HW 1
MFE 408: Fixed Income
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Qn1

Assume the face value of the bond at maturity is 100

```
ytm <- 0.01 # semiannual payment
cf <- c(rep(5,49),105) # cf of 5 dollar every 6 month

discount <- 1 / (ytm + 1)^seq(1,50,1)
price <- sum(cf * discount)
cat("Bond price is ", price)
```

```
## Bond price is  256.7845
```

Qn2

```
cf <- 6.25
discount <- 0.07

price <- cf/(discount/2)
cat("willing to pay ", price)
```

```
## willing to pay  178.5714
```

Qn3

Assume the first month you receive both downpayment and monthly payment

```
# figure out the cash flow
cf <- rep(14000,120)
r <- 0.04/12
d <- (1+r)^(seq(119,0,-1))
accSum <- sum(d * cf) + 1000000*(1+r)^120
cat("Accummulative sum is ", accSum)
```

```
## Accummulative sum is  3552330
```

Qn4

```
g <- 0.03
div <- 10
r <- 0.12
p <- div /(r - g)
cat("Willing to pay: ", p)
```

```
## Willing to pay:  111.1111
```

Qn 5

```
cf1 <- 1000
r <- 0.1
cf <- cf1 * (1+r)^seq(0,5,1)
cf <- c(cf, rep(1610.51,24))

discount <- 1/(1.035^(seq(1,30,1)))
pv <- sum(cf * discount)

cat("PV: ", pv)
```

```
## PV:  27826.05
```

Qn 6

```
annuity <- 10000
# 25 years in total
n <- 25
discount <- 1/1.03^(seq(1,n,1))
cf <- rep(annuity, n)
pv_21 <- sum(cf * discount)
pv <- pv_21 /((1.03)^20)
cat("Annuity PV: ", pv)
```

```
## Annuity PV:  96412.38
```

Qn 7

```
library(FinCal)
FV <- 1000
r <- 0.03
cf1 <- c(rep(FV*r, 39),1030)
cf <- c(-893.22, cf1)

cat("YTM is: ", irr(cf)*2)
```

```
## YTM is:  0.06996525
```

Arbitrage Case 1

To find the arbitrage opportunity, we need to find 2 treasury bonds, 1 treasury strip principle bond expiring on the same day.

Step 1:Create a replicated bond of the smaller coupon bond, using the strip coupon bond and the higher coupon bond.

Step 2: Compare the price of the replicated and actual bond, long the cheaper and short the expensive

We will illustrate with one example:

From the slides, we know that current Date: 9th Jan 2015 Bonds with Maturity Date: 11/15/22 Previous coupon payment: 11/15/2014

Treasurey strip: 86.15

Find clean price Coupon Bond A with rate 1.625: 98.57812 Coupon Bond B with rate 7.625: 143.0625

Find dirty price

```
bondAPrice <- 98.57812
bondARate <- 1.625

bondBPrice <- 143.0625
bondBRate <- 7.625

daysSincePayment <- as.numeric(as.Date('01/09/2015','%m/%d/%Y') - as.Date('11/15/2014','%m/%d/%Y'))
daysInBetweenPayments <- 182

dirty_priceA <- (daysSincePayment/daysInBetweenPayments) * (bondARate/2) + bondAPrice
dirty_priceB <- (daysSincePayment/daysInBetweenPayments) * (bondBRate/2) + bondBPrice
cat("Dirty price for A: ", dirty_priceA,"\n")
```

```
## Dirty price for A:  98.82366
```

```
cat("Dirty price for B: ", dirty_priceB)
```

```
## Dirty price for B:  144.2146
```

To generate a low coupon bond

$$a * 0 + (1 - a) * 7.625 = 1.625$$

```
a <- (7.626 -1.625) / 7.625
cat("a: ",a,"\n")
```

```
## a:  0.7870164
```

```
bondPrice <- a * 86.15 + (1-a) * dirty_priceB
cat("Bond price for the replicated bond is: ",bondPrice,"\n")
```

```
## Bond price for the replicated bond is:  98.51681
```

```
cat("This price is lower than the dirty price for bond with coupon rate 1.625, which is ",dirty_priceA)
```

```
## This price is lower than the dirty price for bond with coupon rate 1.625, which is  98.82366
```

Hence, We long the replicated bond using 0 coupon and 7.625 coupon bond, and short actual 1.625 coupon bond to realize a profit of 0.31

Arbitrage Case 2