

# Statistics



1. “You are driving at 65 km/h”
2. “You will (most likely) reach destination in 30 mins”

## Descriptive statistics

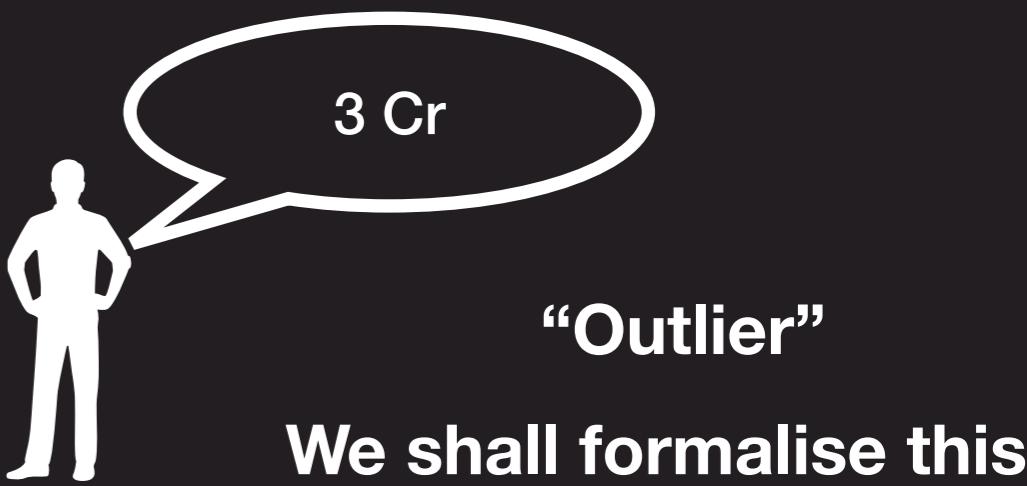
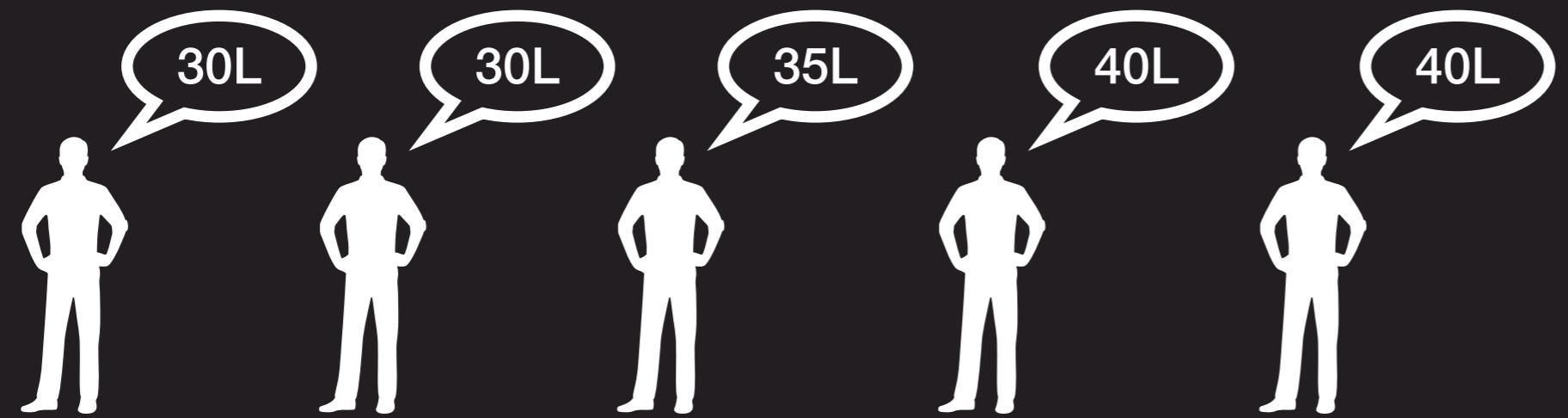
Summarise data

Central tendency, variability

## Inferential statistics

Drawing conclusions from observations

Confidence interval, hypothesis test, regression



$$\text{Mean} = \frac{(30 + 30 + 35 + 40 + 40)}{5} = 35L$$

Another word for mean is “Average”

$$\text{New Mean} = \frac{30 + 30 + 35 + 40 + 40 + 300}{6} = 79L$$

**Crucial observation: Median is more robust to outliers**

**Median = 35L**

**Central value (if unique)**

**$N = 5$ , odd**

**New Median = 37.5L**

**Average of 2 central values**

$$\frac{35 + 40}{2}$$

**$N = 6$ , even**

## Median

10, 20, 30, 40, 50, 60, 70

Middle number: 40; Median = 40

10, 20, 30, 40, 50, 60, 70, 80

Two middle numbers: 40, 50; Median =  $(40 + 50)/2 = 45$

**Quiz** There are 4 people whose average age is 24.

We know the age of three people: 20, 22, and 28.

What is the median age of these 4 people?

$$\frac{20 + 22 + 28 + x}{4} = 24$$

$$x = 4 * 24 - (20 + 22 + 28)$$

$$x = 26$$

20, 22, 26, 28

$$\text{Median} = \frac{22 + 26}{2} = 24$$

## Mode

90, 90, 90, 80, 90, 70, 95, 90

**Mode = 90**

**Mode is the most frequently occurring number, if such a number exists**

2, 2, 3, 3, 4

**We call this bi-modal with 2 and 3 as the modes**

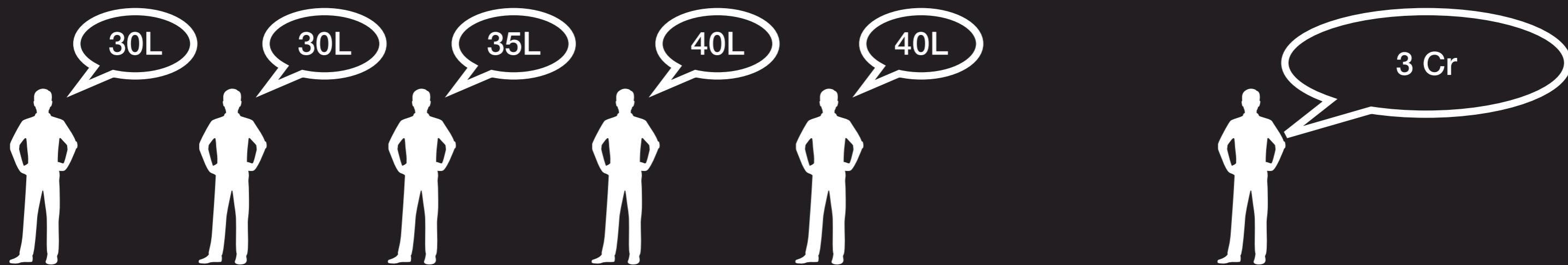
## Range

Suppose a cricketer has scored as follows

20, 25, 60, 100

We say the range =  $100 - 20 = 80$

Consider again the example of salaries

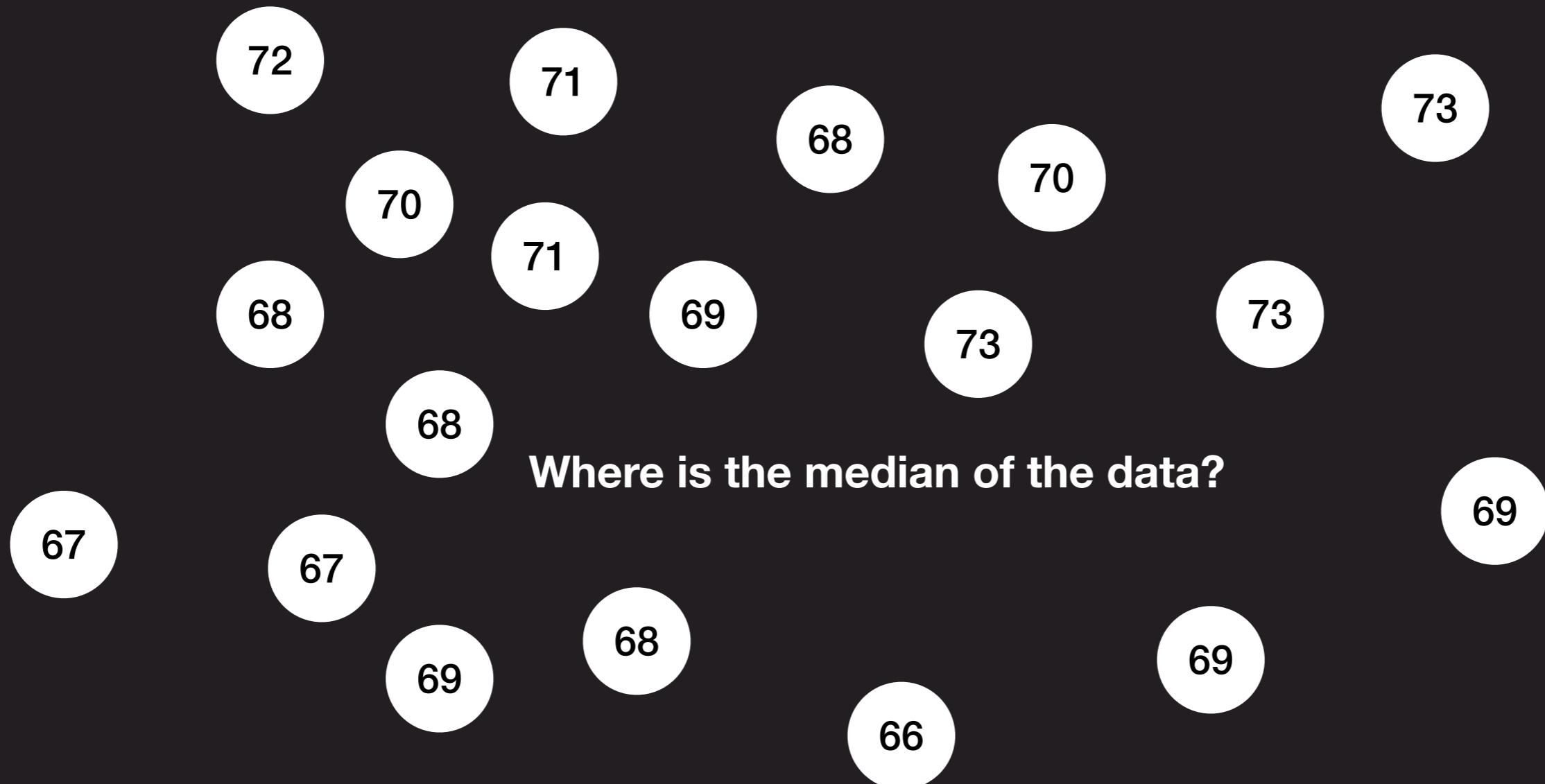


Here range =  $300 - 30 = 270$  L

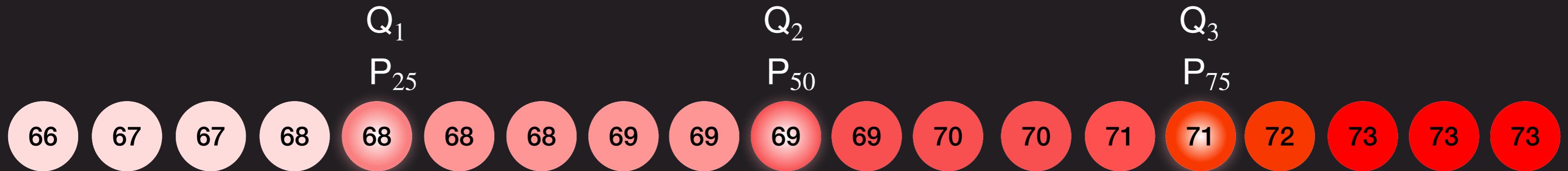
Sometimes, simply giving range may not make sense

# Percentiles, Quartiles, Inter Quartile range (IQR)

Sort the data!



## Percentiles, Quartiles, Inter Quartile range (IQR)



**What is the median of the first half?**

**What fraction of data is less than this? 25 %**

**Another name for this is 25<sup>th</sup> percentile - P<sub>25</sub>**

**What is the median of the second half?**

**What fraction of data is less than this? 75 %**

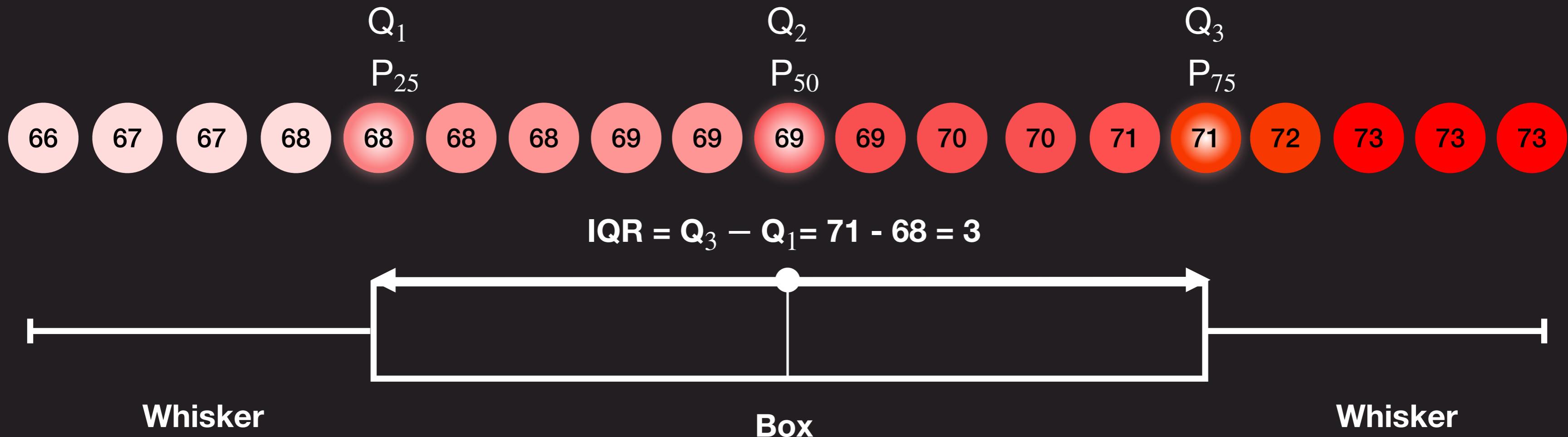
**Another name for this is 75<sup>th</sup> percentile - P<sub>75</sub>**

**Where is the median of the data?**

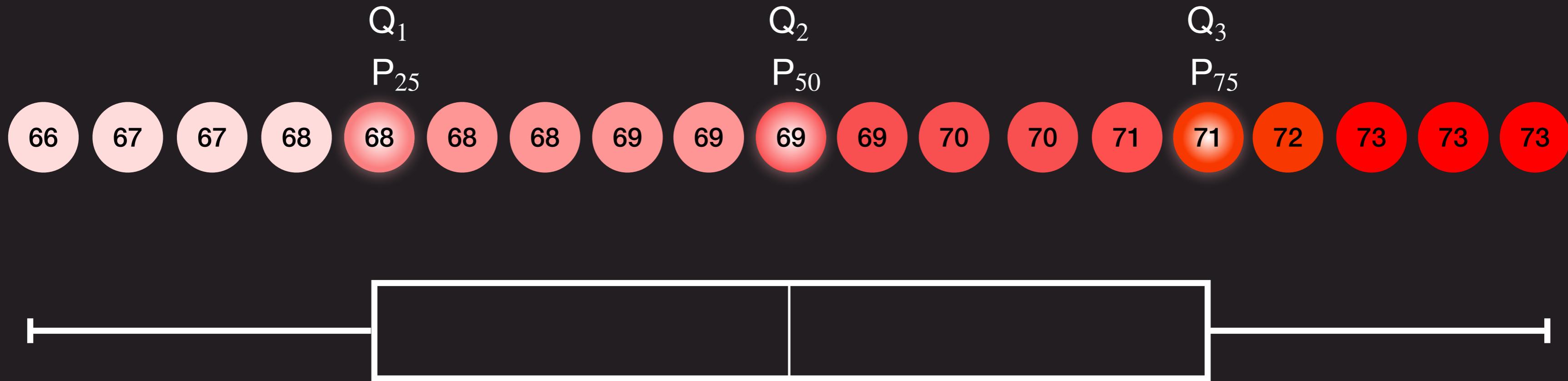
**What fraction of data is less than this? 50 %**

**Another name for this is 50<sup>th</sup> percentile - P<sub>50</sub>**

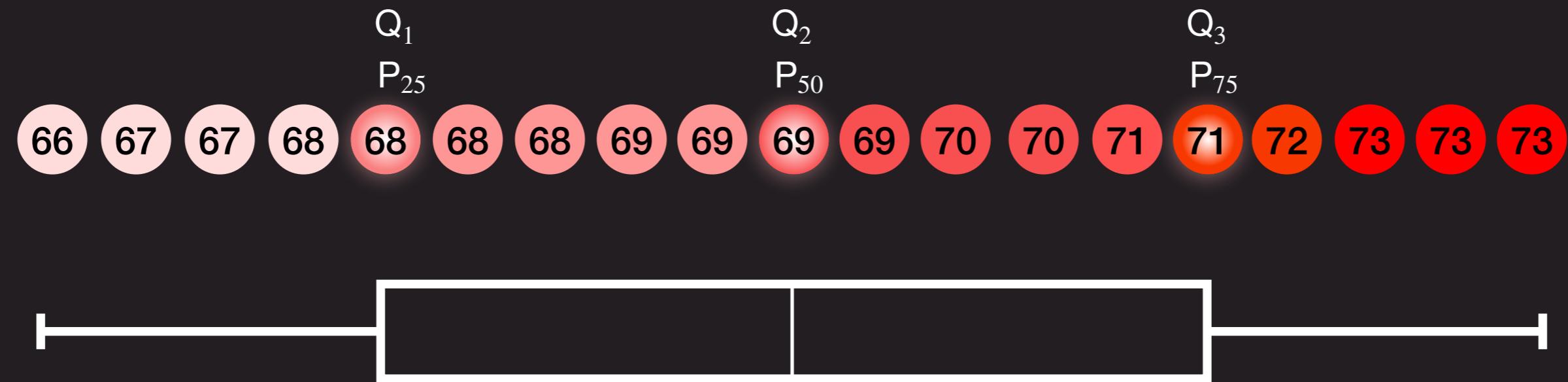
## Percentiles, Quartiles, Inter Quartile range (IQR)



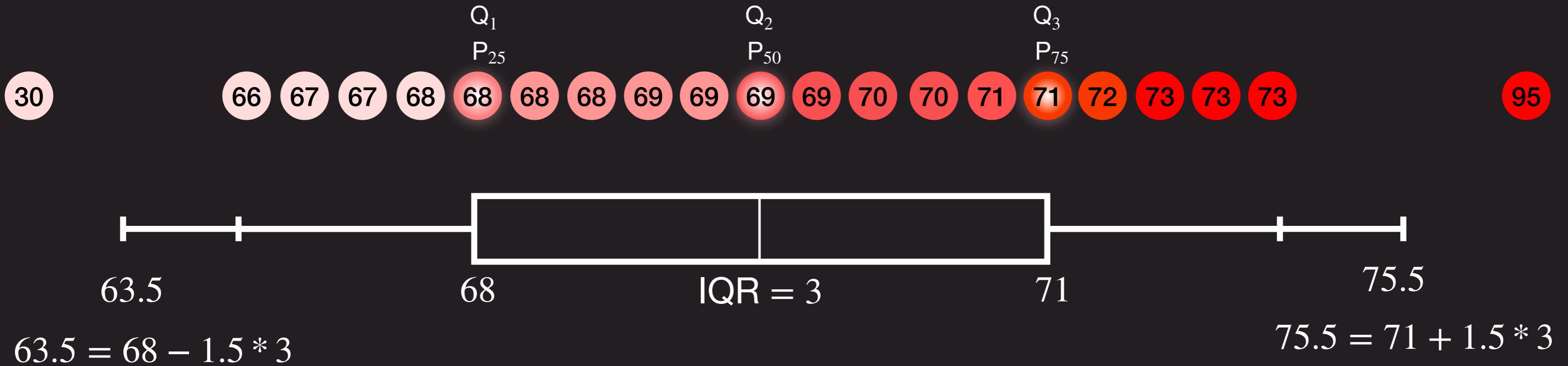
## Percentiles, Quartiles, Inter Quartile range (IQR)



## Percentiles, Quartiles, Inter Quartile range (IQR)

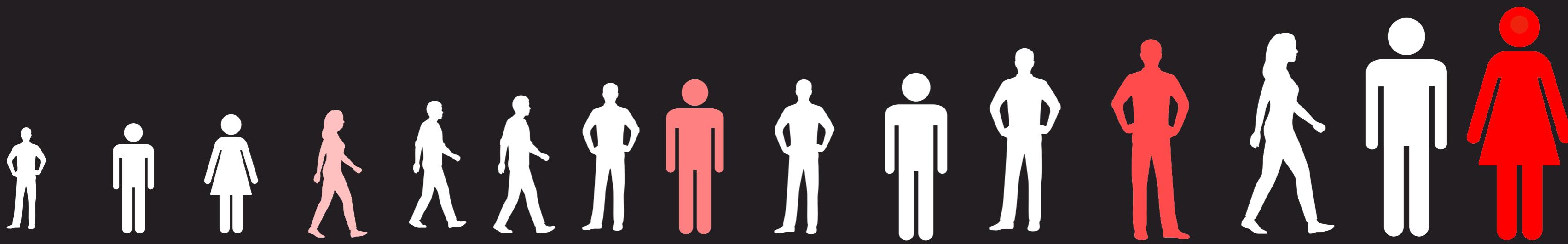


## Percentiles, Quartiles, Inter Quartile range (IQR)



**Outlier are points outside [63.5, 75.5]**

**Outlier are points outside  $[Q_1 - 1.5 * \text{IQR}, Q_3 + 1.5 * \text{IQR}]$**



**0.25**

**What fraction of  
people are shorter  
than this lady?**

**0.5**

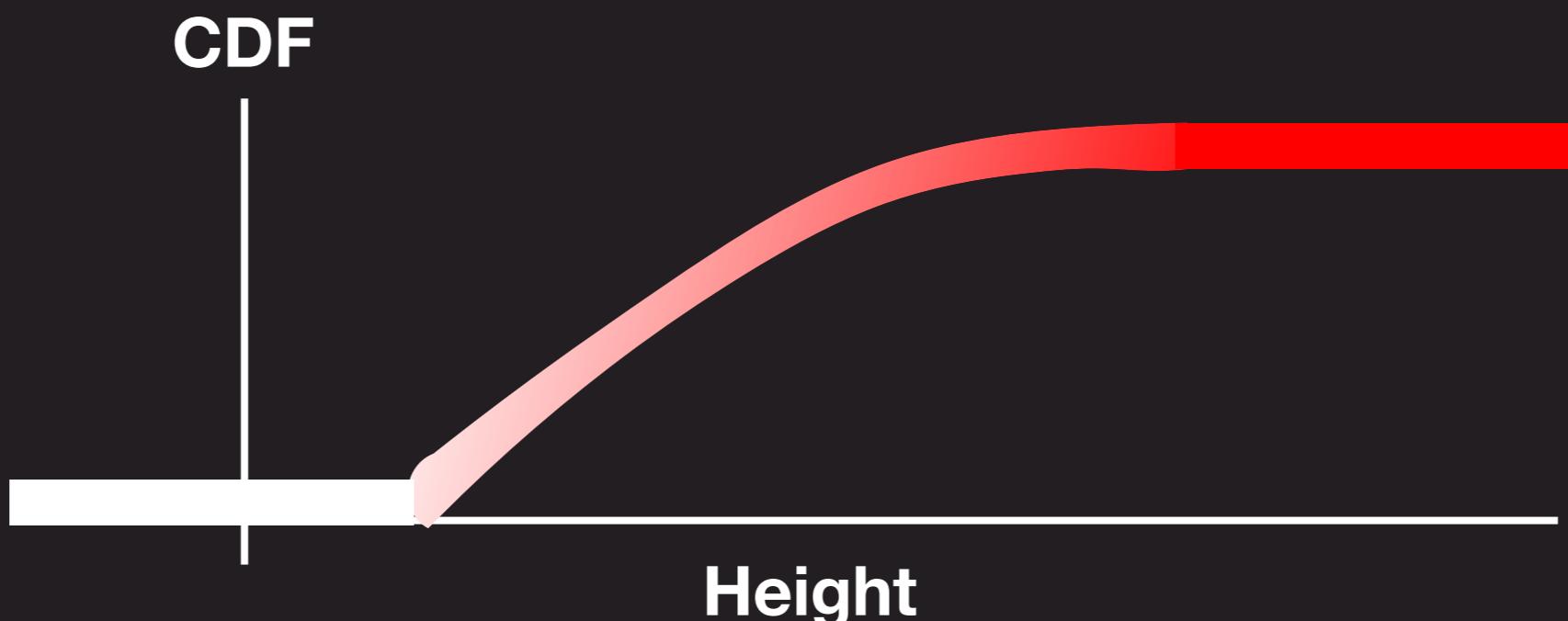
**What fraction of  
people are shorter  
than this guy?**

**0.75**

**What fraction of  
people are shorter  
than this guy?**

**1**

**What fraction of  
people are shorter  
than this lady?**



## Simple Arithmetic

Original salary 30, 32, 35, 35, 38

Mean = 34

Median = 35

Mode = 35

Range = 38 - 30 = 8

IQR = 36.5 - 31 = 5.5

### Effect of addition

After 5 L bonus

35, 37, 40, 40, 43

Mean = 39

Median = 40

Mode = 40

Range = 43 - 35 = 8

IQR = 41.5 - 36 = 5.5

### Effect of multiplication

Salary in Yen: 1 Rs = 1.76 Yen

52.8 , 56.32, 61.6 , 61.6 , 66.88

Mean = 59.8

Median = 61.6

Mode = 61.6

Range = 66.88 - 52.8 = 14.08

IQR = 64.24 - 54.56 = 9.68

## Variance



### How to define Error?

$$10 \text{ mm} + 5 \text{ mm} + (-5 \text{ mm}) + (-10 \text{ mm}) = 0 \text{ mm} \quad \times$$

$$(10 \text{ mm})^2 + (5 \text{ mm})^2 + (-10 \text{ mm})^2 + (-5 \text{ mm})^2 = 250 \text{ mm}^2 \quad \checkmark$$

$$\text{Variance} = \frac{250}{4} \text{ mm}^2$$

$$\text{Std dev} = \sqrt{\frac{250}{4}} \text{ mm}$$

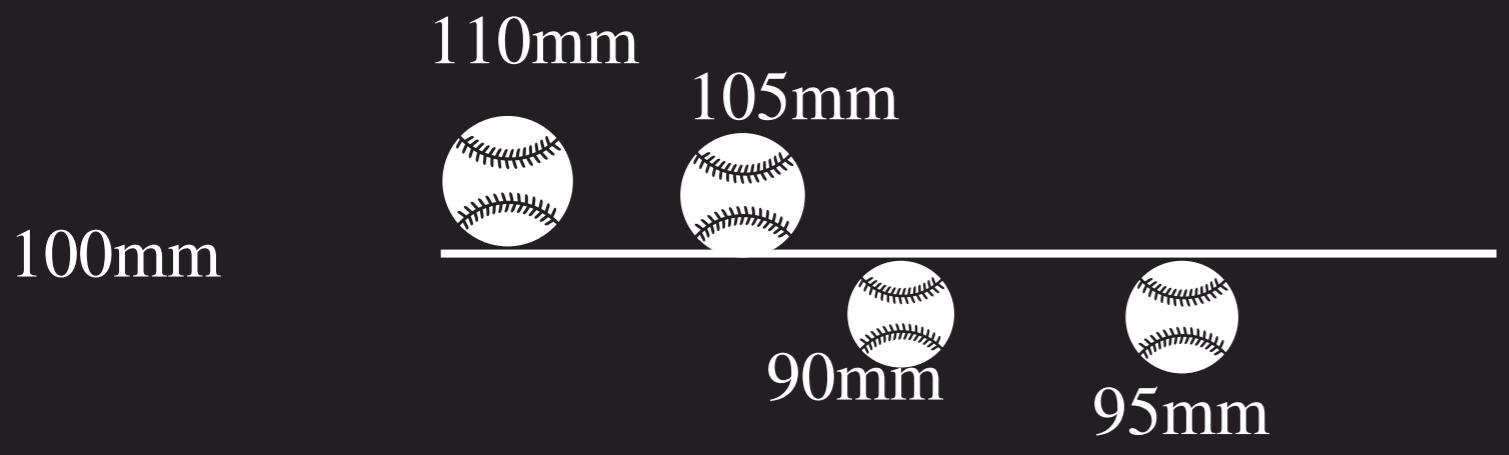
$$(50 \text{ mm})^2 + (40 \text{ mm})^2 + (-50 \text{ mm})^2 + (-40 \text{ mm})^2 = 8200$$

$$\text{Variance} = \frac{8200}{4} \text{ mm}^2$$

$$\text{Std dev} = \sqrt{\frac{8200}{4}} \text{ mm}$$

# Variance

M1



$$10 \text{ mm} + 5 \text{ mm} + (-5 \text{ mm}) + (-10 \text{ mm}) = 0 \text{ mm}$$

$$(10 \text{ mm})^2 + (5 \text{ mm})^2 + (-10 \text{ mm})^2 + (-5 \text{ mm})^2 = 250 \text{ mm}^2$$

$$\text{Variance} = \frac{250}{4} \text{ mm}^2$$

$$\text{Std Dev} = \sqrt{\frac{250}{4}} \text{ mm}$$

$$\text{Variance} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2}{4}$$

$$\text{Std Dev} = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2}{4}}$$

$x_1$	110
$x_2$	105
$x_3$	95
$x_4$	90
$\bar{x}$	100

$$\text{Std Dev} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n}} = \sigma$$

$$\text{Variance} = \frac{\sum_i (x_i - \bar{x})^2}{n} = \sigma^2$$