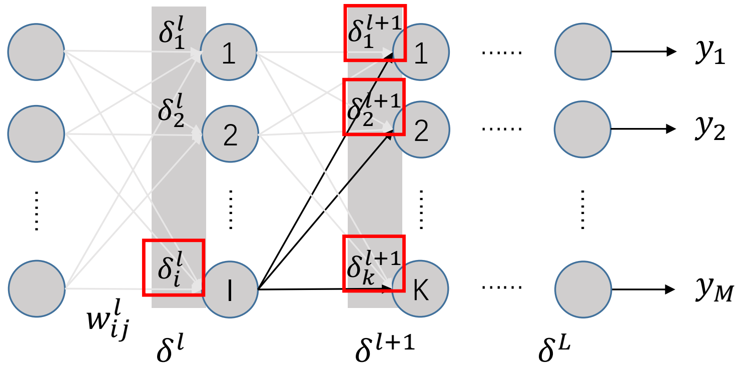

$$\frac{\partial Loss(\theta)}{\partial w_{ij}^l} = \boxed{\frac{\partial Loss(\theta)}{\partial z_i^l}} \frac{\partial z_i^l}{\partial w_{ij}^l}$$

$$\delta_i^l = \frac{\partial Loss(\theta)}{\partial z_i^l}$$



# Backpropagation：Multi-Layer Perception




$$\delta_i^l = \frac{\partial Loss(\theta)}{\partial z_i^l} = \frac{\partial Loss(\theta)}{\partial z_1^{l+1}} \frac{\partial z_1^{l+1}}{\partial a_i^l} \frac{\partial a_i^l}{\partial z_i^l} + \cdots + \frac{\partial Loss(\theta)}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial a_i^l} \frac{\partial a_i^l}{\partial z_i^l}$$

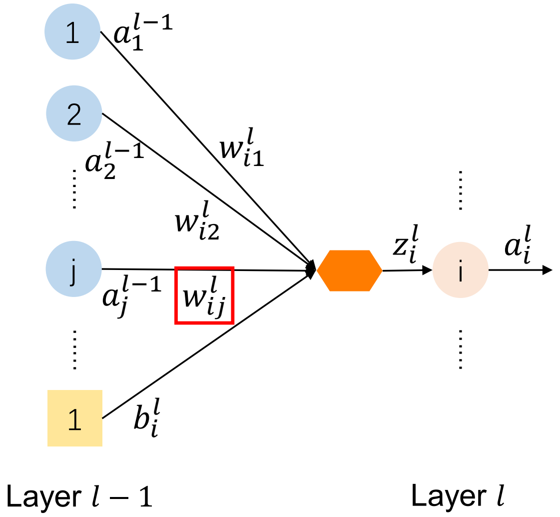
$$= \frac{\partial a_i^l}{\partial z_i^l} \sum_k \frac{\partial Loss(\theta)}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial a_i^l} = \frac{\partial a_i^l}{\partial z_i^l} \sum_k \boxed{\frac{\partial z_k^{l+1}}{\partial a_i^l}} \delta_k^{l+1}$$

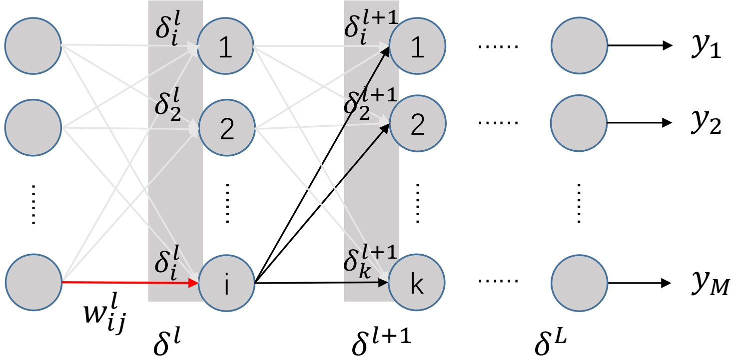
$$= \boxed{\sigma'(z_i^l) \sum_k w_{ki}^{l+1} \delta_k^{l+1}}$$

$$z_k^{l+1} = \sum_i w_{ki}^{l+1} a_i^l + b_k^{l+1}$$

$$\delta^l = \sigma'(z^l) \odot \left( \left( W^{l+1} \right)^T \delta^{l+1} \right)$$

# Backpropagation：Multi-Layer Perception



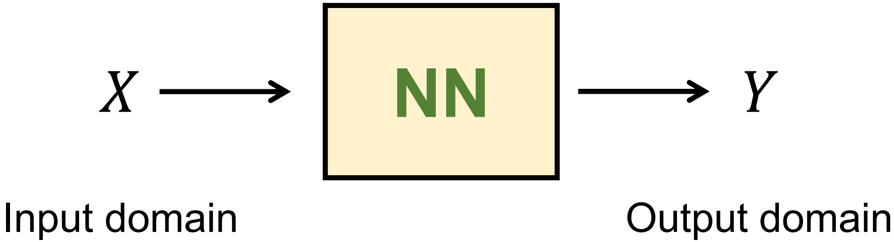




$$\frac{\partial Loss(\theta)}{\partial w_{ij}^l} = \frac{\partial Loss(\theta)}{\partial z_i^l}\frac{\partial z_i^l}{\partial w_{ij}^l} = \boxed{\delta_i^l}\boxed{\frac{\partial z_i^l}{\partial w_{ij}^l}}$$

$$\boxed{\delta_i^l = \sigma'(z_i^l)\sum_k w_{ki}^{l+1}\delta_k^{l+1}}$$

$$\boxed{\frac{\partial z_i^l}{\partial w_{ij}^l} = \begin{cases} a_j^{l-1} & l > 1 \\ x_j & l = 1 \end{cases}}$$

# Universal Function Approximator




- Input domain: document, word, image, voice, etc.
- Output domain: probability distribution, single label, etc.

# Universal Function Approximator

The learning algorithm is to map the input domain $X$ into the output domain $Y$

$$f : X \longrightarrow Y$$

- Handwriting Recognition

$f($  $) =$ “1”

- Speech Recognition

$f($  $) =$ “Hello, MIRA”

In fact, the neural networks are universal function approximators!

# Universal Function Approximator

$$y = f(x;\theta) = \sigma(W^L \dots \sigma(W^2\sigma(W^1x + b^1) + b^2) \dots + b^L)$$

Different model parameters $W$ and $b$ determine different mappings.

Standard multilayer feedforward networks with as few as one hidden layer using arbitrary 压缩 squashing functions are capable of approximating any Borel measurable function from one finite dimensional space to another to any desired degree of accuracy.
------*‘Multilayer feedforward networks are universal approximators’*

Pick a function $f$ = pick a set of model parameters $\theta$

# Universal Function Approximator

- A good function: The output of the function is close to the label.

$$f(x;\theta) \sim y$$

- An example loss function:

$$Loss = \sum_{k} ||y_k - f(x_k;\theta)||^2$$

where $k$ is the number of training examples

# Commonly Used Loss Functions

- Square loss

$$Loss = \left(1 - f(x;\theta)\right)^2$$

- Hinge loss

$$Loss = \max(0,1 - yf(x;\theta))$$

- Logistic loss

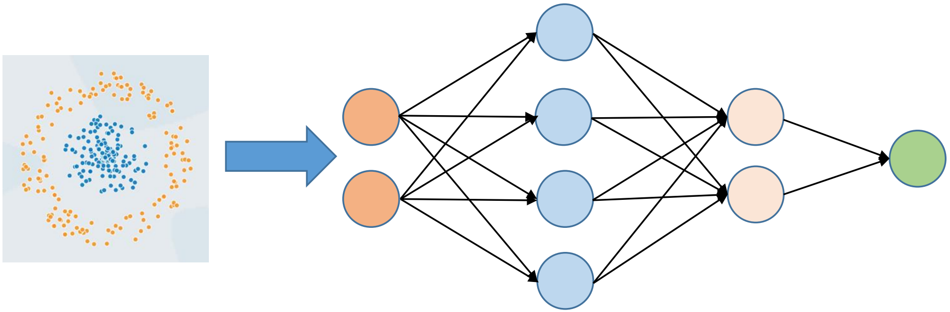
$$Loss = -y\log(f(x;\theta))$$

- Cross entropy loss

$$Loss = -\sum y\log(f(x;\theta))$$

# Demonstration 示范

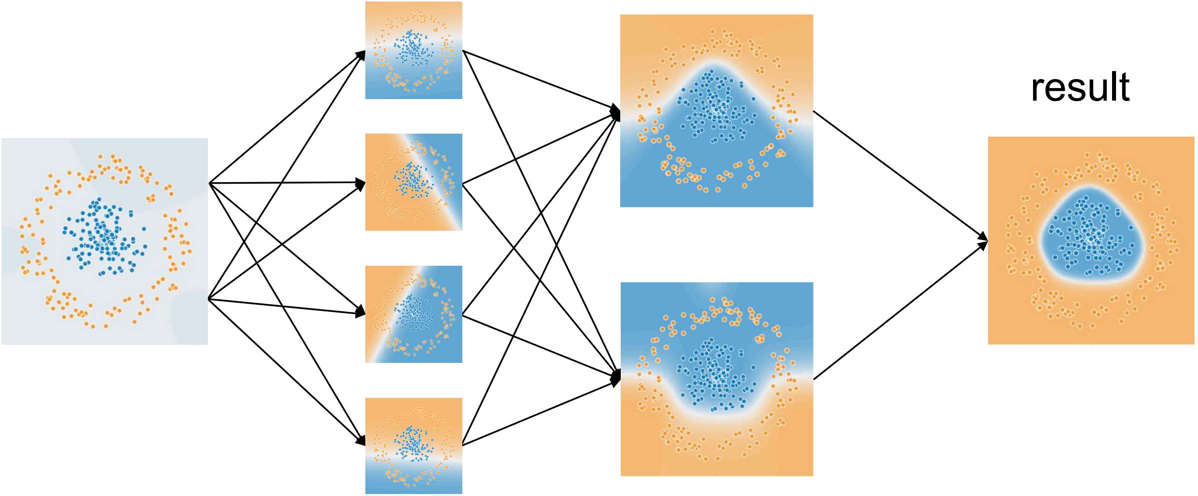
Classification Problem



The input is the coordinates of the points.

# Demonstration

Classification Problem: 500 Epoches




An epoch= one forward pass and one backward pass of all the training examples



http://playground.tensorflow.org

# Tips

# Deeper is Better?

**Deeper $\overset{?}{=}$ Better performance**

# Deeper is Better?

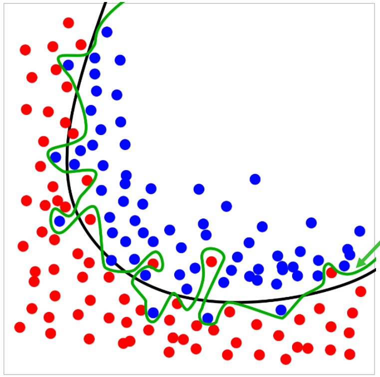
| Model | Depth(layers) | Performance(error rate) |
|---|---|---|
| AlexNet[Hinton, at. al. 2012] | 8 | 16.4% |
| GooLeNet[Simonyan, at. al. 2014] | 22 | 6.7% |
| ResNet[Kaiming He, at. al. 2015] | 152 | 3.57% |

基准.

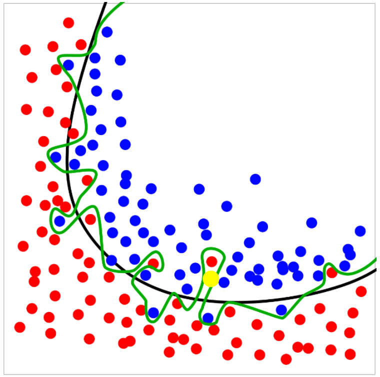
Dataset: ImageNet, which is a benchmark dataset for image classification.

Deep structure can capture complex patterns more efficiently than the shallow one.

# Overfitting



The generalization performance of this model can be poor.



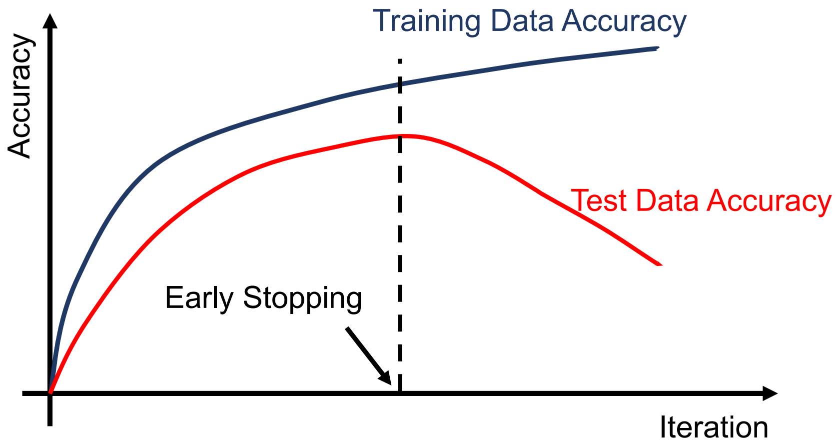
The predicted label is ~~red~~!

Which one is better?

A good model is the one that generalizes well on the unseen data.

# Preventing Overfitting in DNN

- Early Stopping
- Regularization
- Dropout
- …



# Preventing Overfitting in DNN

- Early Stopping
- Regularization
- Dropout
- …

$$Loss'(\theta) = Loss(\theta) + \lambda||\theta||_p$$

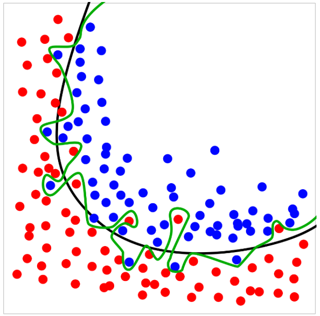
regularization term

➢ $\ell_2$ norm

$$||\theta||_2^2 = (\theta_1)^2 + (\theta_2)^2 + \cdots$$

➢ $\ell_1$ norm

$$||\theta||_1 = |\theta_1| + |\theta_2| + \cdots$$



Small weights usually imply smooth decision boundary.

# L2 Regularization

$$Loss'(\theta) = Loss(\theta) + \lambda \frac{1}{2} ||\theta||_2^2$$

$$||\theta||_2 = (\theta_1)^2 + (\theta_2)^2 + \cdots$$

$$\frac{\partial Loss'}{\partial \theta} = \frac{\partial Loss}{\partial \theta} + \lambda\theta \quad \Rightarrow \quad \theta^{t+1} := \theta^t - \eta \frac{\partial Loss'}{\partial \theta^t}$$

$$= \theta^t - \eta\left(\frac{\partial Loss}{\partial \theta^t} + \lambda\theta^t\right)$$

$$= (1 - \eta\lambda)\theta^t - \eta \frac{\partial Loss}{\partial \theta^t}$$

# L1 Regularization

$$Loss'(\theta) = Loss(\theta) + \lambda||\theta||_1$$

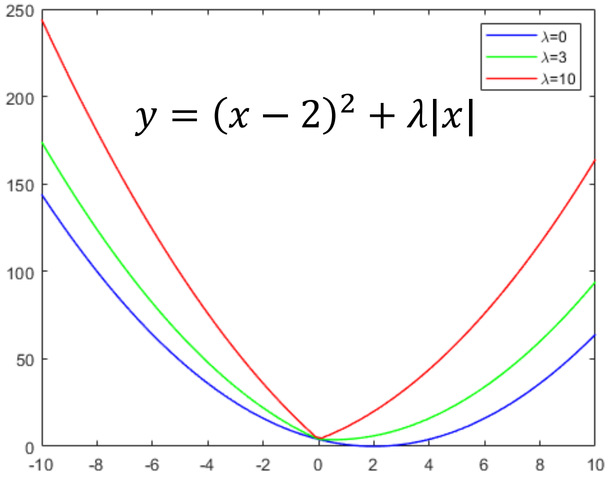
$$||\theta||_1 = |\theta_1| + |\theta_2| + \cdots$$

$$\frac{\partial Loss'}{\partial \theta} = \frac{\partial Loss}{\partial \theta} + \lambda * sgn(\theta)$$

$$\theta^{t+1} := \theta^t - \eta \frac{\partial Loss'}{\partial \theta^t}$$

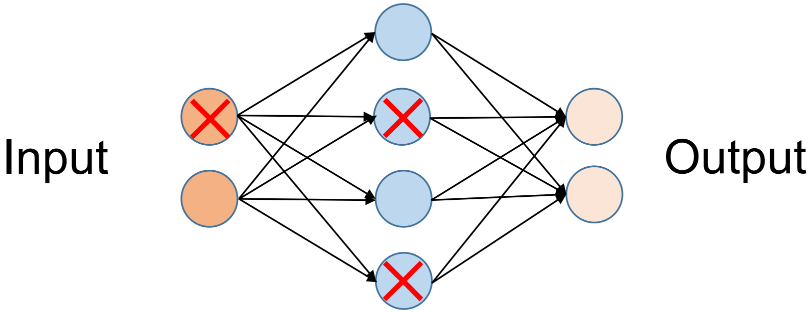
$$= \theta^t - \eta\left(\frac{\partial Loss}{\partial \theta^t} + \lambda sgn(\theta^t)\right)$$

$$= \theta^t - \eta\lambda sgn(\theta^t) - \eta \frac{\partial Loss}{\partial \theta^t}$$



# Preventing Overfitting in DNN

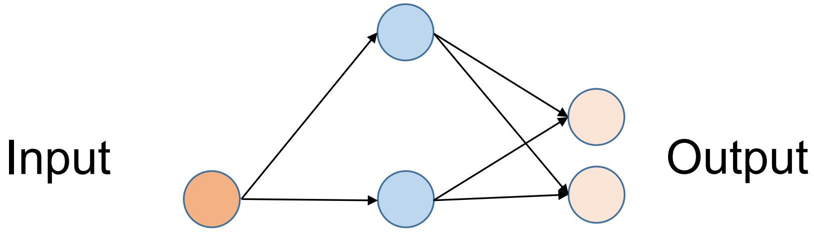
- Early Stopping
- Regularization
- Dropout
- …




Training: We drop each neuron with probability p

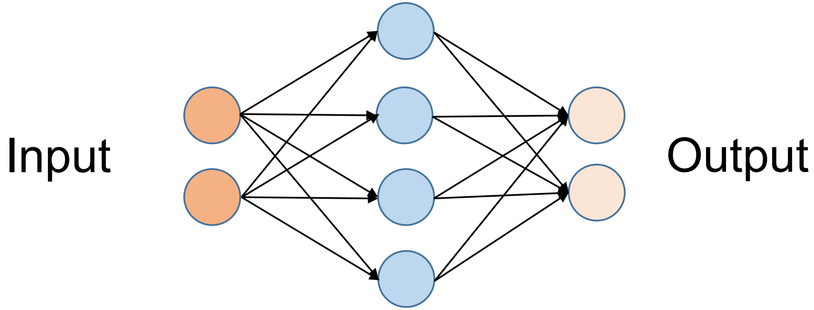
丢弃.

# Dropout

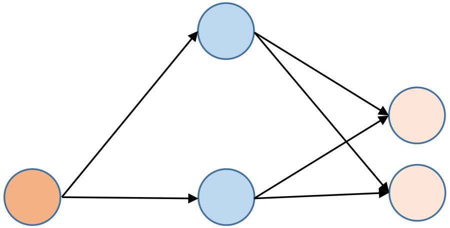



Training: We dropout each neuron with probability p. Then, we train the resulting network for one iteration.

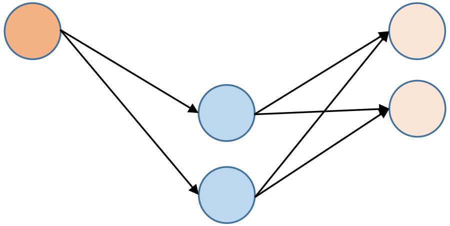
# Dropout




In each iteration, we will not update the weights of the links connecting to the dropped neurons.



Iteration 1



Iteration 2

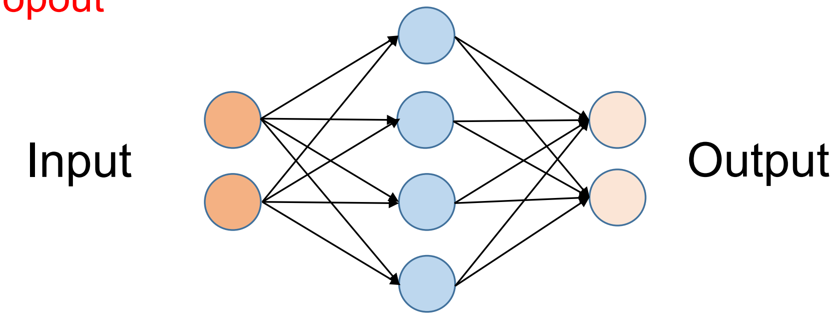
……

……

An iteration = a *batch* of training data passing through the network

# Dropout

Testing: No dropout

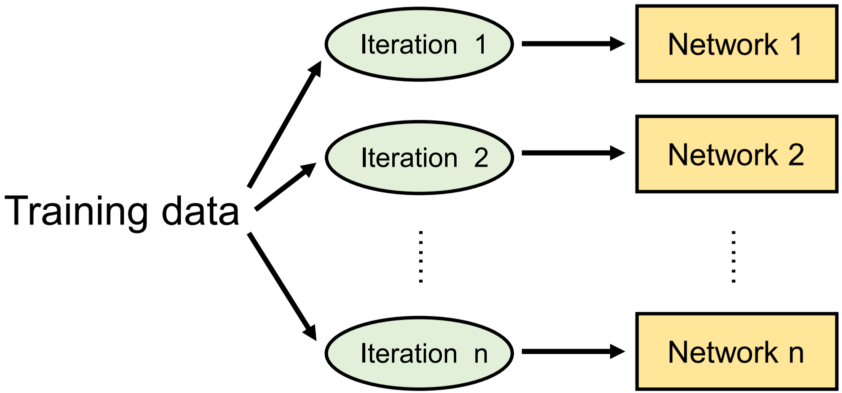



$$W_{test} = (1 - p)W_{train}$$

Why?

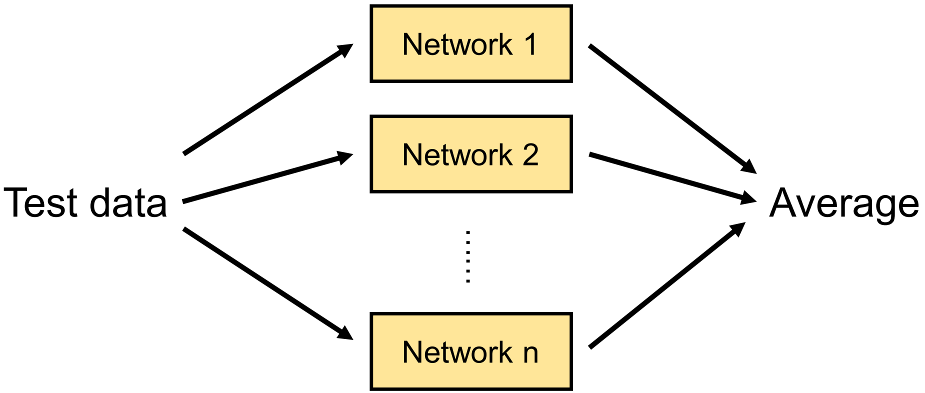
# Why Dropout

Dropout is a kind of ensemble



# Why Dropout

Dropout is a kind of ensemble



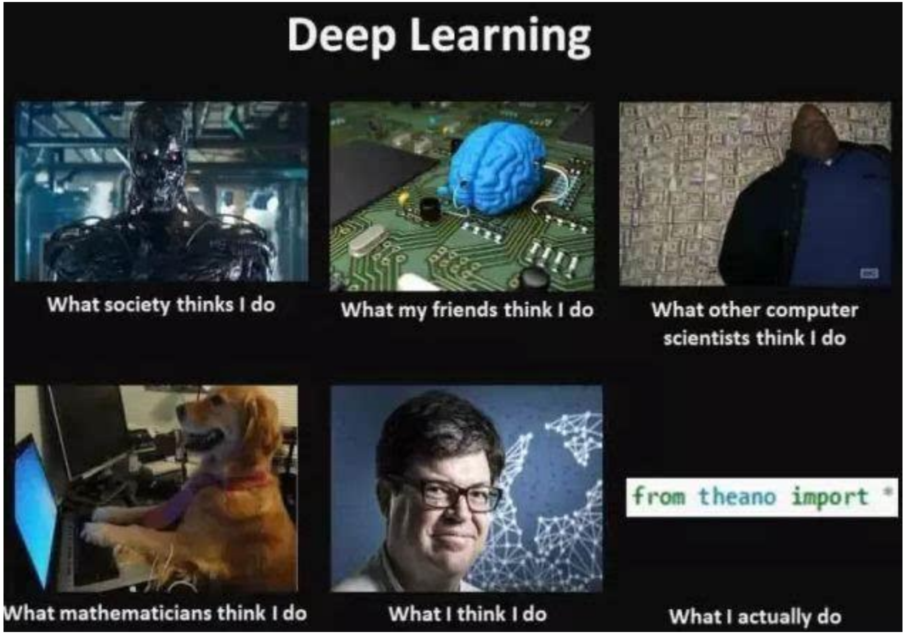

With N neurons, there are $2^N$ possible sub-networks.

- The average can relieve overfitting
- Dropout can learn more robust patterns



http://deeplearning.cs.cmu.edu/slides/lec6.stochastic_gradient.pdf

# Design Deep model




http://www.sohu.com/a/100823811_114877

# Questions