

MFE 409: Financial Risk Management

Problem set 1

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due 4/8 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and submit it on CCLE before Monday April 9 at midnight.

1 VaR for an exponential distribution

1. Assume an asset value in 10 days follows an exponential distribution with mean W_0 . Derive a formula for the value-at-risk for confidence c and reference level W_0 for the asset. Apply with $W_0 = 200$ and $c = 99.9\%$.
2. Now assume you are short this asset. Compute your value-at-risk for confidence c and apply with the same numerical values.
3. Comment on the similarity or difference between the results in the two questions.
4. Repeat the previous questions with expected shortfall.

2 VaR for mixtures

You have to allocate \$1bn in the stock market. You are discussing with your partner regarding the volatility of returns. She has a view that, in line with historical averages, the volatility of returns will be of $\sigma_1=12\%$ in the next year. However, you believe that volatility will be higher, in the orders of $\sigma_2=20\%$ for the next year. After discussing with your partner, you agree in the following way: stocks returns follow a normal distribution with mean μ and σ_1 with probability π and normal distribution with mean μ and σ_2 with probability $1 - \pi$. For now assume that $\mu = 8\%$ and $\pi = 0.7$.

1. Compute the 1-year 99% VaR with your view, with your partner view, and with the common view. Compare results and provide a very brief explanation as if you were presenting to your manager.

2. To understand the role of π , plot a chart with π between 0 and 1 on the horizontal axis and the corresponding VaR on the vertical axis. Comment on your results.
3. *More challenging.* After presenting your common view to your manager, you are challenged with an alternative view about volatility: σ is time-varying. The volatility trader suggests that a sensible model for sigma is a gamma distribution. Explain in as many details as possible (either derive of formula or use a computer program) how to compute the VaR of your portfolio when returns have a normal distribution conditional on σ and σ is distributed according to a Gamma distribution.

3 VaR for options

You will need to write code to solve this question.

You are a hedge fund manager with \$100 million of capital, and you can take any long or short position as long as you keep the 99% 10-day VaR for your portfolio under your capital. You have limited liability.

You have access to long and short positions on a stock, 3-month European calls and puts on the underlying, and risk free bonds. You choose your positions today and must hold them for the next 10 days. Assume the risk-free asset has annual rate of return $r = 2\%$, and the stock price S_t has dynamics:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

with $\mu = 7\%$ and $\sigma = 16\%$ (again, in annual units), and the current value $S_0 = 50$. Assume there are 252 trading days in a year.

1. Find a formula for the 10-day VaR for one share of the stock as a function of μ and σ . *Hint: if X is a random variable with quantile c equal to x_0 , what is the quantile c of $g(X)$ if g is a monotone function?*
2. If you can only invest in stocks and bonds and want to maximize the average return on equity (subject to the VaR constraint), which portfolio do you choose?
3. If you can only invest in ATM calls and bonds, which portfolio do you choose?
4. If you can only invest in ATM puts and bonds, which portfolio do you choose?
5. Now you can choose one of the stock, call, or put (with arbitrary strike) to combine with bonds. Plot the optimal portfolio position as well as your expected return for each strike. Which strike and portfolio do you choose? Explain the intuition behind this result.
6. *Extra question.* What happens if you have to respect a constraint on expected shortfall instead of value-at-risk?