$Experiment 1$：有限差分法近似求解

杨乐园 PB18010496

问题描述

利用有喜爱能查分法求下述偏微分方程初值问题在时刻$t=0.3$的近似解：

$$
\begin{cases}
u_t = u_x, & -\infty < x < +\infty, t > 0 \\
u(x,0) = \sin 2\pi x, & -\infty < x < +\infty \\
\text{Periodic boundary condition}, & T = 1
\end{cases}
$$

该方程的精确解为$u(x,t)=sin(2\pi(x+t))$，对时空区域$[0,1]\times[0,1]$均匀剖分如下：

$$
\text{时间：} t_n = n\cdot\Delta t, n = 0,1,2,\ldots,N, \text{时间步长 } \Delta\, t = \frac{1}{N}
$$
$$
\text{空间 }: x_j = j\cdot\Delta x, j = 0,1,2,\ldots,J, \text{时间步长 } \Delta\, x = \frac{1}{J}
$$

问题如下：

$1.1:$ 取 $\Delta\, x=0.02, \Delta t=0.01$,求上述偏微分方程初值问题在时刻$t=0.3$的近似解，并画图比较精确解和精确解 (画图)。
$1.2:$ 取 $\Delta\, x=0.02, \Delta t=0.03$,求上述偏微分方程初值问题在时刻$t=0.3$的近似解，并画图比较精确解和精确解 (画图)。
1.3 :对上述两种实验结果进行描述, 分析并评论。

数值方法

记$v_j^n \approx u(x_j,t_n)$，有导数近似$u_t \approx \frac{u(x,t+\Delta t)-u(x,t)}{\Delta t}$，$u \approx \frac{u(x+\Delta x,t)-u(x,t)}{\Delta x}$，从而由偏微分方程得到相应的离散方程如下：

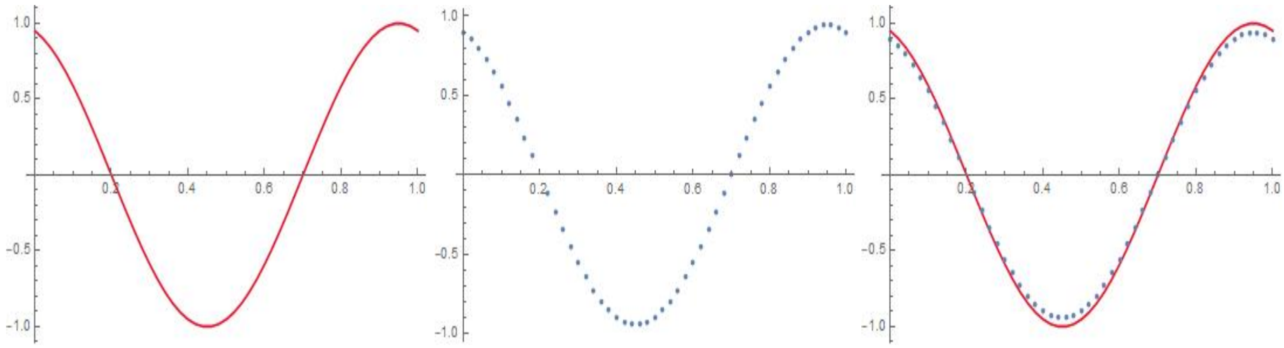
$$
v_j^n = v_j^n + \frac{\Delta t}{\Delta x}(v_{j+1}^n - v_j^n)
$$

其中定解条件为：初始条件：$v_j^0 = sin2\pi x_j$，边界条件：$v_j^n = v_{j+J}^n$。

数值结果

1. $\Delta t = 0.01$

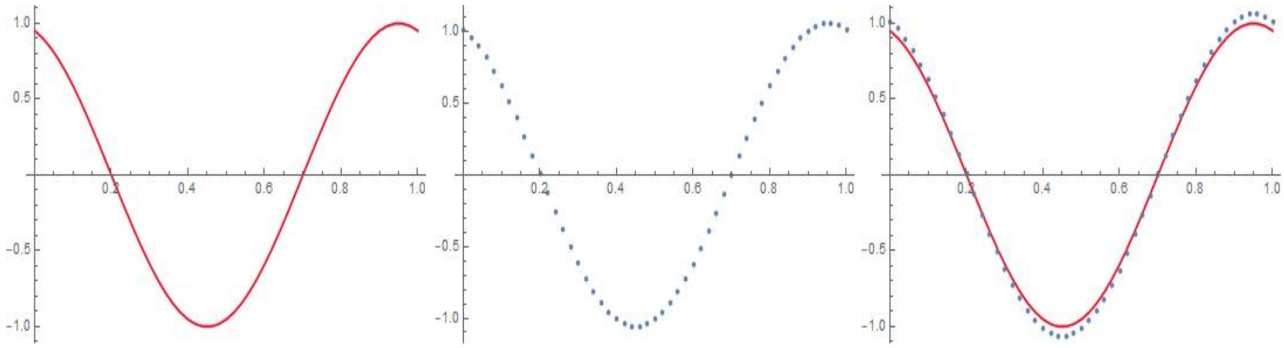
```
deltat=0.01时(x坐标，差分解，精确解)
(0,    0.896337,    0.951057)    (0.02,    0.852768,    0.904827)    (0.04,    0.795749,    0.844328)
(0.06,    0.726182,    0.770513)    (0.08,    0.645162,    0.684547)    (0.1,    0.553967,    0.587785)
(0.12,    0.454036,    0.481754)    (0.14,    0.346944,    0.368125)    (0.16,    0.234381,    0.24869)
(0.18,    0.118122,    0.125333)    (0.2,    3.46945e-17,    1.22465e-16)    (0.22,    -0.118122,    -0.125333)
(0.24,    -0.234381,    -0.24869)    (0.26,    -0.346944,    -0.368125)    (0.28,    -0.454036,    -0.481754)
(0.3,    -0.553967,    -0.587785)    (0.32,    -0.645162,    -0.684547)    (0.34,    -0.726182,    -0.770513)
(0.36,    -0.795749,    -0.844328)    (0.38,    -0.852768,    -0.904827)    (0.4,    -0.896337,    -0.951057)
(0.42,    -0.925771,    -0.982287)    (0.44,    -0.940605,    -0.998027)    (0.46,    -0.940605,    -0.998027)
(0.48,    -0.925771,    -0.982287)    (0.5,    -0.896337,    -0.951057)    (0.52,    -0.852768,    -0.904827)
(0.54,    -0.795749,    -0.844328)    (0.56,    -0.726182,    -0.770513)    (0.58,    -0.645162,    -0.684547)
(0.6,    -0.553967,    -0.587785)    (0.62,    -0.454036,    -0.481754)    (0.64,    -0.346944,    -0.368125)
(0.66,    -0.234381,    -0.24869)    (0.68,    -0.118122,    -0.125333)    (0.7,    -1.38778e-17,    -2.44929e-16)
(0.72,    0.118122,    0.125333)    (0.74,    0.234381,    0.24869)    (0.76,    0.346944,    0.368125)
(0.78,    0.454036,    0.481754)    (0.8,    0.553967,    0.587785)    (0.82,    0.645162,    0.684547)
(0.84,    0.726182,    0.770513)    (0.86,    0.795749,    0.844328)    (0.88,    0.852768,    0.904827)
(0.9,    0.896337,    0.951057)    (0.92,    0.925771,    0.982287)    (0.94,    0.940605,    0.998027)
(0.96,    0.940605,    0.998027)    (0.98,    0.925771,    0.982287)    (1,    0.896337,    0.951057)
```



精确解　　数值解　　对比图

2. $\Delta t = 0.03$

```
deltat=0.03时(x坐标，差分解，精确解)
(0,    1.01025,    0.951057)    (0.02,    0.96183,    0.904827)    (0.04,    0.898243,    0.844328)
(0.06,    0.82049,    0.770513)    (0.08,    0.729797,    0.684547)    (0.1,    0.627595,    0.587785)
(0.12,    0.515495,    0.481754)    (0.14,    0.395265,    0.368125)    (0.16,    0.268802,    0.24869)
(0.18,    0.1381,    0.125333)    (0.2,    0.00522024,    1.22465e-16)    (0.22,    -0.127742,    -0.125333)
(0.24,    -0.25869,    -0.24869)    (0.26,    -0.385558,    -0.368125)    (0.28,    -0.506346,    -0.481754)
(0.3,    -0.619148,    -0.587785)    (0.32,    -0.722186,    -0.684547)    (0.34,    -0.813835,    -0.770513)
(0.36,    -0.892649,    -0.844328)    (0.38,    -0.957385,    -0.904827)    (0.4,    -1.00702,    -0.951057)
(0.42,    -1.04078,    -0.982287)    (0.44,    -1.05812,    -0.998027)    (0.46,    -1.05878,    -0.998027)
(0.48,    -1.04274,    -0.982287)    (0.5,    -1.01025,    -0.951057)    (0.52,    -0.96183,    -0.904827)
(0.54,    -0.898243,    -0.844328)    (0.56,    -0.82049,    -0.770513)    (0.58,    -0.729797,    -0.684547)
(0.6,    -0.627595,    -0.587785)    (0.62,    -0.515495,    -0.481754)    (0.64,    -0.395265,    -0.368125)
(0.66,    -0.268802,    -0.24869)    (0.68,    -0.1381,    -0.125333)    (0.7,    -0.00522024,    -2.44929e-16)
(0.72,    0.127742,    0.125333)    (0.74,    0.25869,    0.24869)    (0.76,    0.385558,    0.368125)
(0.78,    0.506346,    0.481754)    (0.8,    0.619148,    0.587785)    (0.82,    0.722186,    0.684547)
(0.84,    0.813835,    0.770513)    (0.86,    0.892649,    0.844328)    (0.88,    0.957385,    0.904827)
(0.9,    1.00702,    0.951057)    (0.92,    1.04078,    0.982287)    (0.94,    1.05812,    0.998027)
(0.96,    1.05878,    0.998027)    (0.98,    1.04274,    0.982287)    (1,    1.01025,    0.951057)
```



精确解　　数值解　　对比图

讨论

通过对比两结果可以发现：对于$\Delta t = 0.01$时，数值求解结果在区间边界处整体偏小，而在区间中段时却整体偏大；而对于$\Delta t = 0.03$时，却正好相反，即数值求解结果在区间边界处整体偏大，而在区间中段时却整体偏小。