Design and Realization of the Doughnut-shaped Karnaugh Map Spatial Structure

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***Abstract*—This paper innovatively proposes a Doughnut-shaped Karnaugh Map spatial structure, which can intuitively show the principle of adjacency between the minimum items in combinational logic circuit. Firstly, we analyze the difficulties in the process of combinational logic circuit design, and analyze the Karnaugh map method used in logic function simplification. Then, we analyze the steps of Karnaugh map reduction in detail, and define and analyze the concepts of minimum term and Karnaugh circle. Then, we propose this kind of donut Karnaugh map structure, which can clearly show the adjacency of the edge of planar Karnaugh map. Finally, we verify the important role of this method in the process of circuit design through the design experiment of combined logic circuit of referee voting device.**

Keywords—Karnaugh Map, Doughnut-shaped Karnaugh Map, Spatial structure, Combinational logic circuit, Minimum items

I. INTRODUCTION

In the field of signal analysis and engineering control design, combinational logic circuit in digital circuit plays an important role [1,2]. Designing a set of excellent combinational logic circuit can not only simplify the design idea, but also reduce the design cost and improve the operation efficiency of the system. In order to use the least electronic devices to achieve a certain logic function, it is necessary to simplify the logic function expression to obtain the simplest logic expression [3,4]. The simplification of logical expression mainly includes logical algebra method and Karnaugh map method [5]. The simplification process of logic algebra method is very complex, which needs a solid foundation of logic algebra. The simplification process of Karnaugh map method is intuitive and clear, which is a better method.

Karnaugh map is first proposed by American engineer Maurice Karnaugh [5]. It plays an important role in the analysis and design of digital circuits. It is an excellent method to design logic electronic circuits. It has many advantages when the input variables are less than 5 [6-8]. Karnaugh map is a more intuitive method for variables between 2 and 4. If the number of variables exceeds 4, dimension reduction is needed to reduce the number of variables to less than 4. As a special method to describe logic function, Karnaugh map divides the logic variables of logic function into two groups. The value combination of each group of variables is arranged according to the rule of cyclic code to form a grid graph. Each grid in the graph corresponds to a minimum term of the logic variable.

In the design process of combinational logic circuit, the simplest and or formula is usually obtained by simplifying the logic function with algebraic method or Karnaugh map method [9-11]. The simplest circuit can be composed of basic “and gate” circuit and “or gate” circuit, and the simplest chip can be selected to optimize the implementation, so as to minimize the structure of physical connection.

The difficulty of Karnaugh map simplification lies in that it is a table of plane structure and meets the principle of adjacent edges, which brings a lot of inconvenience in the process of design and simplification [12]. In order to overcome this difficulty, this paper proposes the Doughnut-shaped Karnaugh Map spatial structure, which can clearly show the adjacent relationship of the logical minimum.

II. THE PRINCIPLE OF KARNAUGH MAP SIMPLIFICATION

The simplification of Karnaugh map is based on the concept of minimum term. If there are *n* variables, each of the “and terms” combined by them, appears once and only once in the form of original variable or inverse variable. These “and terms” can be called the minimum terms of *n* variables. For example, if the function contains *n* variables, it can form 2^n minimum terms, denoted as m_n respectively.

For example, there are $2^2 = 4$ minimum terms of the two variables situation, which are

$$\overline{A}\overline{B} = m_0,\quad \overline{A}B = m_1,\quad A\overline{B} = m_2,\quad AB = m_3 \qquad (1)$$

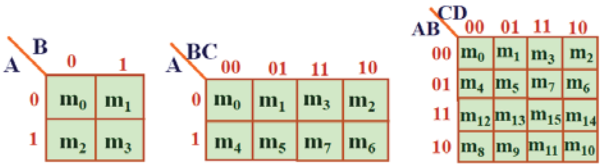
There are $2^3 = 8$ minimum terms in the three variables situation, which are (We only list two minimum terms, and the middle 6 minimum terms are omitted.)

$$\overline{A}\overline{B}\overline{C} = m_0......ABC = m_7 \qquad (2)$$

There are $2^4 = 16$ minimum terms in the four variables situation, which are (We only list two minimum terms, and the middle 14 minimum terms are omitted.)

$$\overline{A}\overline{B}\overline{C}\overline{D} = m_0......ABCD = m_{15} \qquad (3)$$

Then, we can draw the **Empty Karnaugh map** and fill the minimum term into each Karnaugh map square. In this way, the corresponding relationship between Karnaugh map and minimum term is established, as shown in **Fig. 1**.



(a) Bivariate empty Karnaugh Map; (b) Three-variable empty Karnaugh map; (c) Four-variable empty Karnaugh map

Fig. 1 The structural relationship between empty Karnaugh map and the minimum terms

From the empty Karnaugh map in Fig.1, we can see that the Karnaugh map, as a variant of truth table, has some special advantages. The Karnaugh map corresponding to the logic function of n variables has 2^n squares, which are consistent with the number of minimum terms. The two minimum terms adjacent to any geometric position in Karnaugh map are logically adjacent. Karnaugh map follows the principle of adjacency; the difference between the two minimum terms is only one variable or independent variable. The most marginal column of Karnaugh map is also adjacent. The top and bottom rows of Karnaugh map are also adjacent. These characteristics of Karnaugh map make it widely used in the process of logic function simplification, digital logic circuit analysis and design [13-15].

On the basis of Karnaugh map, the main steps of Karnaugh map simplification are as follows:

(1) According to the variable number, the empty Karnaugh map with corresponding number of squares is drawn;

(2) According to the logic function expression that needs to be simplified, the minimum items in all function expressions are recorded as 1, and then they are drawn into the Karnaugh map, and the minimum items that have not appeared are filled in 0. We will obtain a binary graph of Karnaugh map filled with 1 and 0;

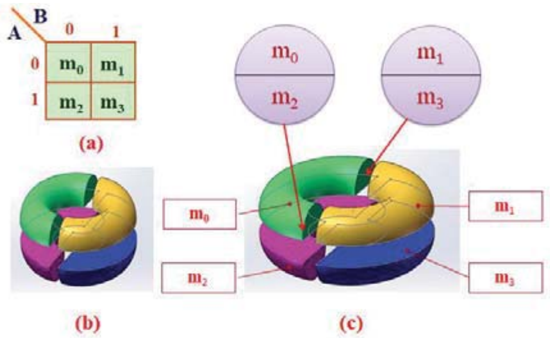
(3) Draw as few Karnaugh loops as possible and circle all 1. In one loop, the number of 1 should be determined according to 1, 2, 4, 8 and 16 adjacent variables, and the principle of maximum Karnaugh loop should be followed;

(4) Write the corresponding expression of the Karnaugh loop. Eliminate all the non variables in the circle, keep the same variable as an “and term”, and finally add (or logic) each “and terms” to obtain the final expression.

III. THE SPATIAL STRUCTURE OF KARNAUGH MAP

Karnaugh map is arranged by gray code. The geometric adjacency of the minimum term is the logical adjacency, and the adjacent term can be simplified according to the rules. From this point of view, Karnaugh map is a kind of planar expanded grid structure. When drawing Karnaugh circle, we regard Karnaugh map as a structure without boundary. The top row is adjacent to the bottom row, and the left row is adjacent to the right row. Therefore, it becomes a spatial adjacent structure. Many researchers think that this is a structural contradiction, it is difficult to imagine the actual structure of Karnaugh map. Therefore, this paper proposes a Doughnut-shaped Karnaugh Map spatial structure, which is easy to understand and apply.

For the Karnaugh map with two variables, we design its spatial structure, as shown in Fig. 2.

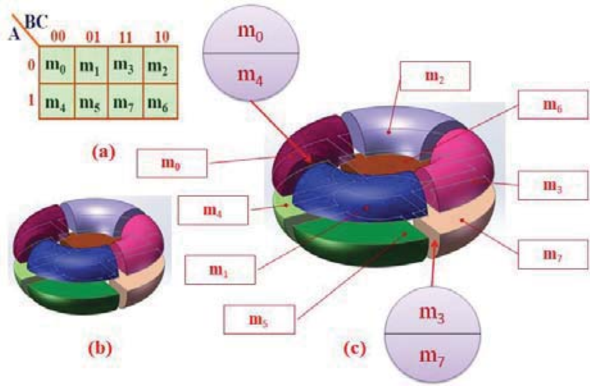


(a) Bivariate empty Karnaugh map; (b) Spatial structure of bivariate Karnaugh map; (c) Correspondence between spatial structure of bivariate Karnaugh map and minimum terms

Fig. 2 Structure of Bivariate Karnaugh map

In Fig.2, map (a) is the empty Karnaugh map, map (b) is the spatial structure of Karnaugh map, and map (c) shows the corresponding relationship between the spatial structure of Karnaugh map and the minimum terms. As can be seen from Fig.2, two variables will produce four minimum terms, each of which is adjacent to the other three minimum terms. It's easier to understand and apply in space.

We further design the spatial structure of Karnaugh map with three variables. As shown in Fig.3.

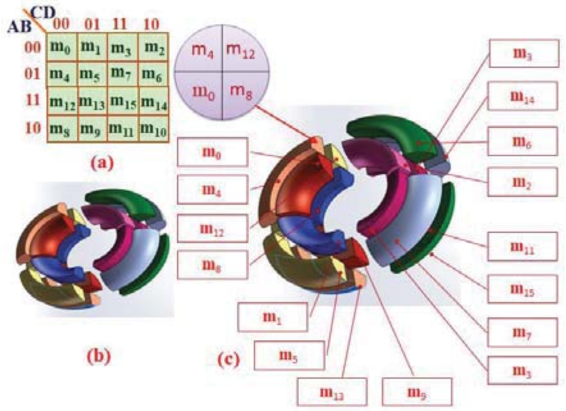


(a) Three-variable empty Karnaugh map; (b) Spatial structure of three-variable Karnaugh map; (c) Correspondence between spatial structure of three-variable Karnaugh map and minimum terms

Fig. 3 Structure of three-variable Karnaugh map

In Fig.3, map (a) shows the three variable empty Karnaugh map, map (b) is the spatial structure of three variable Karnaugh map, and map (c) is the corresponding relationship between the spatial structure of three-variable Karnaugh map and the minimum terms.

Using the same method, we design the spatial structure of Karnaugh map with four variables, as shown in Fig.4.



(a) Four-variable empty Karnaugh map; (b) Spatial structure of four-variable Karnaugh map; (c) Correspondence between spatial structure of four-variable Karnaugh map and minimum terms

Fig. 4 Structure of four-variable Karnaugh map

As can be seen from Fig.4, the four variables will produce 16 minimum terms, which have the most complex spatial structure. The arrangement of these 16 minimum terms also follows the principle of adjacency, corresponding to the planar structure in Fig.4(a), and the spatial structure in Fig.4(c). It can be seen that two adjacent rows or columns are structures with one variable or independent variable difference, and they are also closely connected in space. In the section of Fig.4(c), the four minimum terms m_0, m_4, m_{12}, and m_8 are all adjacent structures, forming a ring, and ensuring that m_0 and m_8 are in contact. Therefore, the spatial structure of the Doughnut-shaped can well show the structure of Karnaugh map and the relationship between the minimum terms.

IV. CASE STUDY

We can verify the effectiveness of this method through the design experiment of a referee voting combinational logic circuit. Through this experiment, we can master the basic concepts and design methods of combinational logic circuit. An enterprise needs to design a referee voting device and use logic control circuit to complete the product skills competition. There are three referees in the competition. After all the contestants show their designed industrial products, the referee will vote whether they can pass the competition. If two or three referees pass, the contestant will pass the competition and enter the next round. If only one referee or no referee votes, the contestant will be refused. This logic control circuit belongs to the combinational logic circuit in the electronic circuit, which needs to use some logic gates, such as the basic and gate, or gate, not gate, etc.

A. Overall variable relationships

We define three referees as A, B and C. Each referee has a vote button. If he agrees to press the button, the result of the button represents 1. If the referee does not agree and does not press the button, it means 0. For instance, $A = 0$ means that referee A pressed the agree button. The final output is F, using an indicator light to indicate 1 or 0. If the majority is in favor, the result is passed and *F* is output 1.

B. List the corresponding truth table

According to the logic relationship given in the experiment, after sorting out, for the logic relationship of three inputs and one output, we can list its corresponding truth table, as shown in Table 1.

TABLE 1 TRUTH TABLE OF COMBINATIONAL LOGIC RELATION

A	*B*	*C*	*F*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	0	1	1

After verification, we find that the output of result F is in line with the logical relationship. When the number of 1 in ABC is more than 2, the output F is 1, indicating that the contestant has passed the selection. The logic results in this table are consistent with the requirements of the experiment. At the same time, this table is also a logical truth table, reflecting the precise logical relationship.

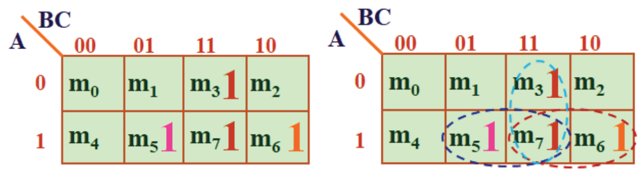
C. List the logical functions

According to the truth Table 1, we can list all cases in which the output F is 1, and each case is represented by a logical multiplier with the structure. The logical relationship between the four cases is "OR", but the logical relationship inside these four cases is "AND". We add all the cases together, because they are a logical relationship of "OR". Within the logic and structure, we can write out what conditions ABC must satisfy at the same time before the result outputs 1. For example, in line 8 of Table 1, only when A = 1 and B = 0 and C = 1, the output result F is equal to 1. So we can write this expression of logical $A\overline{B}C$ (using AND). Finally, we write out the four cases of 1 in Table 1 to obtain the final logical function expression (using OR).

$$F = \overline{A}BC + A\overline{B}C + \mathrm{AB}\overline{C} + ABC \tag{4}$$

D. Karnaugh map simplification

We use Karnaugh map method to get the simplest form. First, we make sure that this is a three variable Karnaugh map, so we first draw an empty map, as shown in Figure 5A. The smallest term in function 4 is expressed into Karnaugh map.



(a) Logic function represents entering Karnaugh map; (b) Draw Karnaugh loops

Fig. 5 Karnaugh map simplifying logic function

Then, according to the principle of Karnaugh loop, we can draw at least 3 Karnaugh loops to circle all 1 in it. Moreover, the number of 1 in each circle should be the most. And the number of 1 must be 2, 4, 8, etc. Then write out the expressions corresponding to the three Karnaugh cycles (AND relationship), and sum the three expressions together (OR relationship). Obtain the final simple form.

$$F = AB + BC + CA \tag{5}$$

E. Draw the logic circuit diagram

According to the simplified logic function expression 5, its corresponding logic diagram can be drawn, as shown in Fig.6.

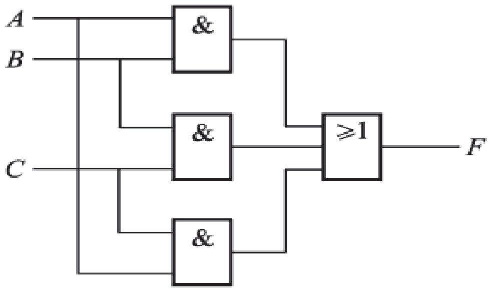


Fig.6 Logic circuit of referee voting device

In Fig.6, A, B and C are the three input variables and F is the output variable. In Fig.6, only two kinds of logic components are used, one is logic "AND" device, and the other is logic "OR" device. Therefore, we can say that by using Karnaugh map to simplify combinational logic circuit, the cost of components can be saved, the complexity of logic circuit can be reduced, and the operation efficiency of logic circuit can be improved.

V. CONCLUSION

(1) This paper analyzes the design process of combinational logic circuit, and analyzes the importance of Karnaugh map to simplify the expression of combinational logic function.

(2) This paper innovatively designs a Doughnut-shaped Karnaugh Map spatial structure, which can intuitively show the minimum term adjacent relationships of Karnaugh map.

(3) The effectiveness and rationality of the proposed method are verified by a combinational logic circuit design experiment of a referee voting device.

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