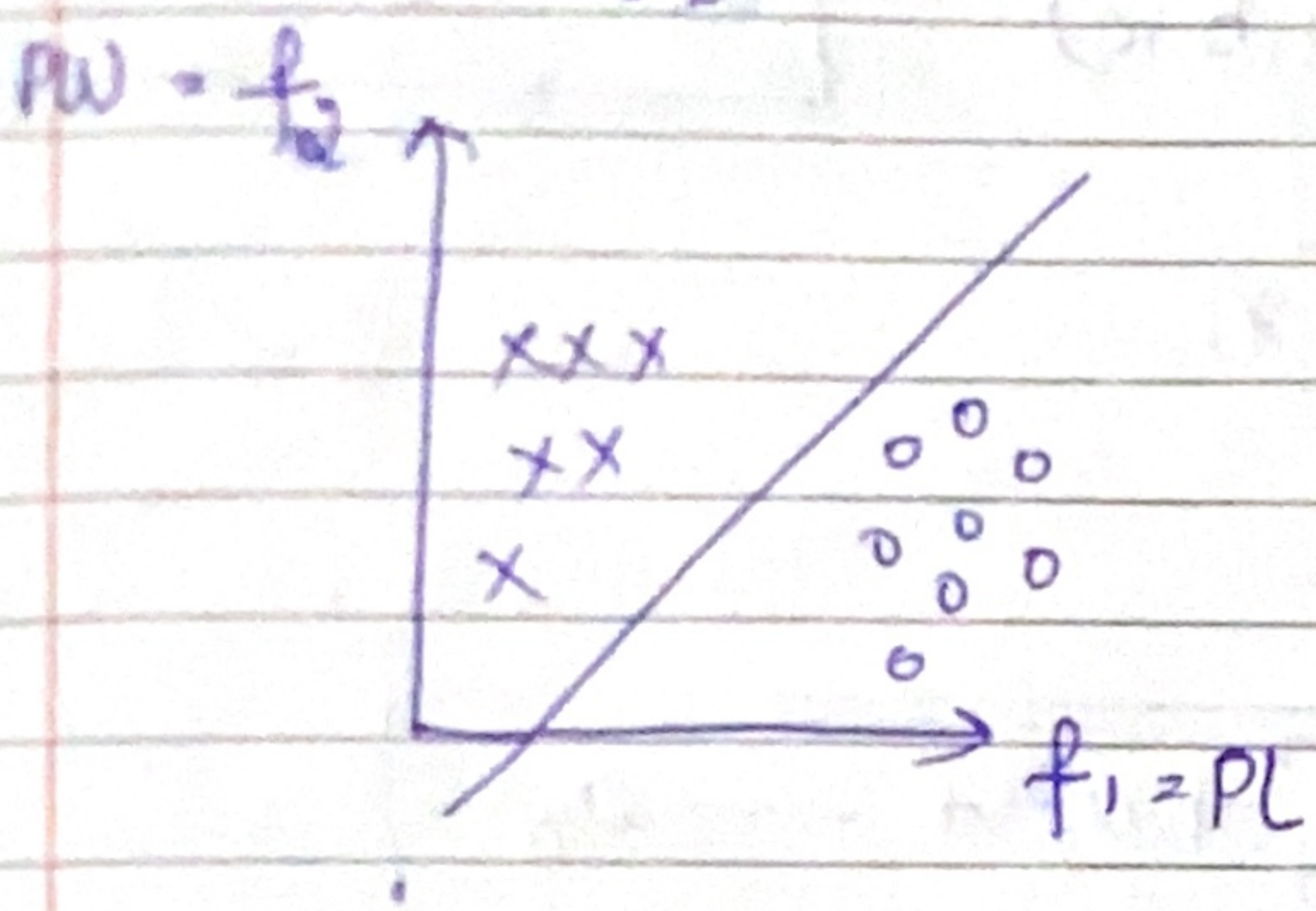
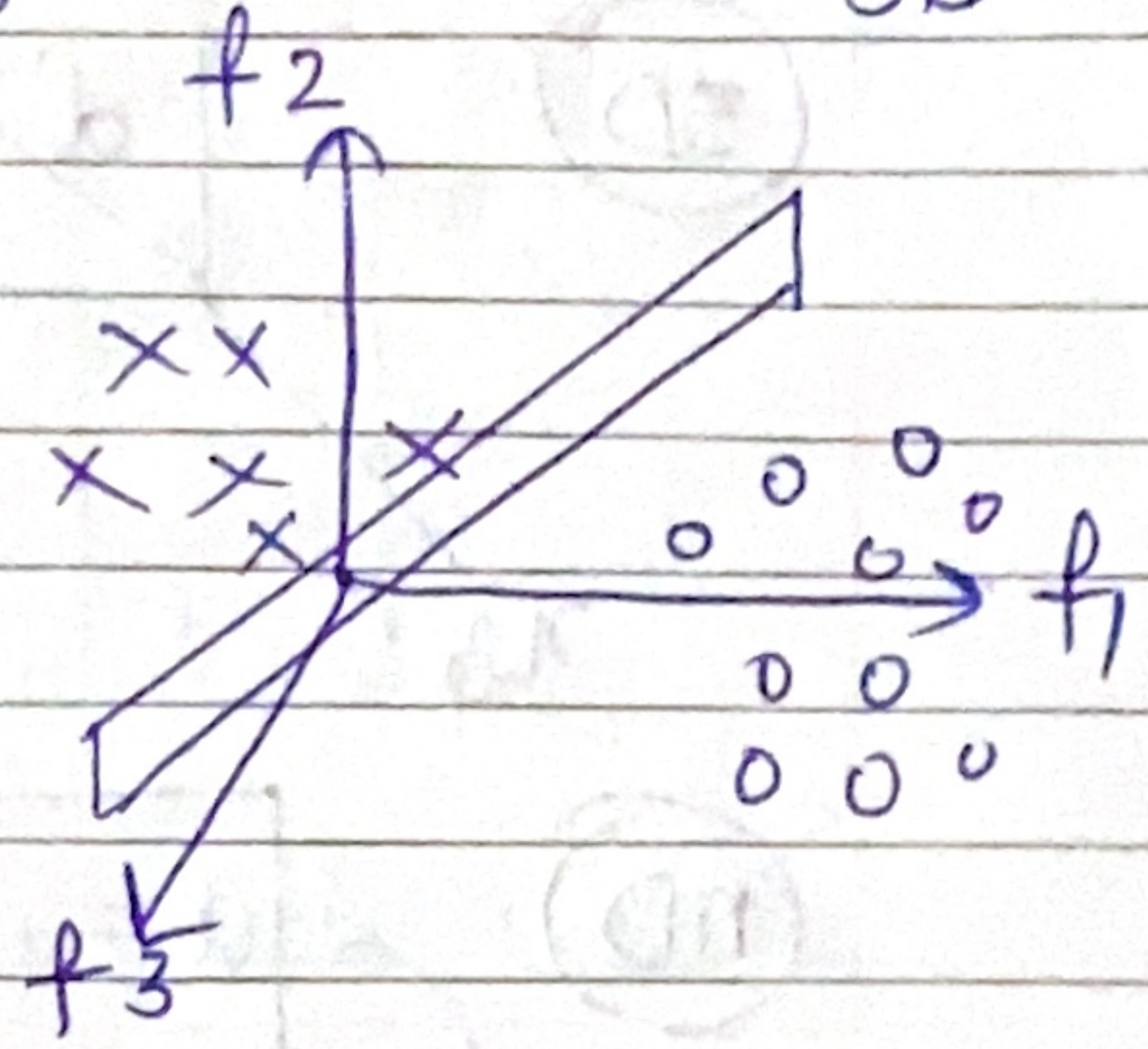
Date :

Page No.

Linear Algebra



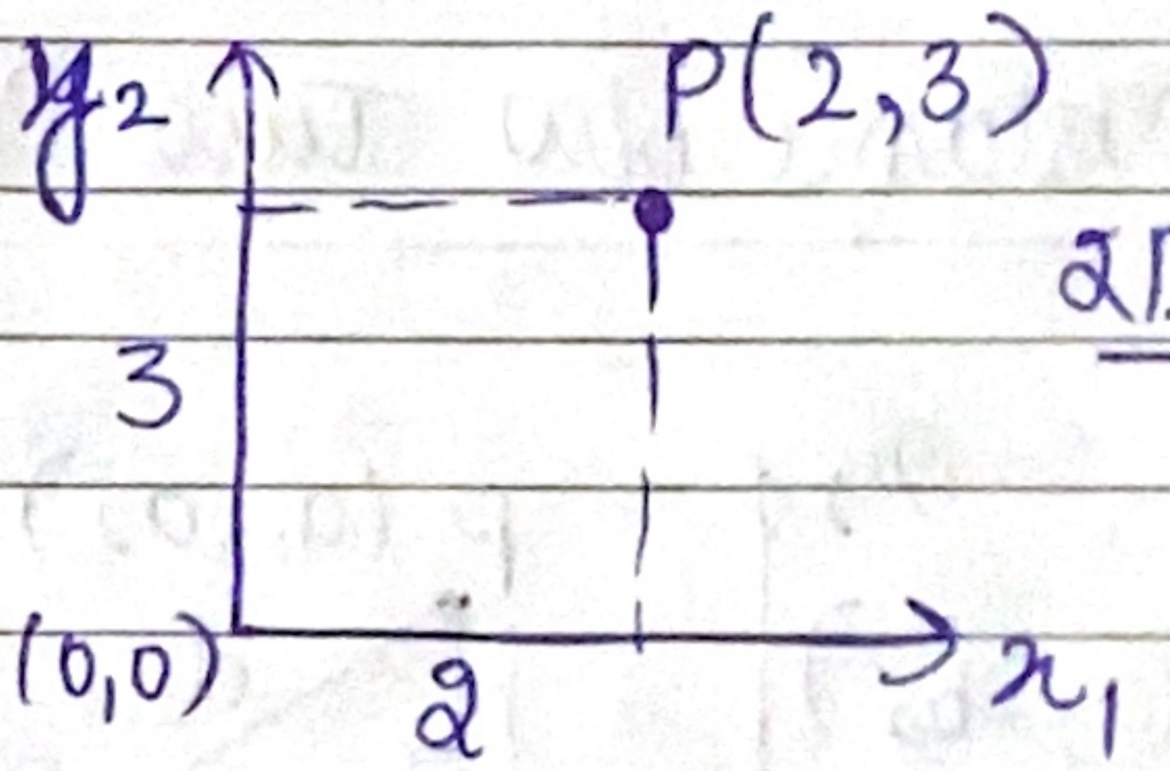


[4D, 5D, 10-D - - - nD]

Point / Vector

P = [2, 3]

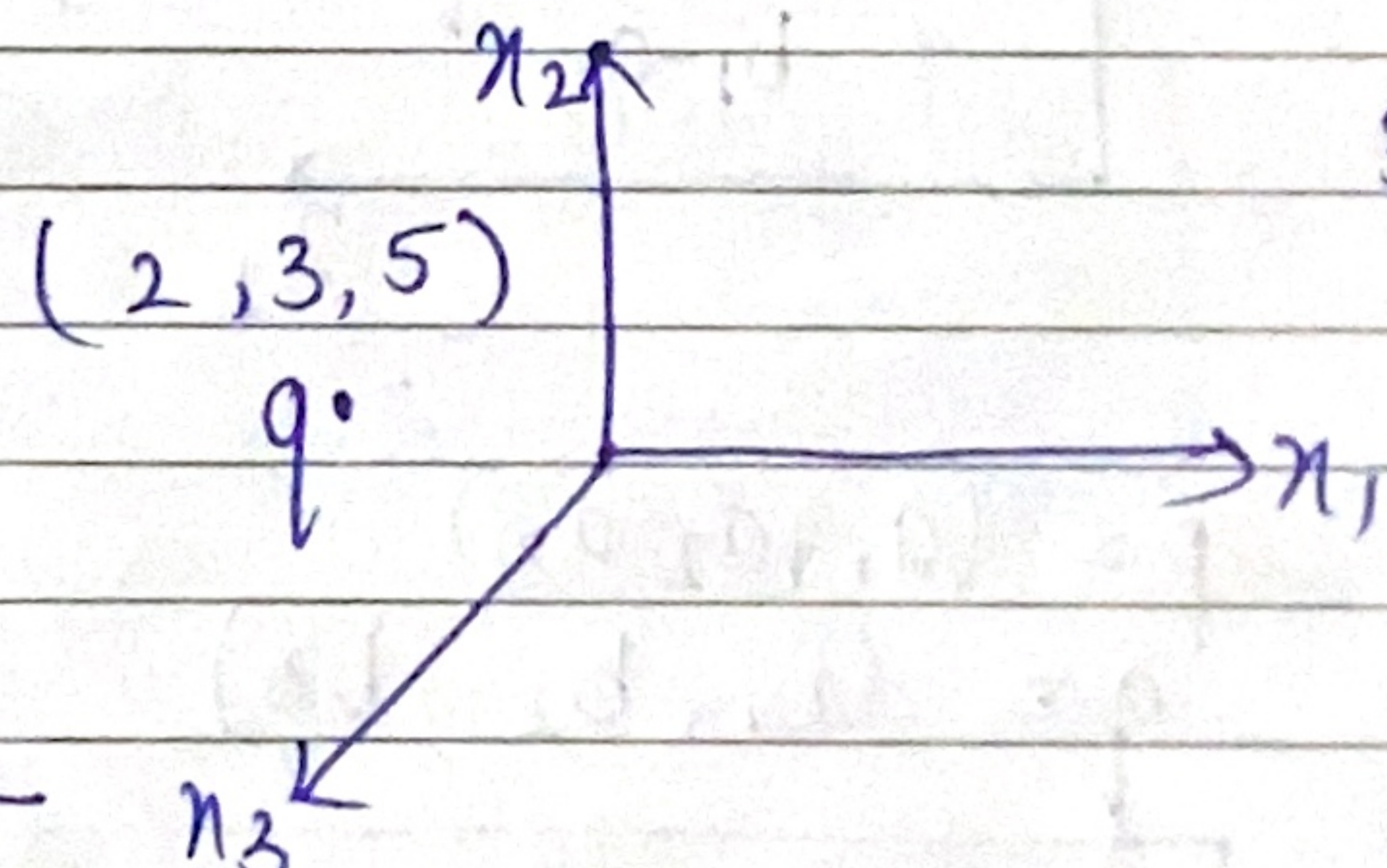
2D vector



2D Space

q = [2, 3, 5]

3D vector

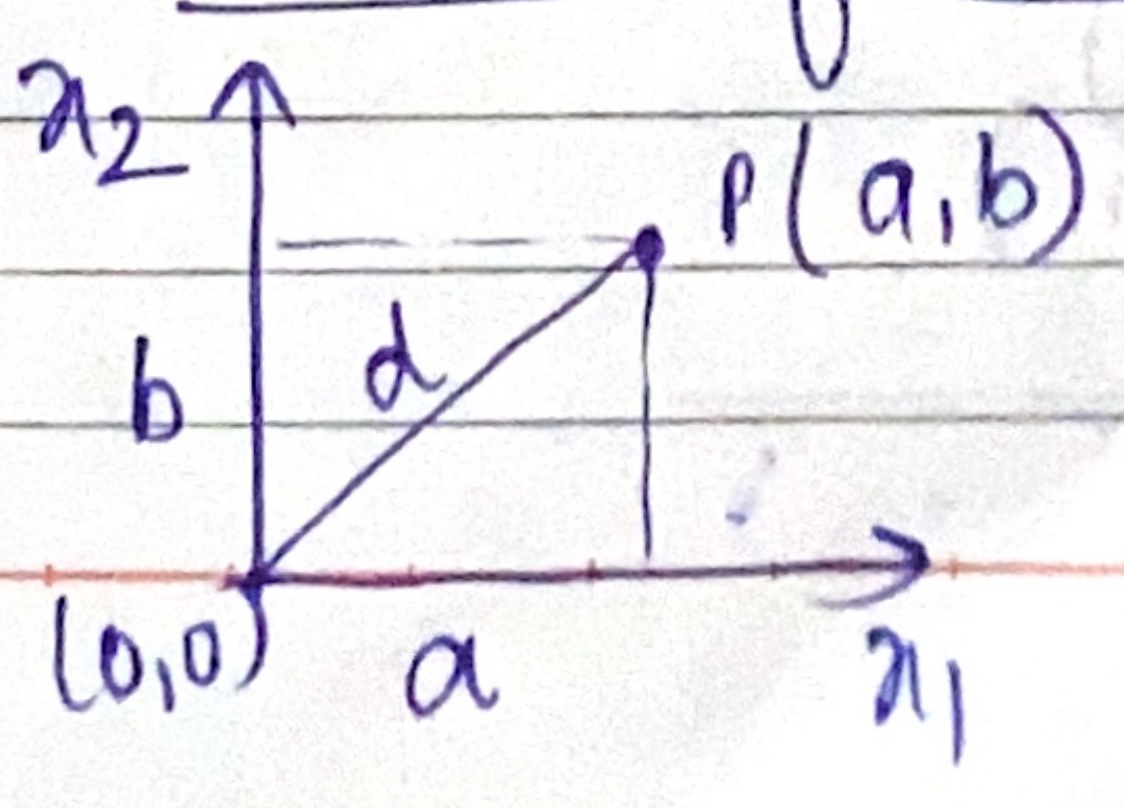


3D Space

n - dim Point

x = [2, 3, 4, 1, 5 - - - - n]

Distance of Point from origin



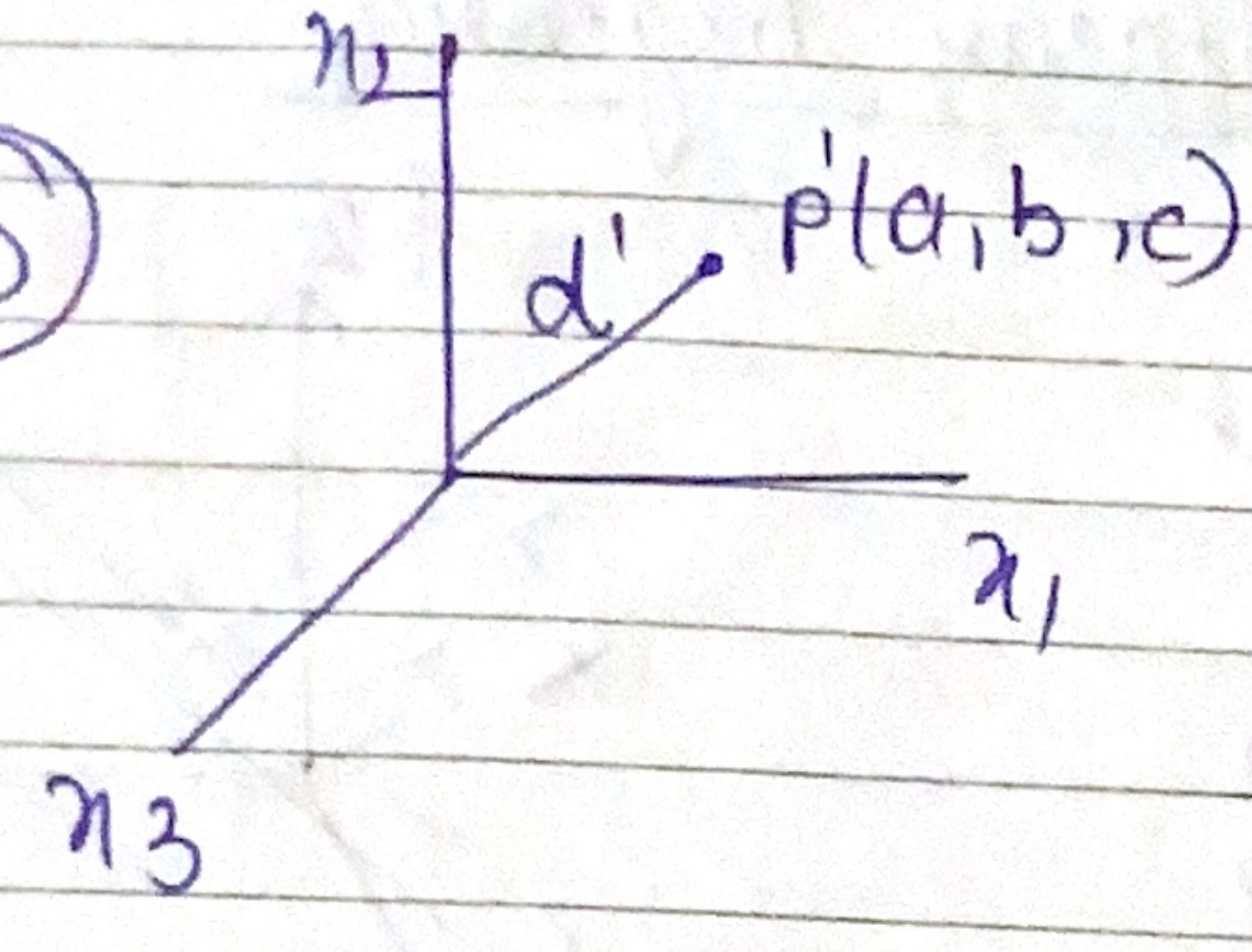
d = dist b/w origin & P

$\boxed{d = \sqrt{a^2 + b^2}}$ (Pythagorus)

Date :

Page No.

3D



$$\boxed{d' = \sqrt{a^2 + b^2 + c^2}}$$

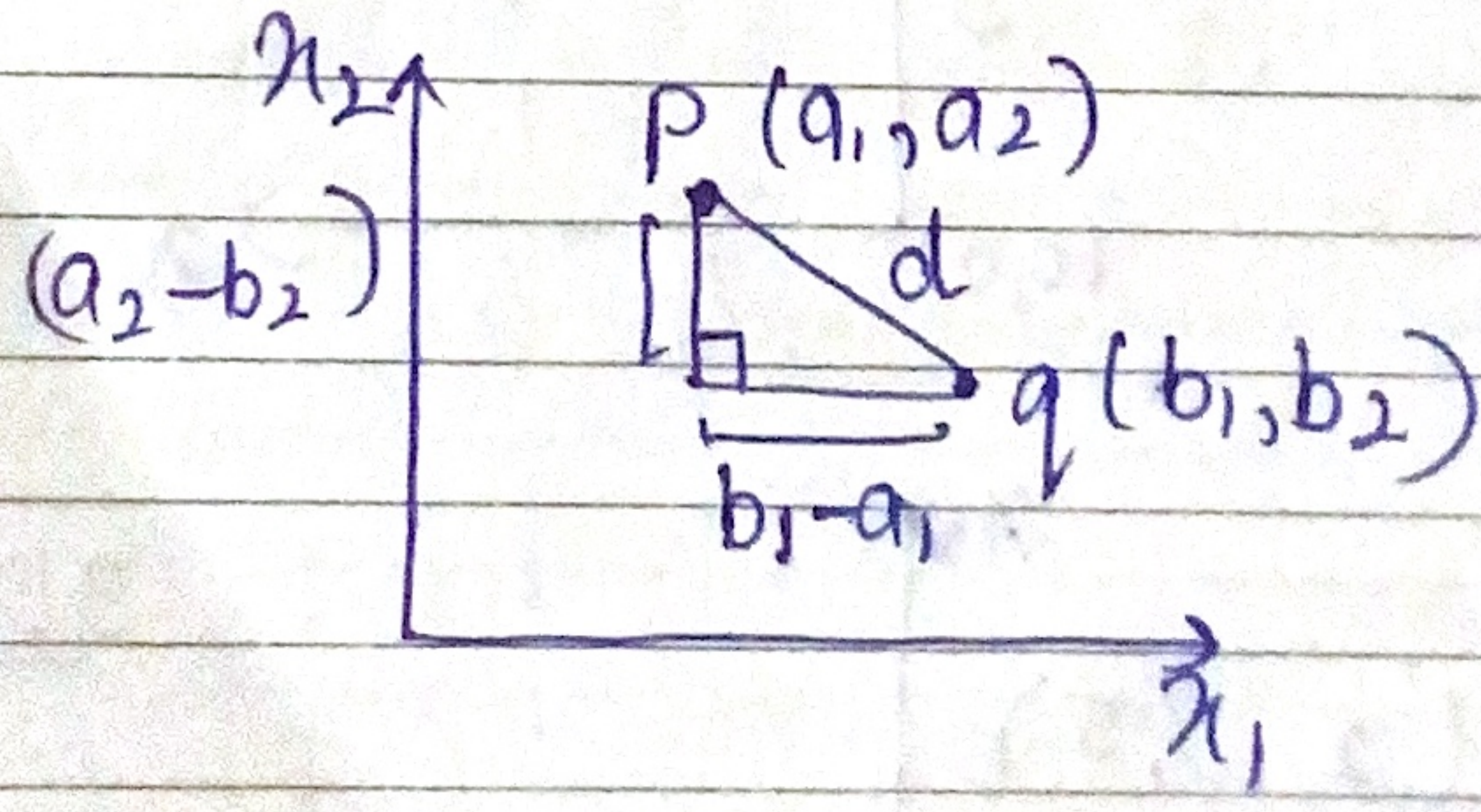
nD

$$\boxed{d = \sqrt{a_1^2 + a_2^2 + \dots a_n^2}}$$

$$p = [a_1 a_2, a_3 \dots a_n]$$

Distance b/w Two Points

2D



$$\boxed{d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2^2)}}$$

3D

$$p = (a_1, a_2, a_3)$$

$$q = (b_1, b_2, b_3)$$

$$\boxed{d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}}$$

nD

$$p = (a_1, a_2 \dots a_n)$$

$$q = (b_1, b_2 \dots b_n)$$

$$d_{pq} = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

Date :
Page No.

Row Vector $A = [a_1, a_2, a_3 - - - a_n]_{1\times n}$

Column Vector $B = \begin{bmatrix} b_1 \\ b_2 \\ | \\ | \\ | \\ b_n \end{bmatrix}_{n\times 1}$

Dot Product & Angle b/w Two Vectors

Addition $a = [a_1, a_2 - - - a_n]$
$b = [b_1, b_2 - - - - b_n]$

$c = a+b = [a_1+b_1, a_2+b_2 - - - a_n+b_n]$

Multiplication Dot Product   Cross Product ✗

$a \cdot b = a_1b_1 + a_2b_2 + a_3b_3 + - - a_nb_n$

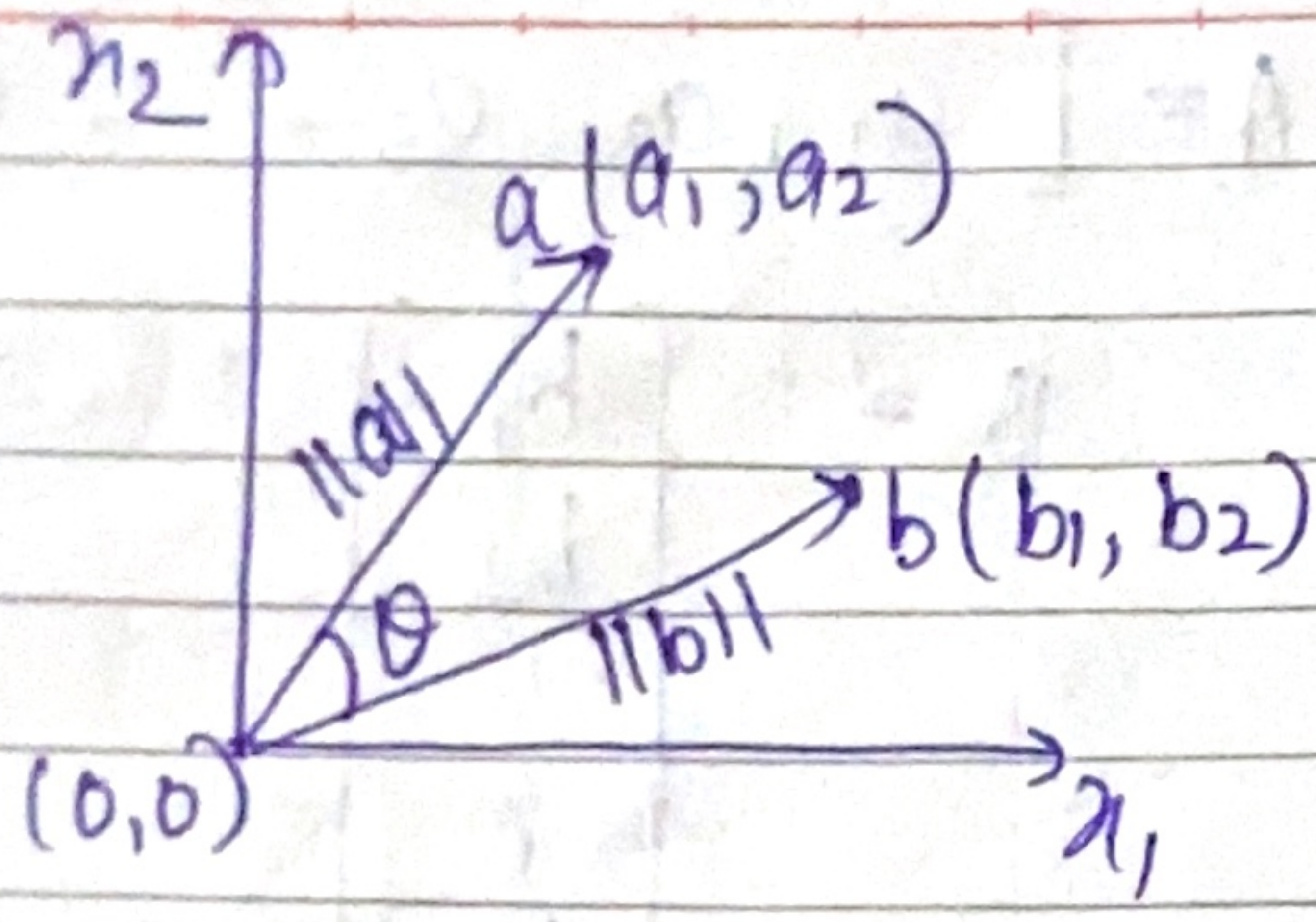
$= [a_1, a_2, - - - a_n]_{1\times n} \begin{bmatrix} b_1 \\ b_2 \\ | \\ b_n \end{bmatrix}_{n\times 1}$ $n=n$

Transpose

$a = \begin{bmatrix} a_1 \\ a_2 \\ | \\ a_n \end{bmatrix}$ $a^T = [a_1, a_2 - - - a_n]$

$a \cdot b = a^T b$
$= \sum_{i=1}^{n} a_i b_i$

Date :
Page No.



$$\boxed{a \cdot b = \|a\| \, \|b\| \cos\theta}$$

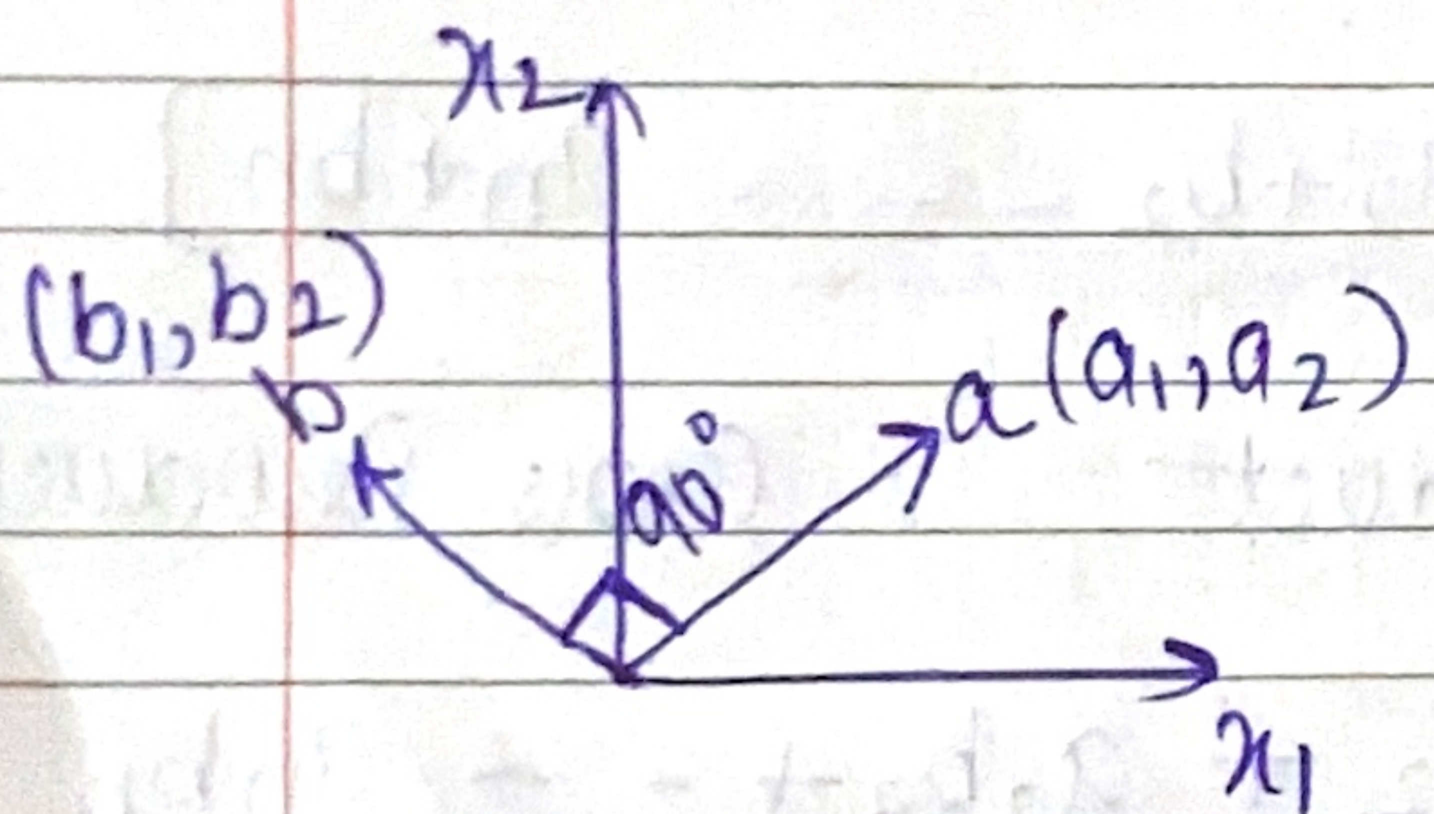
↓

lenght of a
(dist of a from origin)

$$\|a\| = \sqrt{a_1^2 + a_2^2}$$

$$a \cdot b = a_1 b_1 + a_2 b_2 = \|a\| \, \|b\| \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left\{\frac{a_1 b_1 + a_2 b_2}{\|a\| \, \|b\|}\right\}}$$



$a \cdot b = \|a\| \, \|b\| \cos 90°$

$a \cdot b = 0$

$\vec{a}$ is $\perp$ to $\vec{b}$

$a = [a_1, a_2 \; - - - \; a_n]$

$b = [b_1, b_2 \; - - - \; b_n]$

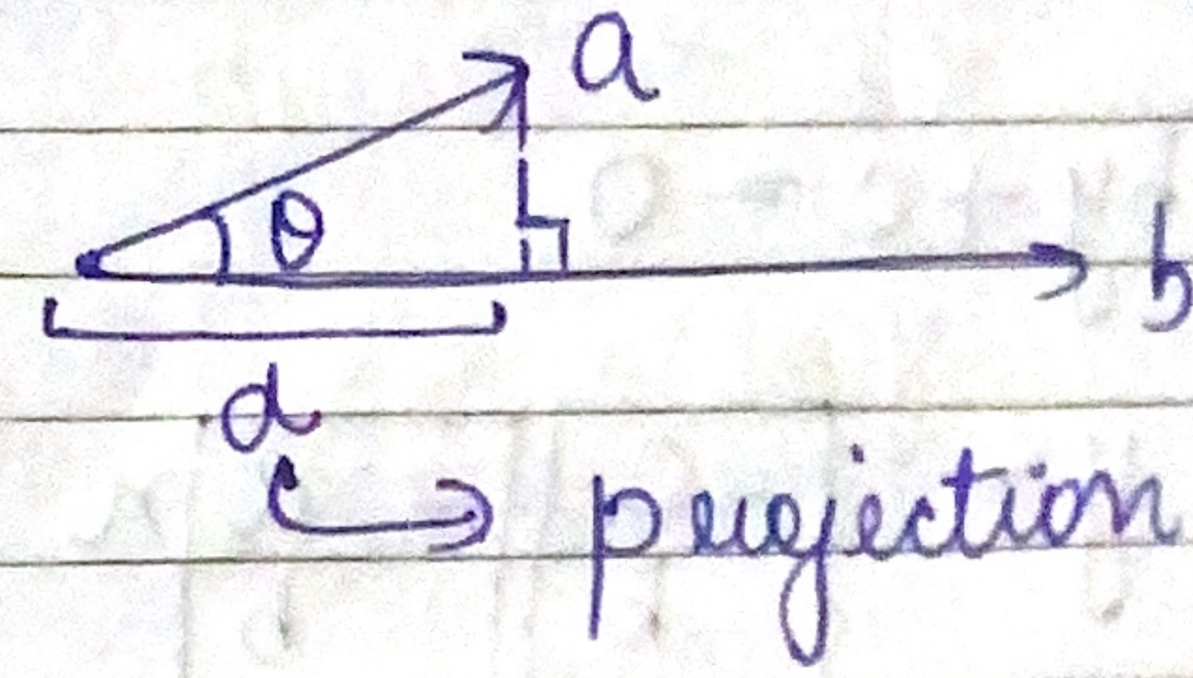
$$a \cdot b = \|a\| \, \|b\| \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{\sum_{i=1}^{n} a_i b_i}{\|a\| \, \|b\|}\right)}$$

$$\boxed{a \cdot b = \sum_{i=1}^{n} a_i b_i = 0 \implies a \perp b}$$

$$a \cdot a = a_1 a_1 + a_2 a_2 + \cdots a_n a_n$$
$$= a_1^2 + a_2^2 + \cdots a_n^2$$
$$= \|a\|^2$$

Projection



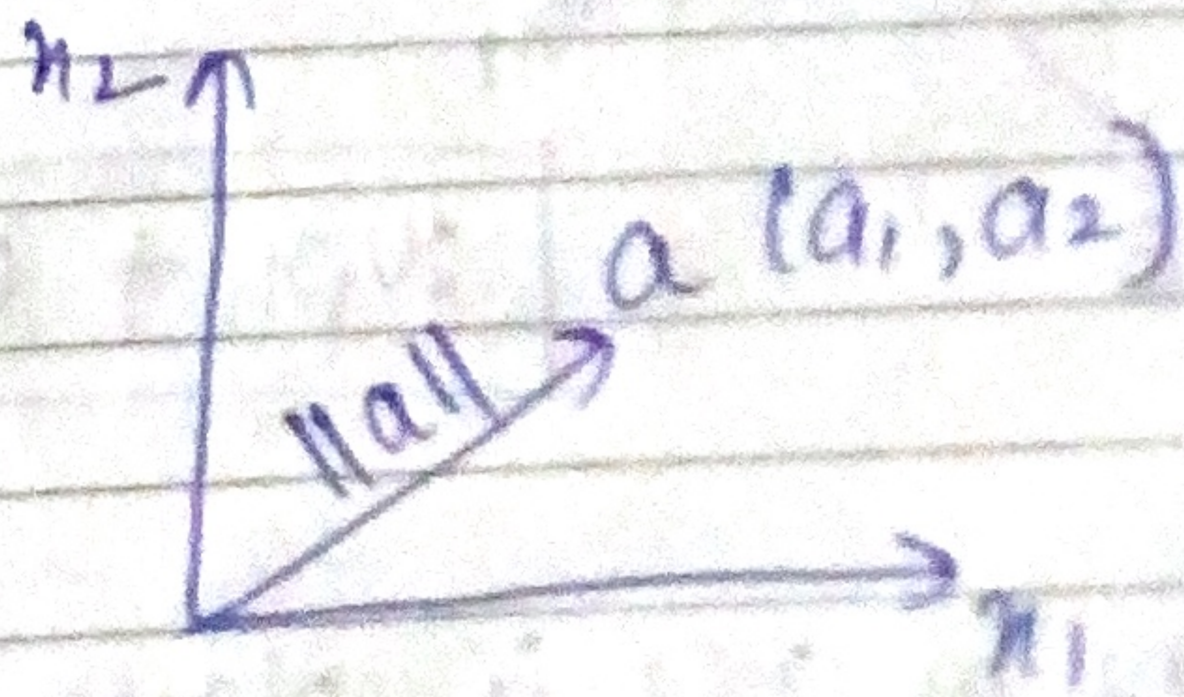
Projection of a on b $= d = \|a\| \cos\theta$ —— ①

$$a \cdot b = \sum_{i=1}^{n} a_i b_i = \|a\| \|b\| \cos\theta$$

$$\boxed{d = \frac{a \cdot b}{\|b\|}} = \frac{\|a\| \cancel{\|b\|} \cos\theta}{\cancel{\|b\|}} = \|a\| \cos\theta$$

Unit Vector

$$\hat{a} = \frac{a}{\|a\|}$$



① $\hat{a}$ is in same direction as a

② $\|\hat{a}\| = 1$

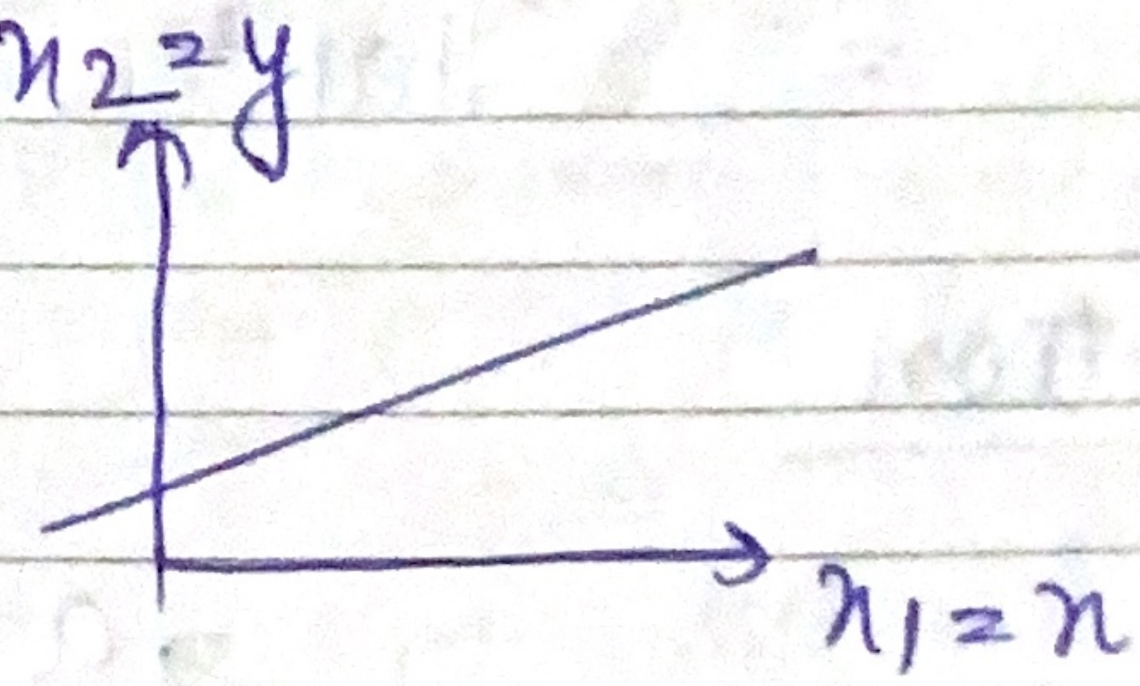
Equation of Line

Date :
Page No.

Line :

(2D)

$y = mx + c$



$ax + by + c = 0$

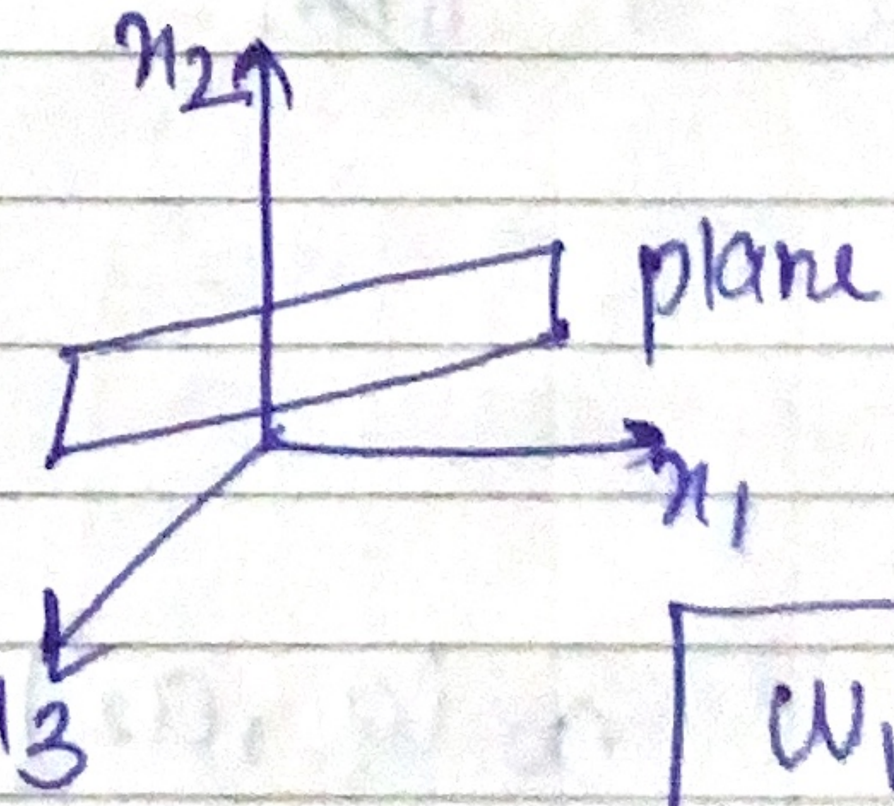
$y = \boxed{\frac{-c}{b}} \boxed{-\frac{a}{b}} x \qquad y = c + mx$

$\frac{-c}{b} \downarrow c \qquad -\frac{a}{b} \downarrow m$

$ax_1 + bx_2 + c = 0$

$w_1x_1 + w_2x_2 + w_0 = 0 \longrightarrow 2D$

(3D)



plane $= ax + by + cz + d = 0$

$$\boxed{w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0}$$

Line in 2D is plane in 3D

(nD) Hyperplane

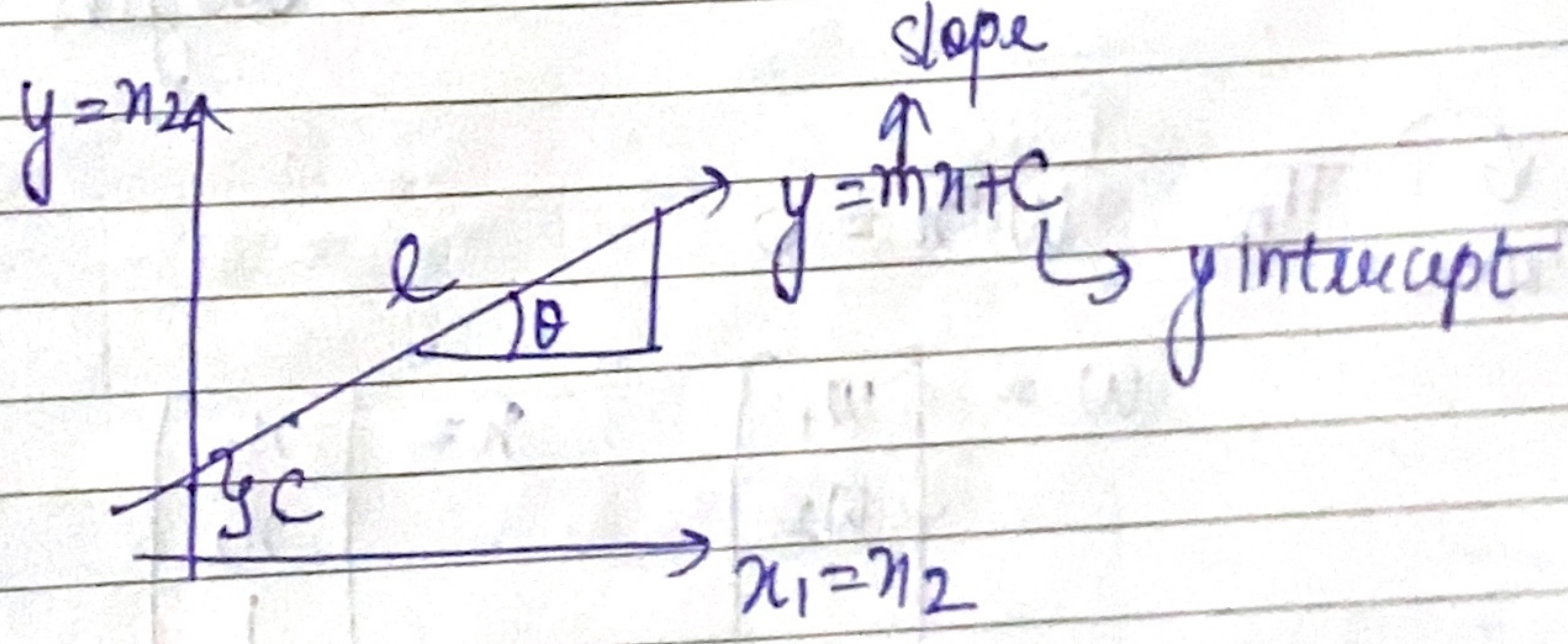
$w_0 + w_1x_1 + w_2x_2 + \cdots w_nx_n = 0$

$$\boxed{w_0 + \sum_{i=1}^{n} w_ix_i = 0}$$

Vector notation $\leftarrow$ $w_0 + \underbrace{[w_1, w_2 \dots w_n]}_{W_{1\times n}} \overbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}^{x_{n\times 1}} = 0$

$$W_{n\times 1} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}_{n\times 1} \qquad X_{n\times 1} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

π_n: $\boxed{w_0 + w^T x = 0}$



(2D) $w_1 x_1 + w_2 x_2 + w_0 = 0$

$$x_2 = -\frac{w_0}{w_2} - \frac{w_1 x_1}{w_2}$$

$\downarrow$

$$y = c + mx$$

If l passes through origin $\Rightarrow c = 0$

Date :

Page No.

& pass through origin

line : 2D = $w_1x_1 + w_2x_2 = 0$

plane : 3D = $w_1x_1 + w_2x_2 + w_3x_3 = 0$

Hyperplane : nD = $w_1x_1 + w_2x_2 + \cdots w_nx_n = 0$

$\boxed{w^Tx = 0}$ equation of plane passing through origin

$\boxed{w^Tx + w_0 = 0}$ not passing through origin

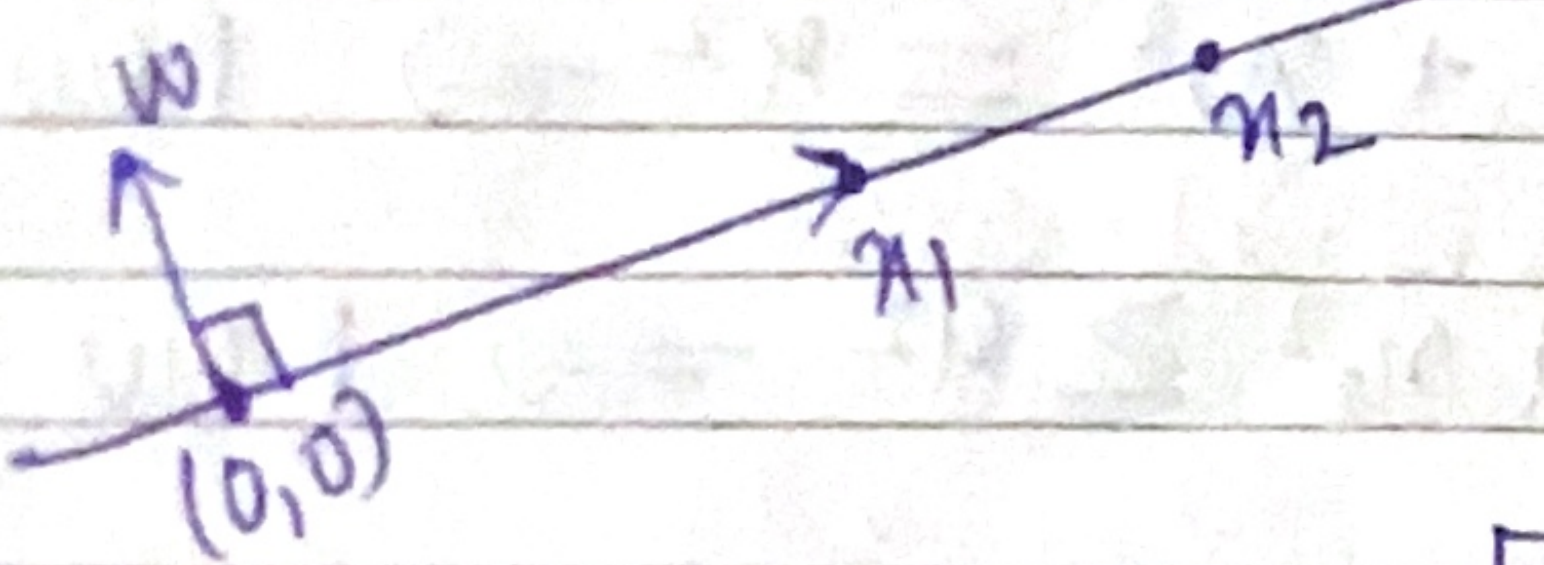
Plane π_n : $w^Tx = 0$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$w \cdot x = w^Tx = \|w\| \|x\| \cos\theta_{wx} = 0$$

$$w \perp x \Rightarrow \theta_{wx} = 90^\circ$$

If $w \perp \pi$ then $w.x_1 = 0$ for all $x_i \in \pi$

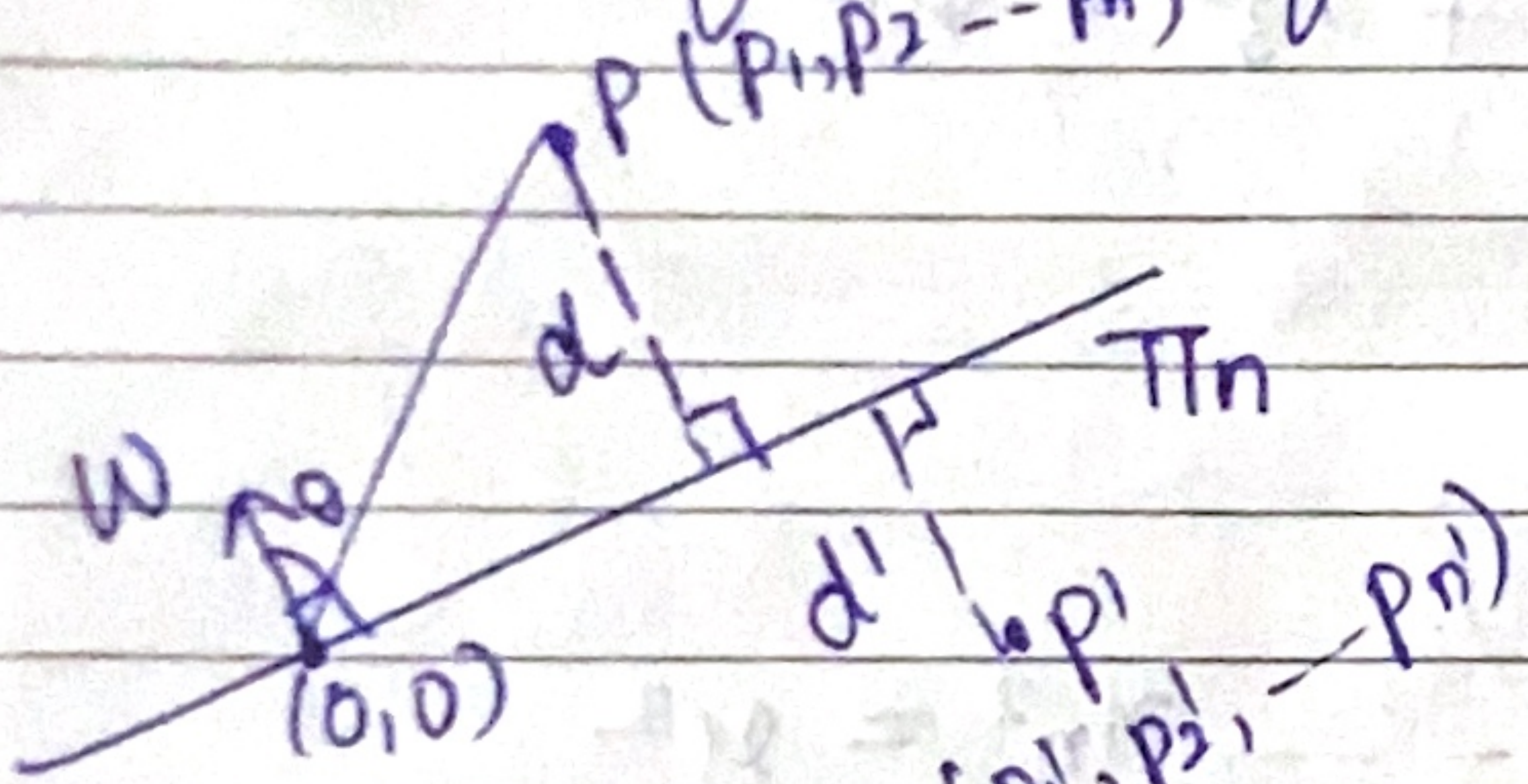


$$w.x_1 = 0$$

$$\boxed{w^T x = 0}$$

$$\hat{w} = \frac{w}{||w||} \qquad \hat{w}.x_i = 0 \;\forall\, x_i \in \pi$$

Distance of Point from Plane - Hyperplane



$$w^T x = 0$$

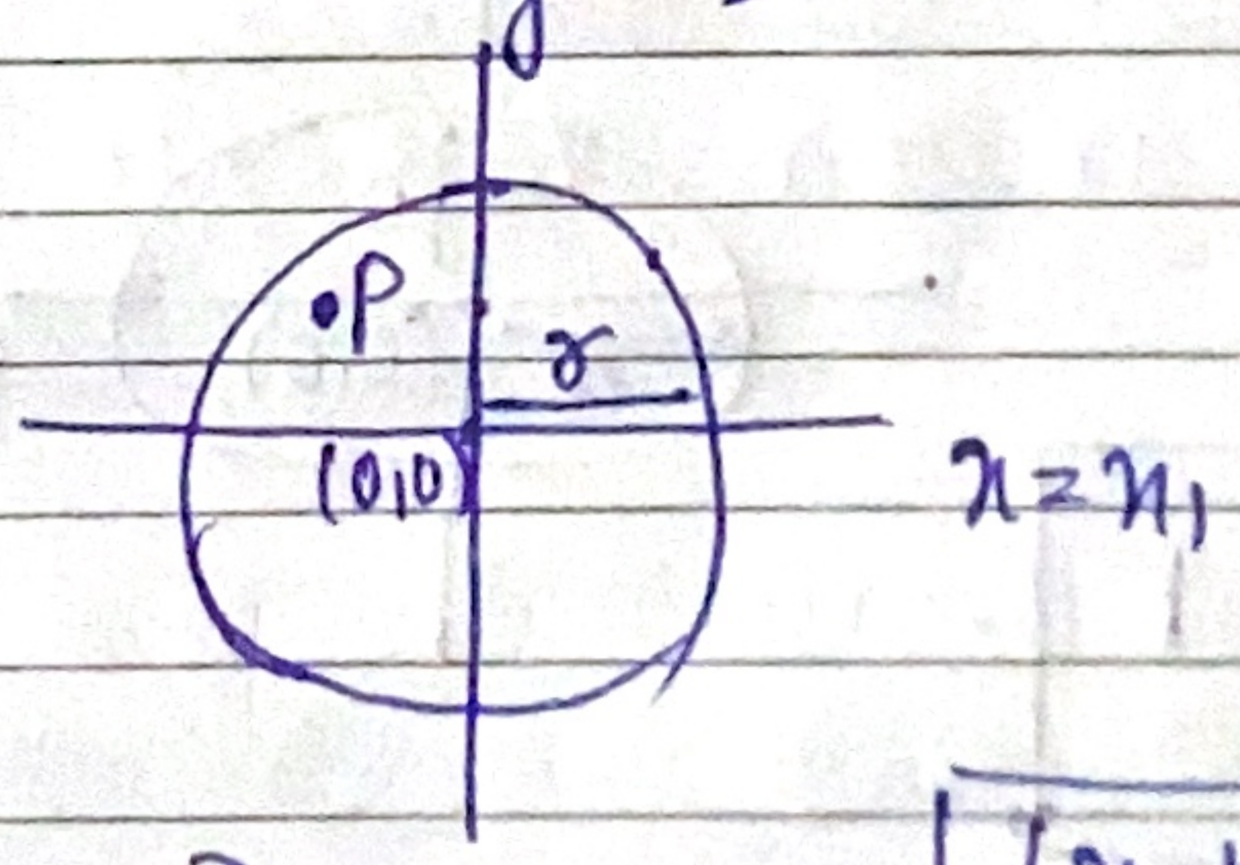
$$w_{n\times 1} \qquad P_{n\times 1}$$

$$\boxed{d = \frac{w^T P}{||w||}}$$

$$\boxed{d = \frac{w.P}{||w||}} \quad +ve \quad \theta < 90^\circ$$

$$\boxed{d' = \frac{w.P'}{||w||}} \quad -ve \quad \theta > 90^\circ$$

Circle



$P(x_1, x_2)$

Centre is origin

$$\boxed{x^2 + y^2 = r^2}$$

(h, k)

$$\boxed{(x - h^2) + (y - k)^2 = r^2}$$

(2D)

$x_1^2 + x_2^2 \leq r^2 \Rightarrow$ P lies inside circle

$x_1^2 + x_2^2 \geq r^2 \Rightarrow$ P lies outside

$x_1^2 + x_2^2 = r^2 \Rightarrow$ P lies on circle

(3D) x_1, x_2, x_3

Sphere

$$x_1^2 + x_2^2 + x_3^3 = r^2$$

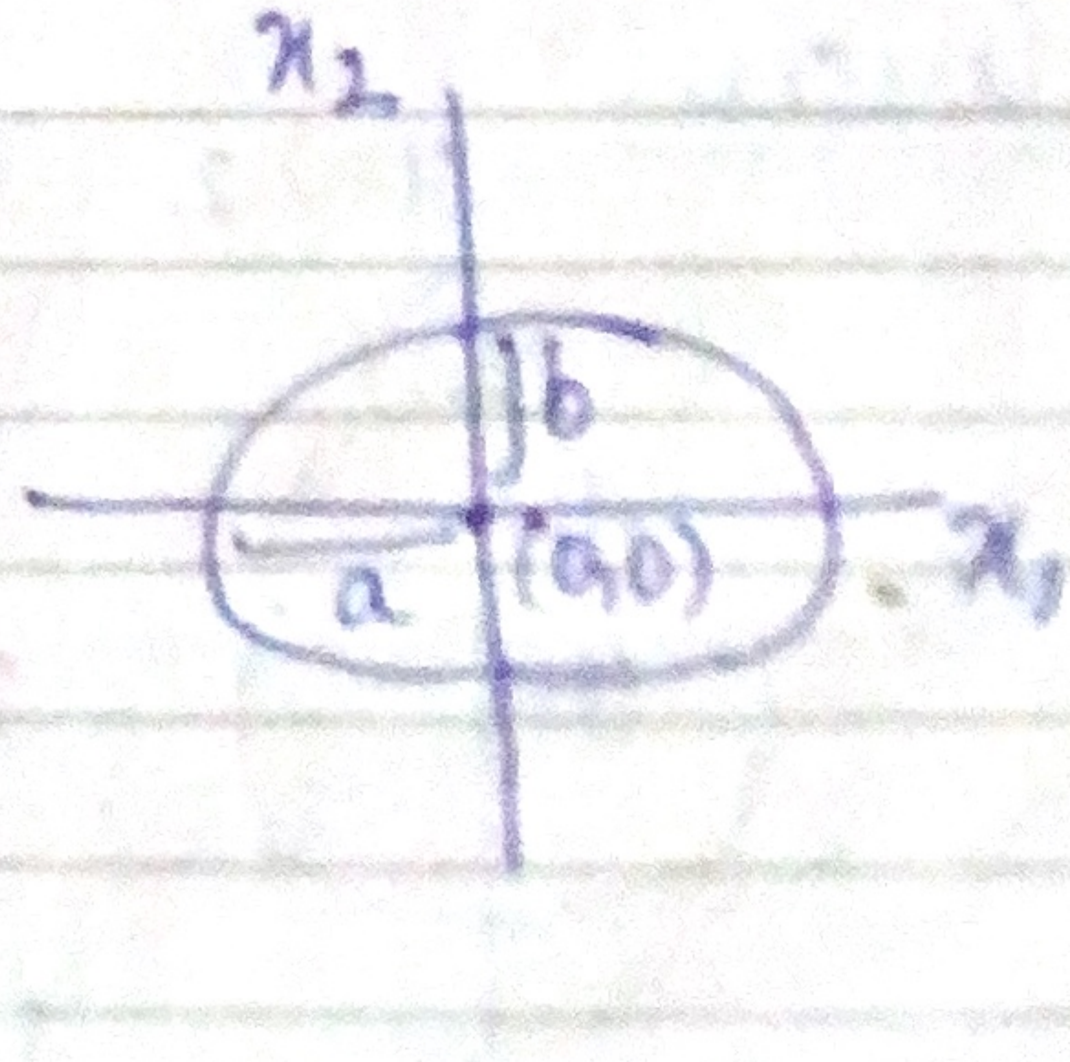
(nD) $x_1, x_2 - - x_n$

Hypersphere $x_1^2 + x_2^2 + - - - x_n^2 = r^2$

$$\sum_{i=1}^{n} x_i^2 = r^2$$

$\sum_{i=1}^{n} x_i^2 < r^2$ P lies inside hypersphere

Ellipse



(2D)

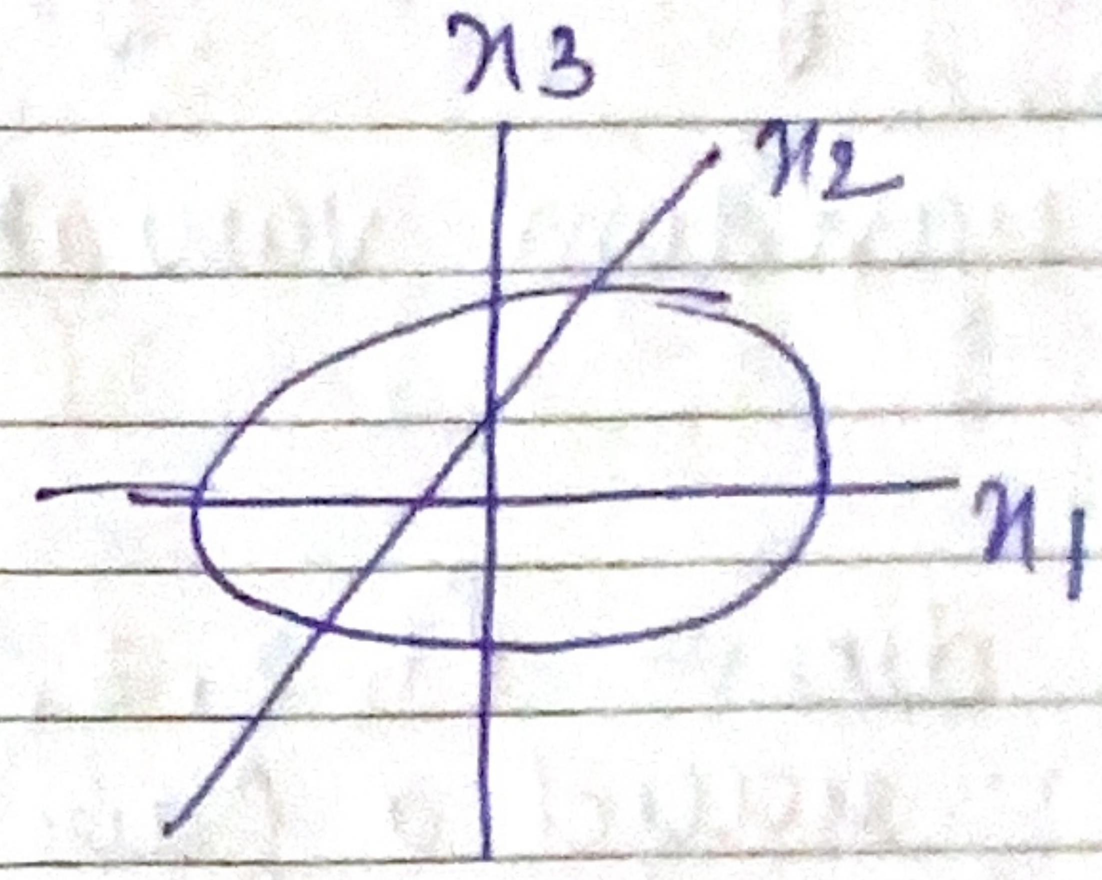
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1 \quad \text{P lies inside ellipse}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} > 1 \quad \text{P lies outside ellipse}$$

(3D) ellipsoid

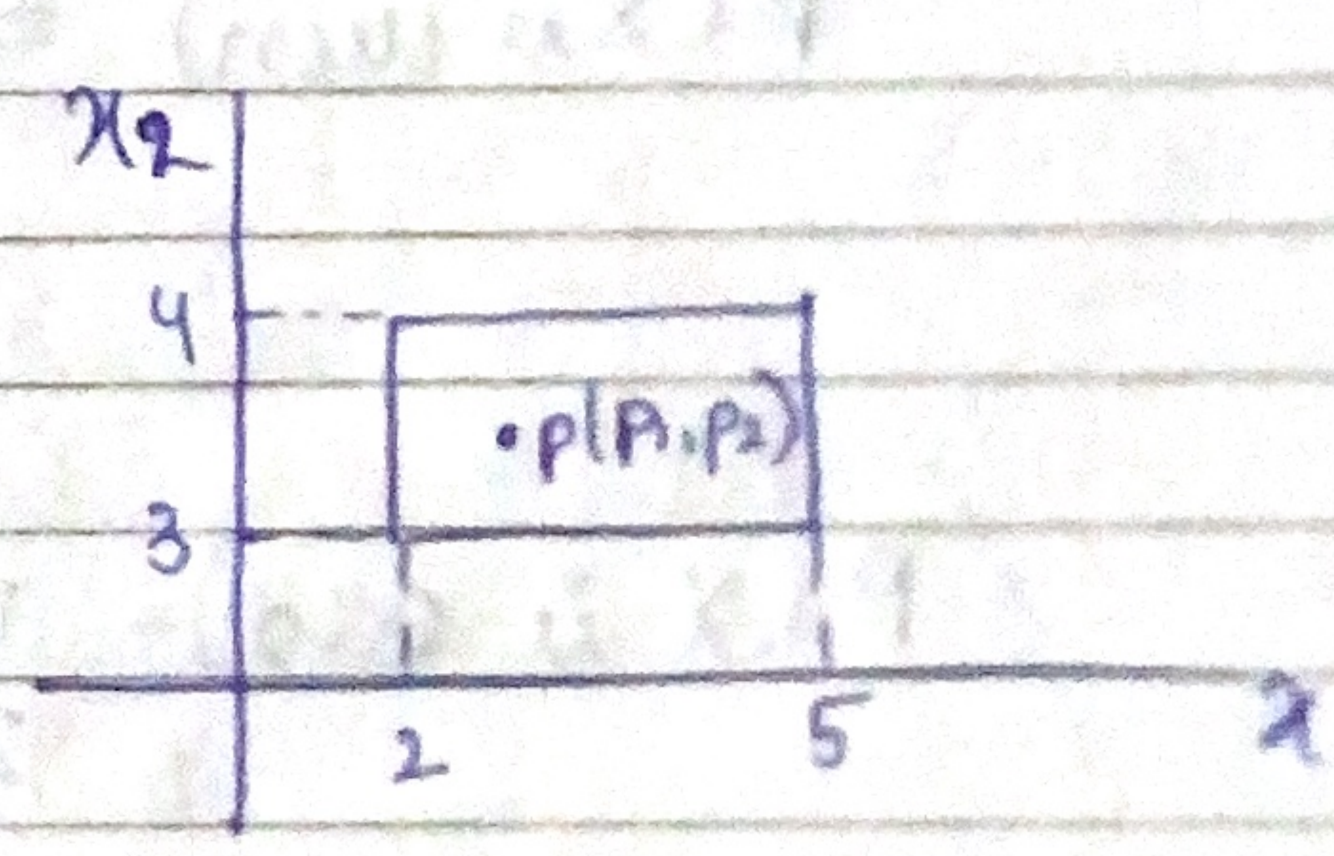
$$\boxed{\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1}$$



(nD)

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \cdots + \frac{x_n^2}{a_n^2} = 1 \text{ — hyperellipsoid}$$

Square, Rectangle



(2D)

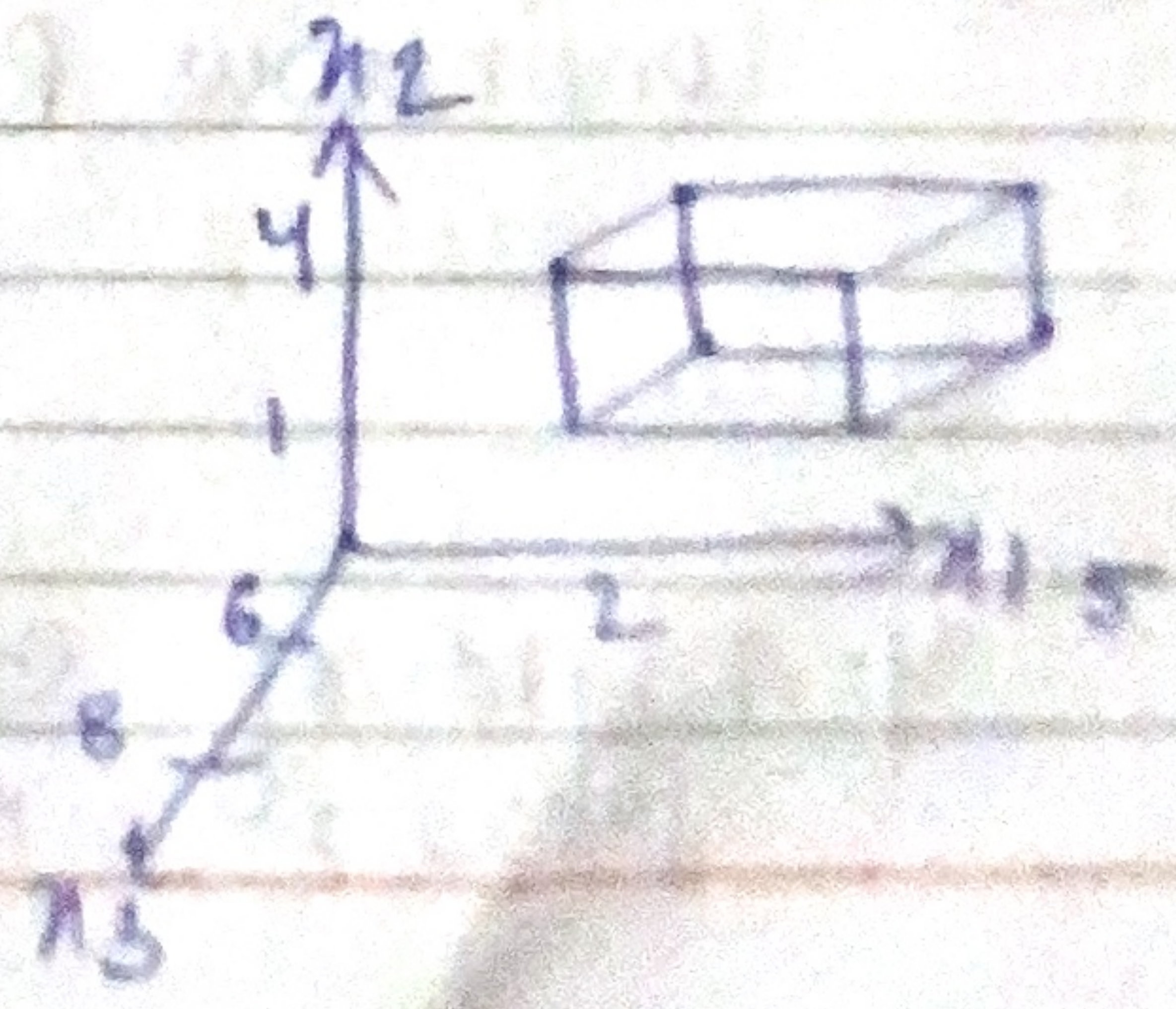
if $p_1 < 5$ & $p_1 > 2$

if $p_2 > 3$ & $p_2 < 4$

then p lies inside rectangle

axis parallel rectangle

(3D) Cuboid



(nD) hypercuboid