HW 2
MFE 409: Financial Risk Measurement and Management
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Group 9
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Qn1

1.

Technique used to compute VaR

The model is based on changes in the market over the previous 260 days with confidence level of 99% * The incorporation of typical risk factors interest rates credit spreads exchange rates securities prices commodities prices and the corresponding volatility.

- The incorporation of correlations between risk factors in order to account for the knock-on effects of risk diversification.

What data is used to compuate VaR? Is more recent data weighted more heavily?

In the report, the market risk identified includes:

- Interest rate risk
- Currency risk
- Price risk
- Credit spread risk
- Options (intrinsic volatility and correlation risk)

The data mainly comes from Fixed income and equities team.

Time horizon

previous 260 days and 10 days horizon

Confidence level

99%

Number of VaR exceptions in 2008

Daily losses exceeded VaR 7 times in 2008 reflecting unprecedented shocks and exceptionally high volatility in the financial markets.

Any changes to VaR methodology made as a result of the financial crisis?

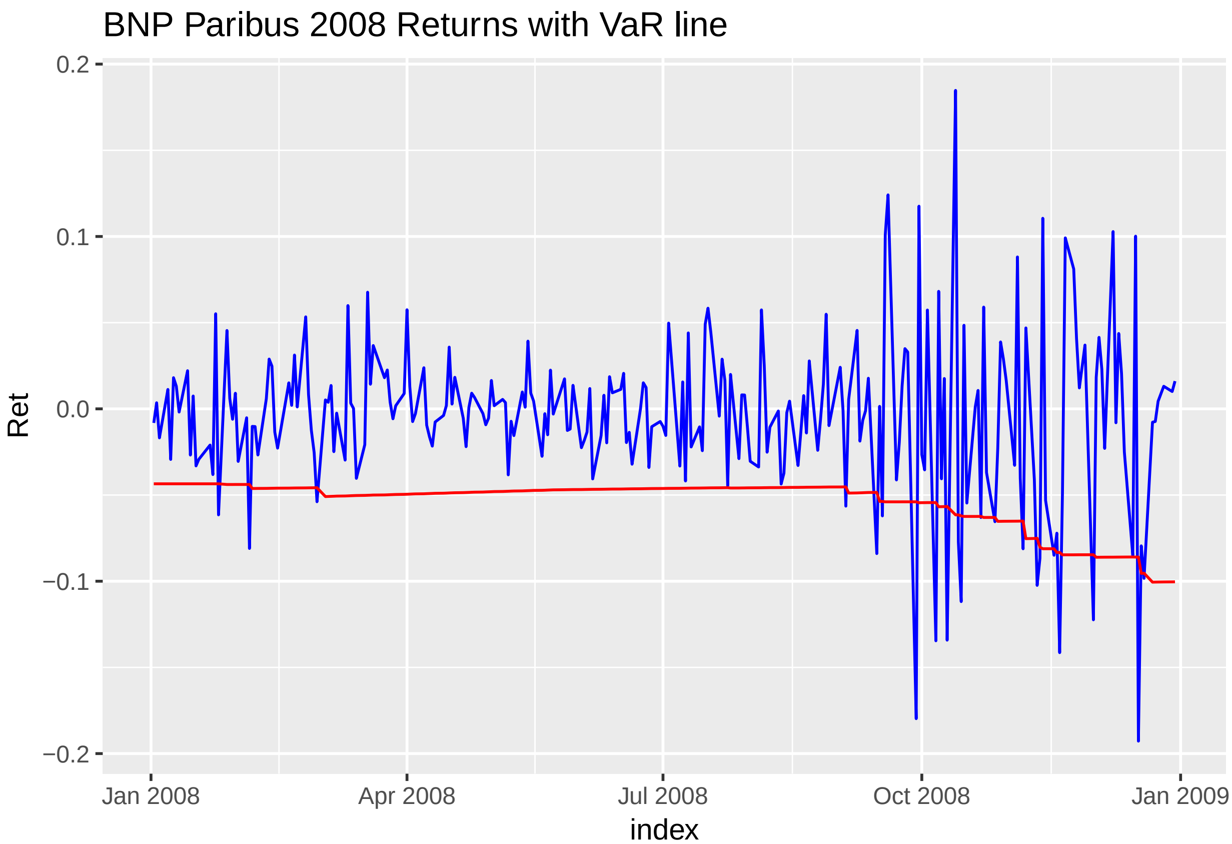
In response to the exreme volatility in the financial markets, the firm started to test the accuracy of its model through variety of techniques including regular comparison over long-term horizon between actual daily losses on capital market transactions and 1-day VaR at 99%

2.

(a)

Plot the Daily Stock Return and VaR. I use expanding window as the way to calculate the daily VaR

```
suppressMessages(library(data.table))
suppressMessages(library(ggplot2))
suppressMessages(library(foreign))
suppressMessages(library(lubridate))
suppressMessages(library(quantmod))
suppressMessages(library(knitr))

rm(list=ls())
# Download data and set as data.table from quantmode
suppressMessages(BNPQY <- getSymbols("BNPQY",auto.assign = FALSE,from='2006-01-01',to='2008-12-31'))
BNPQY <- as.data.table(BNPQY)

# find the daily return
BNPRet_all <- BNPQY[, Ret:=c(NA,diff(log(BNPQY.Adjusted)))][-1,]

# subset stock in 2008
BNPRet_08 <-BNPRet_all[index >= "2008-01-01"]

calculateDailyVaR <- function(Ret_08, Ret_all, c){
  # initialize daily VaR Dataframe
  dailyVaDTr <- data.frame(Date= as.Date(character()),VaR = double())

  for( i in Ret_08$index){
    historical_price <- Ret_all[index < as.Date(i)]
    var <- quantile(historical_price$Ret, 1-c)
    dailyVaDTr <- rbind(dailyVaDTr, c(as.Date(i), as.double(var)))
  }
  return(dailyVaDTr)
}

# set c
c <- 0.99

dailyVaDTr <- calculateDailyVaR(BNPRet_08, BNPRet_all,c)

colnames(dailyVaDTr) <- c("index", "Daily_VaR")
dailyVaDTr$index <- as.Date(dailyVaDTr$index)

BNPRet_08 <- as.data.frame(BNPRet_08)
BNPRet_08$DailyVar <- dailyVaDTr$Daily_VaR
ggplot(BNPRet_08, aes(index)) + geom_line(aes(y = Ret), color="blue") + geom_line(aes(y = DailyVar), col
```



(b)

We can back test the strategy by counting the number of times the returns have gone less than the daily VaR over 2008

```
count <- length(BNPRet_08$Ret) - sum(BNPRet_08$Ret > BNPRet_08$DailyVar)
cat("In 2008, the total number of times that return has gone below the daily VaR is ", count)
```

```
## In 2008, the total number of times that return has gone below the daily VaR is  21
```

For large n, we run the chisquare test shown in the notes.

```
m <- count
n <- length(BNPRet_08$Ret)
testVal <- -2 * log((c^(n-m)) * ((1-c)^m)) + 2 * log(((1 - m/n)^(n-m)) * ((m/n)^m))
chisqVal <- qchisq(p=0.95, df=1)
cat("Test Value is: ", testVal,"\n")
```

```
## Test Value is:  53.49507
```

```
cat("Chisquare value is: ", chisqVal,"\n")
```

```
## Chisquare value is:  3.841459
```

Hence we reject the null that the VaR is set properly at 99%. Intuitively, this could be due to the financial crisis where the daily return frequently went below VaR.

(c)

In this case, we found 21 exception for the return of BNP stock whereas only 7 exceptions were reported in the annual report. This could be due to the change of BNP's risk measure so that less exceptions were reported.

3.

(a)

To get portfolio returns:

step 1: find the daily return for all stocks

step 2: multiply by the corrresponding amount to get the gain/loss for each period

step 3: sum by the gain/loss for all stocks at each period to obtain the weighted portfolio gain/loss

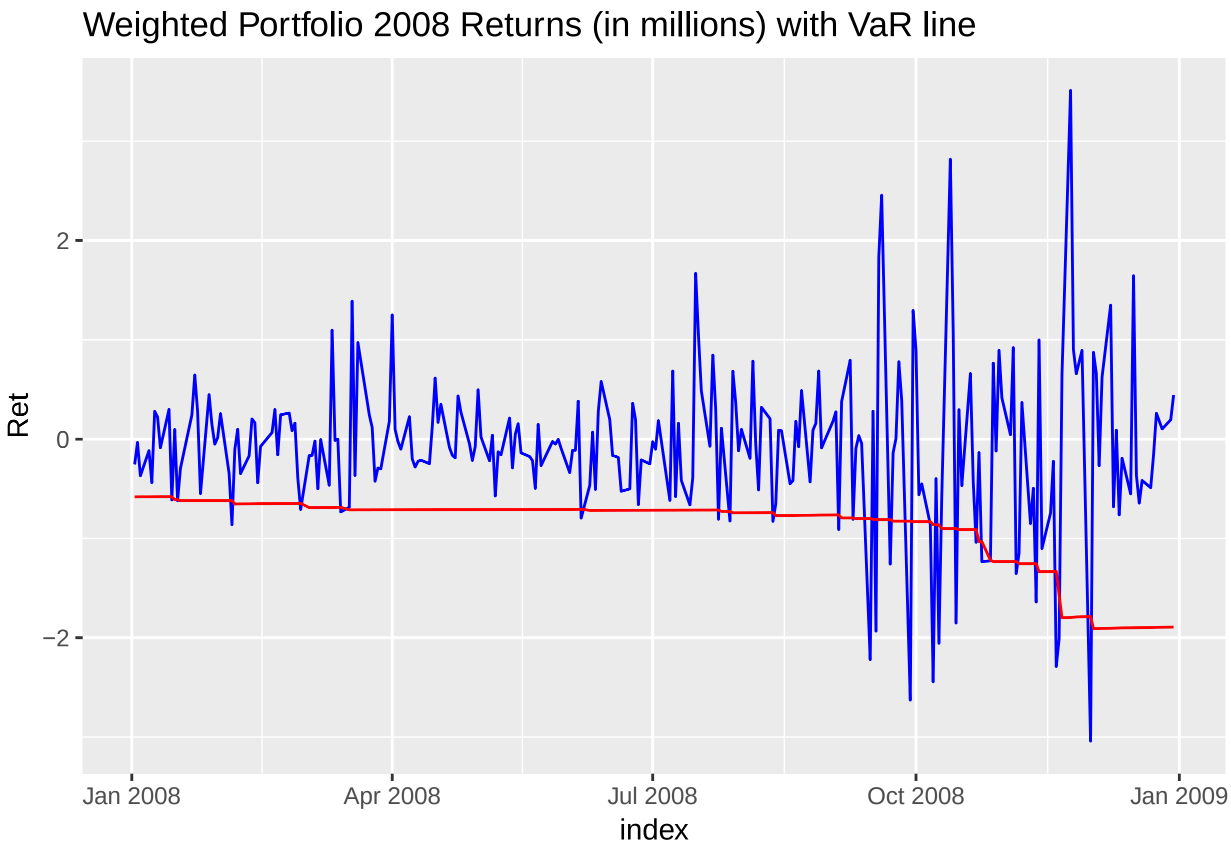
Also, Use the same method to calculate the VaR for the weighted portfolio in 2008.

The following diagram shows the daily VaR in 2008 for the portfolio.

```
# Get stock data from quantmod
symbols <- c("GS", "UBS", "JPM", "C", "BCS", "MS", "DB", "BAC", "BNPQY", "CS")
suppressMessages(getSymbols(symbols,auto.assign = TRUE,from='2006-01-01',to='2008-12-31'))
```

```
##  [1] "GS"    "UBS"   "JPM"   "C"     "BCS"   "MS"    "DB"    "BAC"
##  [9] "BNPQY" "CS"
```

```
# combined all Adjusted Prices
allAdjPrice <- data.frame(GS$GS.Adjusted, UBS$UBS.Adjusted, JPM$JPM.Adjusted, C$C.Adjusted, BCS$BCS.Adj

# get all daily return
allRet <- apply(allAdjPrice, 2, function(x){
  diff(log(x))
})

# starting with 1 or 2 mill, find the portfolio return (in mil)
totalAmount <- c(1,2,1,2,1,2,1,2,1,2)

baseLineRet <- totalAmount * allRet

# portfolio return from 2006 to 2008
baseLineRet <- data.table(
  index = rownames(baseLineRet),
  Ret = apply(baseLineRet, 1, function(x){ sum(x)})
)

# portfolio return in 2008
baseLineRet_08 <- baseLineRet[baseLineRet$index >= "2008-01-01"]

# set c
c <- 0.99

# calculate daily VaR
dailyVaDTr <- calculateDailyVaR(baseLineRet_08, baseLineRet, c)
colnames(dailyVaDTr) <- c("index", "Daily_VaR")

# convert index to dates
dailyVaDTr$index <- as.Date(dailyVaDTr$index)

baseLineRet_08$dailyVaR <-dailyVaDTr$Daily_VaR
baseLineRet_08$index <- as.Date(baseLineRet_08$index )

ggplot(baseLineRet_08, aes(x = index)) + geom_line(aes(y = Ret), color="blue") + geom_line(aes(y = dail
```



To calcualate the portfolio VaR, we experiment on two dataset: 1. Daily Return of portfolio from 2006 to 2008 2. Daily Return of portfolio in 2008

The idea is to see the impact on VaR in response to longer horizon and the financial crisis.

If We consider the the portfolio VaR to be the 1% quantile of all returns from 2006 to 2008

```
cat("Portfolio VaR is: ", abs(quantile(baseLineRet$Ret, 0.01))," mils")
```

```
## Portfolio VaR is:  1.891331  mils
```

If We consider the the portfolio VaR to be the 1% quantile of all returns from just in 2008

```
cat("Portfolio VaR is: ", abs(quantile(baseLineRet_08$Ret, 0.01))," mils")
```

```
## Portfolio VaR is:  2.364828  mils
```

This may be due to the financial crisis where the banks suffer a lot of huge losses (negative shocks to the stock prices)

(b)

Increment of the weight by $1 for each firm and calculate the DVaR and CVaR.

DVaR is obtained by the change in portfolio return divided by the change in weight.

CVaR is the product of DVaR and the corresponding weights

Code and summary below:

```
DVaR <- c()
DVaR_08 <- c()
for(i in 1:10){
  totalAmount <- c(1,2,1,2,1,2,1,2,1,2)
```

```
  totalAmount[i] <- totalAmount[i] + (1/1000000)
  portVal <- totalAmount * allRet

  portVal = data.table(
  index = rownames(allRet),
  Ret = apply(portVal, 1, function(x){ sum(x)})
  )
  # finding DVaR based on data in 2008
  portVal_08 <- portVal[portVal$index >= "2008-01-01"]
  DVaR_i_08 <- (abs(quantile(portVal_08$Ret, 0.01)) - abs(quantile(baseLineRet_08$Ret, 0.01)))/ (1/10000
  DVaR_08 <- rbind(DVaR_08, as.numeric(DVaR_i_08))

  # finding DVaR based on data from 2006 to 2008
  DVaR_i <- (abs(quantile(portVal$Ret, 0.01)) - abs(quantile(baseLineRet$Ret, 0.01)))/ (1/1000000)
  DVaR <- rbind(DVaR, as.numeric(DVaR_i))
}

totalAmount <- c(1,2,1,2,1,2,1,2,1,2)
CVaR <- DVaR * totalAmount
CVaR_08 <- DVaR_08 * totalAmount
```

DVaR and CVaR using data from 2006 to 2008:

```
summary <- cbind(DVaR, CVaR)
colnames(summary) <- c("DVaR", "CVaR")
rownames(summary)<- c("GS","UBS", "JPM", "C", "BCS", "MS", "DB", "BAC", "BNPQY", "CS")
kable(summary,col.names = c("DVaR","CVaR"),caption = "DVaR and CVaR using data from 2006 to 2008")
```

Table 1: DVaR and CVaR using data from 2006 to 2008

	DVaR	CVaR
GS	0.1149092	0.1149092
UBS	0.0958063	0.1916126
JPM	0.0209894	0.0209894
C	0.1433954	0.2867908
BCS	0.0879615	0.0879615
MS	0.2258437	0.4516875
DB	0.0916076	0.0916076
BAC	0.1249620	0.2499241
BNPQY	0.1419626	0.1419626
CS	0.1269431	0.2538863

DVaR and CVaR using data in 2008

```
summary_08 <- cbind(DVaR_08, CVaR_08)
colnames(summary_08) <- c("DVaR_08", "CVaR_08")
rownames(summary_08)<- c("GS","UBS", "JPM", "C", "BCS", "MS", "DB", "BAC", "BNPQY", "CS")
kable(summary_08,col.names = c("DVaR","CVaR"),caption = "DVaR and CVaR using data in 2008")
```

Table 2: DVaR and CVaR using data in 2008

	DVaR	CVaR
GS	0.1367676	0.1367676

	DVaR	CVaR
UBS	0.1220829	0.2441659
JPM	0.1579018	0.1579018
C	0.1365545	0.2731090
BCS	0.1731730	0.1731730
MS	0.2087190	0.4174381
DB	0.1986692	0.1986692
BAC	0.1446068	0.2892137
BNPQY	0.0495857	0.0495857
CS	0.2124023	0.4248045

(c)

CVaR shows how much each bank has an impact on the overall portfolio VaR. The result shows that Morgan Stanley has the highest and second highest impact on the portfolio VaR from both datasets. Credit Sussie has the highest impact of portfolio VaR in 2008. This could mean that its worth putting more weights for Morgan Stanley and Credit Sussie in a crisis. However, we also need to factor in the profits that each bank is making to get a better answer. We shall address this in terms of RAROC in the next part.

This is confirmed by the sum of CVaR, which is 1.891332 mils and 2.364828 mils, which is equal to the portfolio VaR correspondingly

```
# sum of CVaR using data from 2006 to 2008
sum(summary[,2])
```

```
## [1] 1.891331
```

```
# sum of CVaR using data in 2008
sum(summary_08[,2])
```

```
## [1] 2.364828
```

(d)

We can use RAROC (Risk Adjusted Rate of Return on Capital) To get the profit, we use the mean of the portfolio return from 2006 to 2008 or portfolio return in 2008 based on the same weights given in the previous questions.

Using portfolio return in 2008 to determine profit:

```
allRet_08 <- allRet[rownames(allRet) >= "2008-01-01",]
profits_08 <- apply(allRet_08,2,mean) * totalAmount
raroc_08 <- profits_08/CVaR_08
colnames(raroc_08) <- c("RAROC_08")
rownames(raroc_08)<- c("GS","UBS", "JPM", "C", "BCS", "MS", "DB", "BAC", "BNPQY", "CS")
raroc_08
```

```
##          RAROC_08
## GS    -0.027703576
## UBS   -0.038551037
## JPM   -0.007695364
## C     -0.042429529
## BCS   -0.032889127
## MS    -0.023202695
## DB    -0.021861308
## BAC   -0.029040984
## BNPQY -0.070692018
## CS    -0.013754495
```

Based on the result, we should be overweighting J.P. Morgan in the crisis as it has the biggest RAROC value.

Using portfolio return from 2006 to 2008 to determine profit:

```
profits <- apply(allRet,2,mean) * totalAmount
raroc <- profits/CVaR
colnames(raroc) <- c("RAROC")
rownames(raroc)<- c("GS","UBS", "JPM", "C", "BCS", "MS", "DB", "BAC", "BNPQY", "CS")
raroc
```

```
##              RAROC
## GS    -0.004944753
## UBS   -0.016399677
## JPM   -0.010232344
## C     -0.018216453
## BCS   -0.021352802
## MS    -0.006447908
## DB    -0.011088475
## BAC   -0.011676311
## BNPQY -0.005281756
## CS    -0.005856439
```

In this case for a long horizon, we should be overweighting Goldman, BNP credit Suisse.

Qn2 Expected Shortfall

1. Gain is $X \sim N(\mu, \sigma)$, Hence $X = \mu + \sigma Z$ where $Z \sim N(0,1)$

$$Prob(X \leq VaR_c(X)) = 1 - c$$

Hence

$$= \Phi(1-c)\sigma + \mu$$

where $\Phi(\cdot)$ is the cummulative function of standard normal distribution

From lecture notes we know that $VaR = -(\mu + \Phi^{-1}(1-c)\sigma)$ for loss

The expected shortfall in terms of VaR is:

$$ES = \frac{1}{1-c}\int_c^1 -(\mu + \Phi^{-1}(1-\alpha)\sigma)d\alpha$$
$$= -\frac{1}{1-c}\int_c^1 \Phi^{-1}(1-\alpha)\sigma d\alpha - \mu$$

Let $\alpha = \Phi(y)$ then $d\alpha = \phi(y)dy$ where $\phi(y)$ is the pdf of standard normal distribution

Hence the upper bound now is $\Phi^{-1}(1)$ which is ∞. Substitute $\Phi(y)$ for α :

$$\int_c^1 \Phi^{-1}(1-\alpha)\sigma d\alpha$$
$$= \int_{\Phi^{-1}(c)}^{\infty} \Phi^{-1}(\Phi(-y))\sigma\phi(y)dy$$
$$= \int_{\Phi^{-1}(c)}^{\infty} -y\sigma\phi(y)dy$$

$$= \int_{\Phi^{-1}(c)}^{\infty} -\frac{y\sigma}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

Integrate to get:

$$= -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{z(c)^2}{2}}$$

Put it back the expected shortfall equation:

$$ES = -\frac{1}{1-c}(-\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{z(c)^2}{2}}) - \mu$$

$$= -\mu + \frac{\sigma}{\sqrt{2\pi}(1-c)} e^{-\frac{z(c)^2}{2}})$$

2. General formula:

$$ES = W_0 - \frac{\int_{-\infty}^{W_0 - VaR} Wf(W)dW}{\int_{-\infty}^{W_0 - VaR} f(W)dW}$$

We need to solve the integration in the numerator, denominator is just 1-c

Let $\alpha = F(W)$, then $d\alpha = f(W)dW$, $F(W_0 - VaR) = 1 - c$, then $F^{-1}(1-c) = W_0 - VaR$

also change the upper and lower bound of the integration:

$$\int_{-\infty}^{W_0 - VaR} f(W)dW = \int_{W_0 + VaR}^{\infty} Wf(W)dW$$

$$= \int_{W_0 + VaR}^{\infty} Wf(W)dW = \int_{F^{-1}(c)}^{F^{-1}(1)} Wf(W)dW$$

Where $F^{-1}(1-c) = W_0 - VaR = F^{-1}(c) = W_0 + VaR$

$$\int_{F^{-1}(c)}^{F^{-1}(1)} W d\alpha = \int_c^1 W d\alpha$$

$$= \int_c^1 W d\alpha = \int_c^1 (W_0 - VaR_\alpha) d\alpha$$

$$= W_0(1-c) - \int_c^1 VaR_\alpha d\alpha$$

Bring it back into the expected shortfall equation

$$ES = W_0 - \frac{W_0(1-c) - \int_c^1 VaR_\alpha d\alpha}{1-c}$$

$$= \frac{1}{1-c} \int_c^1 VaR_\alpha d\alpha$$