HW6 Fixed Income

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May 22, 2019

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Question 1

```
suppressMessages(library(data.table))
suppressMessages(library(readxl))
suppressMessages(library(xts))

bdtree = read_xls('/Users/Harley/Desktop/bdttree.xls',col_names = F)

pfilea = read_xlsx('/Users/Harley/Desktop/pfilea.xlsx',col_names = F)
pfilea = sapply(pfilea$X__1[1:30], as.numeric)
voldat = read_xlsx('/Users/Harley/Desktop/voldat.xlsx',col_names = F)
voldat = sapply(voldat$X__1[1:29], as.numeric)
voldat = c(0,voldat)

rate_tree = data.frame(matrix(ncol = 30, nrow = 30))
discount = data.frame(matrix(ncol = 30, nrow = 30))

#Building the rate tree
for (i in 1:30) {
  if (i == 1) {
    remainder = 999
    r_star = 0
    while (remainder > 0) {
      r_star = r_star+0.001
      remainder = 1/(1+r_star/2)-pfilea[1]
    }
    rate_tree[1,1] = r_star
  }

  if (i != 1) {
    r_star = 0
    remainder = 999
    while (remainder > 0) {
      r_star = r_star+0.001
      for (j in 1:i) {
        rate_tree[j,i] = r_star*exp(-2*(j-1)*voldat[i]*sqrt(0.5))
      }

      for (j in i:2) {
        for (k in 1:(j-1)) {
          if (j == i) {
            discount[k,j] = 0.5/(1+rate_tree[k,j]/2)+0.5/(1+rate_tree[k+1,j]/2)
          }
          if (j != i) {
            discount[k,j] = (0.5*discount[k,j+1])/(1+rate_tree[k,j]/2)+(0.5*discount[k+1,j+1])/(1+rate_
```

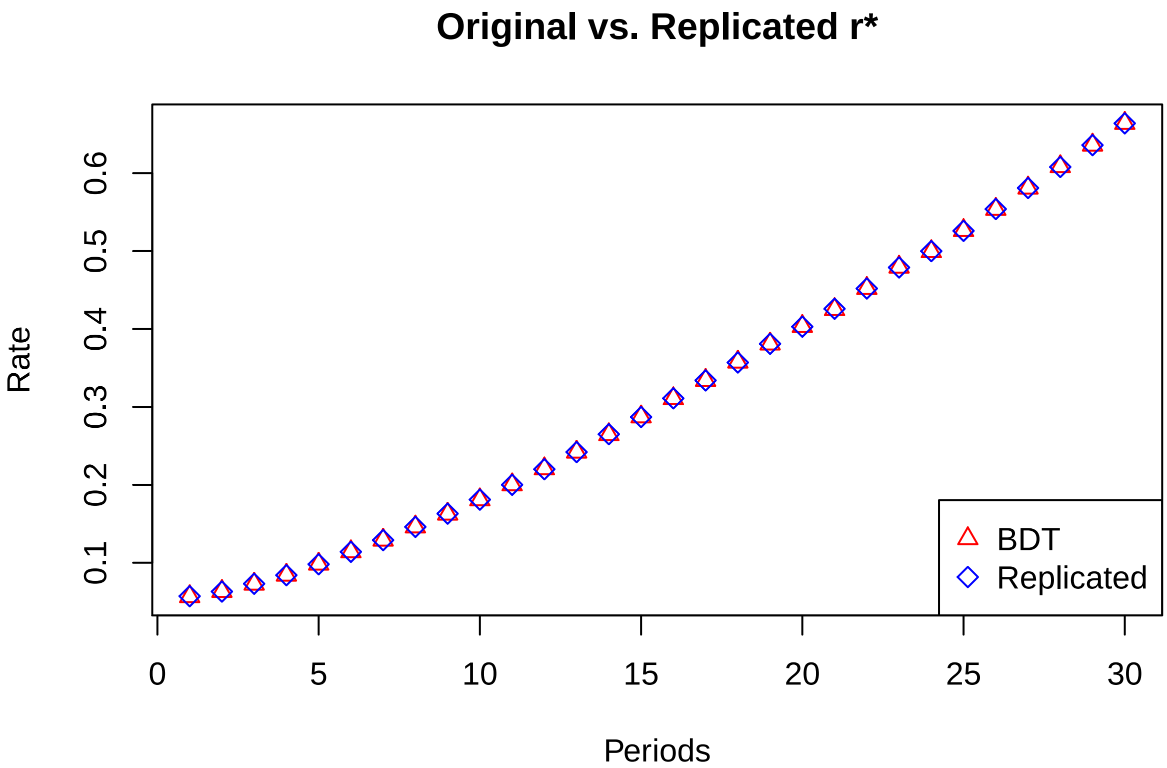
```r
          }

        }
      }
      discount[1,1] = discount[1,2]/(1+rate_tree[1,1]/2)
      remainder = discount[1,1]-pfilea[i]
    }
  }
}

print(rate_tree, digits = 2)
```

```
##       X1    X2    X3    X4    X5    X6    X7    X8    X9   X10   X11   X12
## 1  0.057 0.063 0.073 0.084 0.098 0.114 0.129 0.146 0.163 0.181 0.200 0.220
## 2     NA 0.055 0.062 0.069 0.079 0.091 0.103 0.116 0.130 0.144 0.160 0.177
## 3     NA    NA 0.052 0.057 0.064 0.073 0.082 0.092 0.103 0.115 0.128 0.142
## 4     NA    NA    NA 0.047 0.052 0.058 0.065 0.073 0.082 0.092 0.103 0.114
## 5     NA    NA    NA    NA 0.042 0.046 0.052 0.058 0.065 0.073 0.082 0.092
## 6     NA    NA    NA    NA    NA 0.037 0.041 0.046 0.052 0.058 0.066 0.074
## 7     NA    NA    NA    NA    NA    NA 0.033 0.036 0.041 0.047 0.053 0.060
## 8     NA    NA    NA    NA    NA    NA    NA 0.029 0.033 0.037 0.042 0.048
## 9     NA    NA    NA    NA    NA    NA    NA    NA 0.026 0.030 0.034 0.039
## 10    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.024 0.027 0.031
## 11    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.022 0.025
## 12    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.020
## 13    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 14    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 15    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 16    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 17    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 18    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 19    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 20    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 21    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 22    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 23    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 24    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 25    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 26    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 27    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 28    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 29    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 30    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
##      X13   X14   X15   X16   X17   X18   X19   X20   X21   X22   X23   X24
## 1  0.242 0.265 0.287 0.311 0.334 0.357 0.381 0.403 0.426 0.452 0.479 0.500
## 2  0.195 0.215 0.234 0.254 0.274 0.295 0.316 0.335 0.356 0.379 0.403 0.422
## 3  0.158 0.174 0.190 0.208 0.225 0.243 0.262 0.279 0.297 0.317 0.338 0.356
## 4  0.128 0.141 0.155 0.170 0.185 0.200 0.217 0.232 0.249 0.266 0.284 0.301
## 5  0.103 0.115 0.126 0.139 0.152 0.165 0.180 0.193 0.208 0.223 0.239 0.254
## 6  0.083 0.093 0.103 0.114 0.125 0.136 0.149 0.161 0.174 0.187 0.201 0.214
## 7  0.067 0.075 0.084 0.093 0.103 0.113 0.123 0.134 0.145 0.156 0.169 0.181
## 8  0.054 0.061 0.068 0.076 0.084 0.093 0.102 0.111 0.121 0.131 0.142 0.152
## 9  0.044 0.050 0.056 0.062 0.069 0.077 0.085 0.093 0.101 0.110 0.119 0.129
## 10 0.035 0.040 0.045 0.051 0.057 0.063 0.070 0.077 0.085 0.092 0.100 0.109
```

```
## 11 0.029 0.033 0.037 0.042 0.047 0.052 0.058 0.064 0.071 0.077 0.084 0.092
## 12 0.023 0.027 0.030 0.034 0.038 0.043 0.048 0.053 0.059 0.065 0.071 0.077
## 13 0.019 0.022 0.025 0.028 0.032 0.036 0.040 0.044 0.049 0.054 0.059 0.065
## 14    NA 0.017 0.020 0.023 0.026 0.029 0.033 0.037 0.041 0.045 0.050 0.055
## 15    NA    NA 0.016 0.019 0.021 0.024 0.027 0.031 0.034 0.038 0.042 0.046
## 16    NA    NA    NA 0.015 0.018 0.020 0.023 0.026 0.029 0.032 0.035 0.039
## 17    NA    NA    NA    NA 0.014 0.016 0.019 0.021 0.024 0.027 0.030 0.033
## 18    NA    NA    NA    NA    NA 0.014 0.016 0.018 0.020 0.022 0.025 0.028
## 19    NA    NA    NA    NA    NA    NA 0.013 0.015 0.017 0.019 0.021 0.024
## 20    NA    NA    NA    NA    NA    NA    NA 0.012 0.014 0.016 0.018 0.020
## 21    NA    NA    NA    NA    NA    NA    NA    NA 0.012 0.013 0.015 0.017
## 22    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.011 0.012 0.014
## 23    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.010 0.012
## 24    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA 0.010
## 25    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 26    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 27    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 28    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 29    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
## 30    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA    NA
##       X25    X26    X27    X28    X29    X30
## 1  0.5260 0.5540 0.5810 0.6080 0.6360 0.6640
## 2  0.4452 0.4702 0.4945 0.5189 0.5444 0.5699
## 3  0.3767 0.3990 0.4209 0.4429 0.4659 0.4892
## 4  0.3188 0.3387 0.3582 0.3780 0.3988 0.4199
## 5  0.2698 0.2874 0.3049 0.3227 0.3414 0.3604
## 6  0.2284 0.2439 0.2595 0.2754 0.2922 0.3094
## 7  0.1933 0.2070 0.2208 0.2351 0.2501 0.2656
## 8  0.1636 0.1757 0.1880 0.2006 0.2141 0.2280
## 9  0.1384 0.1491 0.1600 0.1712 0.1832 0.1957
## 10 0.1171 0.1266 0.1362 0.1462 0.1568 0.1680
## 11 0.0991 0.1074 0.1159 0.1247 0.1342 0.1442
## 12 0.0839 0.0912 0.0986 0.1065 0.1149 0.1237
## 13 0.0710 0.0774 0.0839 0.0909 0.0983 0.1062
## 14 0.0601 0.0657 0.0714 0.0776 0.0842 0.0912
## 15 0.0509 0.0557 0.0608 0.0662 0.0720 0.0783
## 16 0.0430 0.0473 0.0518 0.0565 0.0617 0.0672
## 17 0.0364 0.0401 0.0440 0.0482 0.0528 0.0577
## 18 0.0308 0.0341 0.0375 0.0412 0.0452 0.0495
## 19 0.0261 0.0289 0.0319 0.0351 0.0387 0.0425
## 20 0.0221 0.0245 0.0272 0.0300 0.0331 0.0365
## 21 0.0187 0.0208 0.0231 0.0256 0.0283 0.0313
## 22 0.0158 0.0177 0.0197 0.0218 0.0242 0.0269
## 23 0.0134 0.0150 0.0167 0.0186 0.0208 0.0231
## 24 0.0113 0.0127 0.0142 0.0159 0.0178 0.0198
## 25 0.0096 0.0108 0.0121 0.0136 0.0152 0.0170
## 26     NA 0.0092 0.0103 0.0116 0.0130 0.0146
## 27     NA     NA 0.0088 0.0099 0.0111 0.0125
## 28     NA     NA     NA 0.0084 0.0095 0.0107
## 29     NA     NA     NA     NA 0.0082 0.0092
## 30     NA     NA     NA     NA     NA 0.0079
```

```
plot(as.numeric(as.character(bdtree[1,])),pch = 2, col = "red",main = "Original vs. Replicated r*",ylab
points(as.numeric(as.character(rate_tree[1,])),pch=5, col = "blue")
```

```r
legend("bottomright", legend=c("BDT", "Replicated"),
       col=c("red", "blue"), cex=1, pch = c(2,5))
```



As seen from plot above, the replicated rates follow closely to the original values.

Question 2

```r
#Probability weighted expected value
probability = data.frame(matrix(ncol = 30, nrow = 30))
for (i in 1:30) {
  for (j in 1:i) {
    if (i == 1) {
      probability[j,i] = 1
    }
    if (i != 1) {

      if (j == 1) {
        probability[j,i] = probability[j,(i-1)]/2
      }
      if (j == i) {
        probability[j,i] = probability[(j-1),(i-1)]/2
      }
      if (j != i & j != 1) {
        probability[j,i] = probability[j,(i-1)]/2+probability[(j-1),(i-1)]/2
      }

    }
  }
}
```

```
probability_weighted = probability*rate_tree
exp_value = colSums(probability_weighted,na.rm = T)
plot(exp_value,cex = 0.01,main = "BDT Rate vs. Forward Rate",ylab = "Rate",xlab = "Periods")
lines(exp_value,col = "blue")

forward = numeric()
forward[1] = (1/pfilea[1]-1)*2
for (i in 2:30) {
  forward[i] = (pfilea[(i-1)]/pfilea[i]-1)*2
}
lines(forward, col = "red")
legend("bottomright", legend=c("Pfilea-Forward", "Replicated"),
       col=c("red", "blue"), cex=1, pch = c(2,5))
```