MGMT 400 Investment HW4

Project for Lecture 4: Yield Curve Spread Trades

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I. Executive Summary

In this report, we are able to examine the relationship between yield rates for different maturities and use the historical data of FEDS treasury yields [1] in the period from 12/30/1983 to 6/30/2018 to back-test the "flattener strategy" as defined herein. We constructed our positions under the "flattener strategy" by shorting the 2-yr Zero Coupon Bond (as "ZCB") and longing the 10-yr ZCB in a proper risk-hedged ratio where the absolute value of DV01 of both positions are equal. Note that 10% capital is required to hold the position. Namely, cash of 10% of the total value of the short and the long is committed in advance.

At the end of the back test, we have a final capital of $ 868,230.4 (lost $ 131,769.6 | 13.2%).

II. Introduction

A yield curve spread is the yield differential between bonds with different maturities (in the case, 10 yr U.S. Treasury Yield – 2 yr U.S. Treasury Yield). Under the flattener strategy we would short the 2-year Treasury and simultaneously long the 10-year Treasury. By doing so, we are speculating on narrower yield differential between these two maturities.

To construct a risk-neutral position, we consider DV01 as the risk measurement and allocate our position in the short and the long such that:

$$DV01_{2yr} * X = DV01_{10yr} * Y$$

where X denotes the unit of the 2yr ZCB and Y denotes the unit of the 10yr ZCB. As the spotted yield changed every week, we closed and rebalanced the position seeking new (X, Y) so that the risk neutral condition holds.

What's also subjected to the condition is the capital requirement. In order to hold positions (both short and long), we are required to hold capital (cash) that equals to 10% of the trading positions:

$$(P_{2yr} * X + P_{10yr} * Y) = \text{Capital requirement} / 10\%$$

Note: starting capital = $ 1,000,000

After forming the position at the beginning of each week, we would have the cash position, as illustrated below, that either earns or generates interest. We assumed that the interest is in force at 7-day treasury yield, which could be derived from the "*Nelson-Siegel-Svensson" (as "NSS")* model.

$$P_{2yr(t)} * X - P_{10yr(t)} * Y + \text{Capital hold} = \text{Cash position}$$

At the end of each week, we closed out the position by buying X unit of 2-yr ZCB, selling Y unit of 10-yr ZCB and interest either paid to or received from the cash position, as illustrated below:

(1) https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html

$-P_{2yr(t+\Delta t)} * X + P_{10yr(t+\Delta t)} * Y$ + Interest from Cash position + Cash position = New capital

And this amount will be used as capital for investment for next coming week. We evaluated the performance of such strategy by comparing changes (%) of capital from the beginning of the week to the end and cumulated such change to calculate the cumulative return.

III. Analysis and Modeling

Data Source and Processing

We retrieved *"The U.S. Treasury Yield Curve: 1961 to the Present"* by Refet S. Gurkaynak, Brian Sack, and Jonathan H. Wright 2006-28. This dataset provides the daily observed spot yields and parameters for the *"Nelson-Siegel-Svensson"* yield curve model, as defined below:

$$r_t = \beta_0 + \beta_1 \frac{1-e^{-t/\tau_1}}{t/\tau_1} + \beta_2\left[\frac{1-e^{-t/\tau_1}}{t/\tau_1} - e^{-t/\tau_1}\right] + \beta_3\left[\frac{1-e^{-t/\tau_2}}{t/\tau_2} - e^{-t/\tau_2}\right]$$

and all yields and interests are assumed to compound continuously in this case. To process the data in R, we first formatted the data into CSV format with appropriate headers and utilized the *"endpoint"* function within the *"xts"* package in R to pick up dates with weekly increments for our analysis.

Assumptions in Model

When constructing the risk-neutral positions, we used *DV01* as the risk measurement and assume parallel shift in the yield curve, namely

$$\Delta r\ (2yr) = \Delta r\ (10yr).$$

When closing out the position every week, we used the *Nelson-Siegel-Svensson* yield curve model discussed above to derive the theoretical prices for the 2-yr and the 10-yr ZCB's one week later and assumed the market is liquid enough for day-in-force execution.

As interest would either be paid or earned depending on the cash position, we also assumed interest rate equal to the 7-day *"NSS"* yield rate in force continuously.

For simplicity, we assume zero transaction costs. Yet, this is not reflecting the case in practice. The main concern here are chances that costs will out-weigh over profits.

Performance Analysis

We analyzed the return of this strategy by considering the total cash capital. Each week, we used the cash capital as leverage to create a risk-neutral position. The bond prices used in constructing position will be derived from the spotted SVENYxx rate in force continuously. At the end of each week, all positions are closed at the "prorated prices", where yield rates can be derived from the

("NSS") model by substituting time as (2yr - 7/365) and (10yr - 7/365), respectively. The total cash capital at the end of the week will include proceeds received from closing and interest either paid or earned from the cash position. This new balance of cash capital will serve as new capital requirement in order to create a new position (to "rebalance") at the beginning of next week.

We evaluated the performance of the strategy by determining any gain or loss from the cash capital between the beginning balance and the ending balance and plotted the cumulative return VS time as below:

Answer to question #1

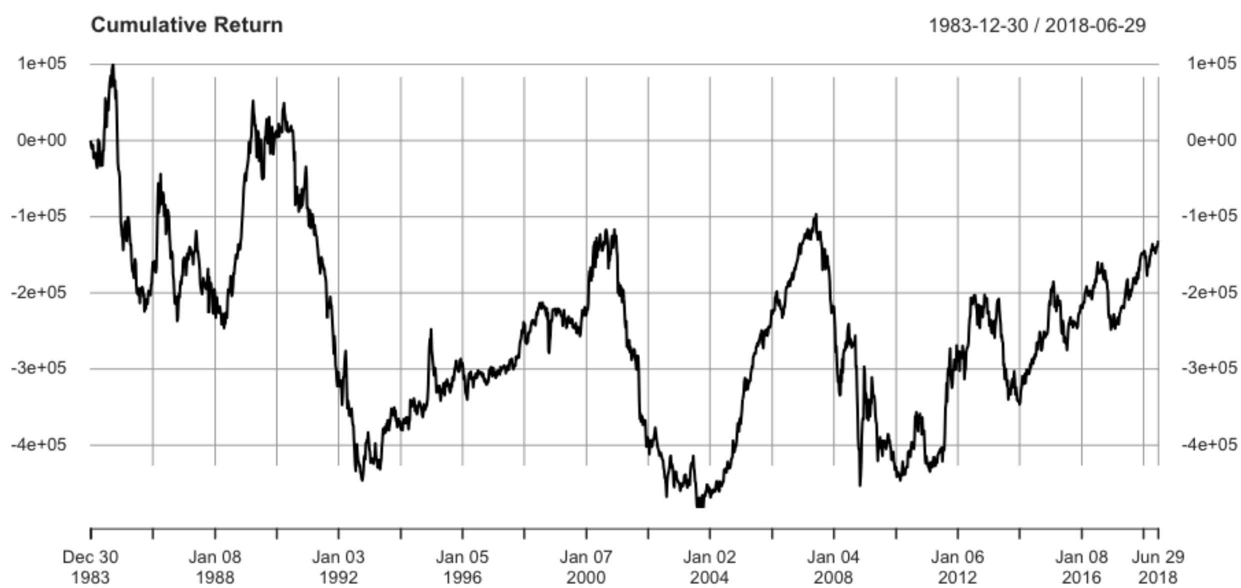

Fig. 1 – Cumulative Return Flattener Strategy

The position is not as intended as to be risk-neutral, however, we only consider *DV01* as the risk measurement. There is unhedged convexity that contributes to the sensitivity of the portfolio and its performance. We examined the convexities of the two ZCB's and noted an increasing divergence as time progressed:

Answer to question #2

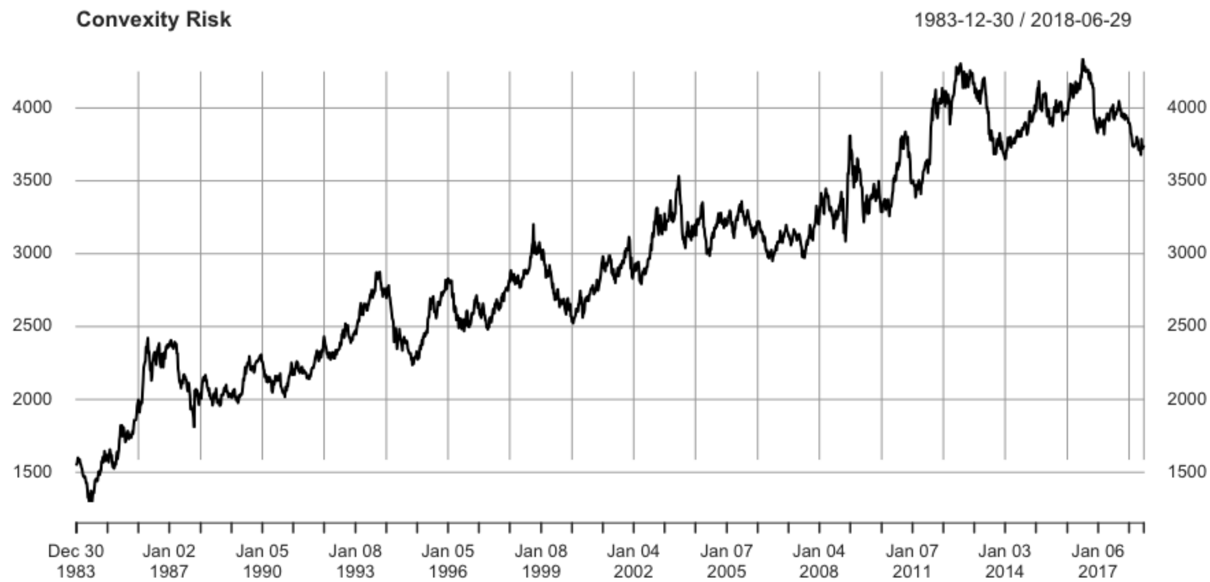

Fig. 2 – Convexity Risk 10 years Bond

In theory, the below illustrated formula shows how portfolio return is contributed by three components: spread return, convexity return and interest (residuals is insignificant). Worth mentioning, we consider Δ® as the difference between the observed SVENYxx rate and our "prorated" interest rates 7-days later under the "NSS" model.

$$P\&L = d_{portfolio} = \text{Spread return} + \text{Convexity return} + \text{Time Return} + \text{Residuals}$$
$$= Y * [-DV01_{10yr} * \Delta(r_{10yr}) + \frac{1}{2} C_{10yr} * P_{10yr} * \Delta(r_{10yr})^2 + TimeReturn_{10yr} + Residuals_{10yr}]$$
$$- X * [-DV01_{2yr} * \Delta(r_{2yr}) + \frac{1}{2} C_{2yr} * P_{2yr} * \Delta(r_{2yr})^2 + TimeReturn_{2yr} + Residuals_{2yr}]$$

Where (Y* $TimeReturn_{10yr}$ - X* $TimeReturn_{2yr}$) is the net interest plus the passage of time effect.

*Time Return = Y * (Bond Price Based on* 10yr - 7/365 - *Bond Price Based on 10 year yield)*
*- X * (Bond Price Based on* 2 yr - 7/365 - *Bond Price Based on 2 year yield) + Interest*

Such discussed effects on P&L can be further decomposed as per illustrated below:

Answer to question #3

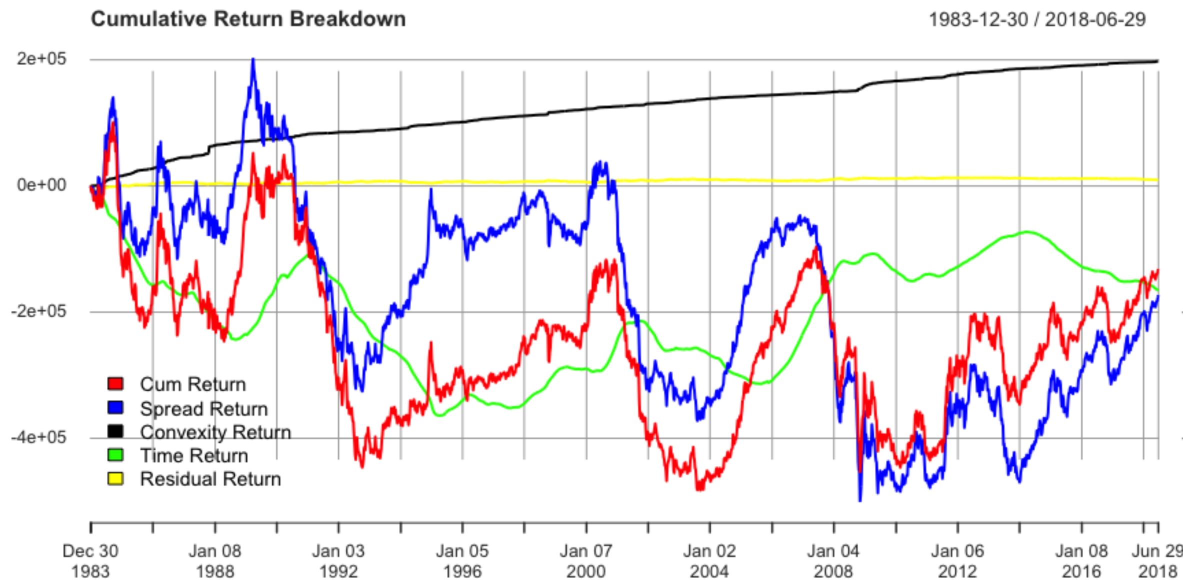

Fig. 3 – Cumulative Return Breakdown

Return	$
Spread	$ -173,054.3
Convexity	$ 197,418.1
Time	$ -166,120.2
Residual	$ 9,986.8
Total (Cumulative)	$ -131,769.6

Fig. 4 – Table of returns break down

"Spread return"

The value of the portfolio constructed at the beginning of each period is only immunized to small parallel shifts in the yield curve. When there's an unparalleled shift in the yield curve, the portfolio value will change. In general, under the flattener strategy, the narrower the spread (in this case between the 2yr and the 10yr), the more return will achieve. The spread, however, from our data ranging from 1983 to 2018, fluctuates and more decreasing periods are observed which caused losses.

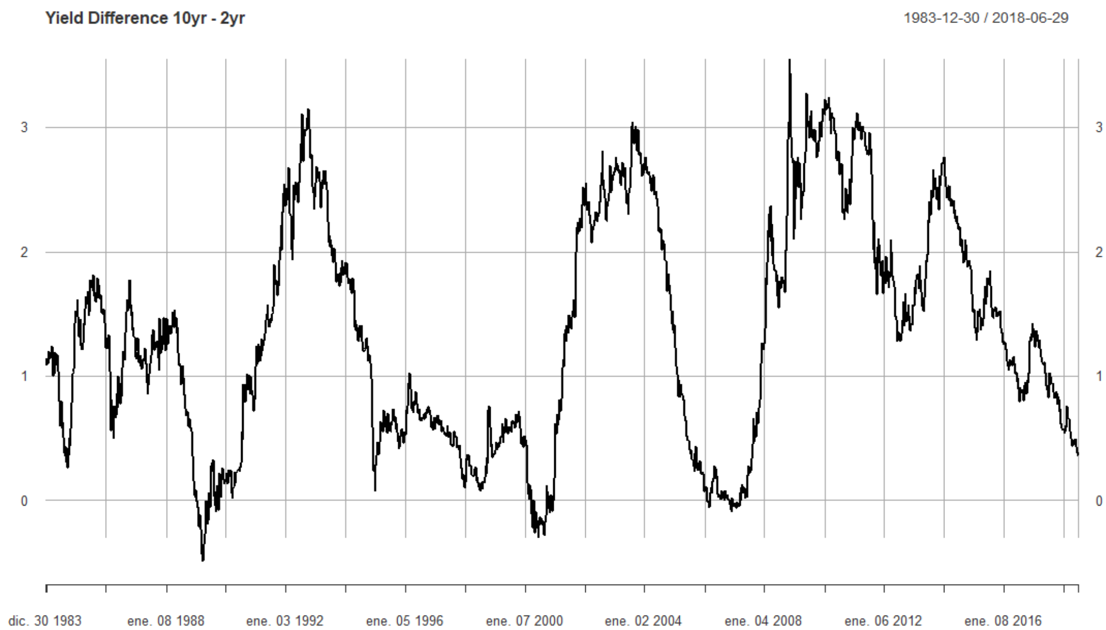

Fig. 5 – Yield Difference 10yr - 2yr

"Convexity return"
Our portfolio is duration hedged but not convexity hedged. The flattener Strategy benefits from convexity of the curve because long-term convexity will dominate short-term convexity.

"Interest return"
Interest return refers to the interest earned on the cash position during the week. Interests were earned on cash positions (value of the short position minus the value of the long position plus the capital requirement) and we assumed such rate at what's derived from the "NSS" model on a "weekly basis" (in fact in our calculation, we counted calendar days).

"Passage of time"
Passage of time is the effect of the maturation of the bonds during the week that we maintain the position. This one was calculated based on the time effect on the bond price as a reflection of the yield reduction because of his maturity.

"Leverage"
Our strategy is leveraged by committing 10% of capital requirement and to magnify trading position by 10 times (namely if one puts \$1M as capital commitment, one would form a position of \$10M in value combined with both long and short). To examine how leverage would impact the return of the strategy, we compare the return of two portfolio under the same strategy, but different leverage ratios, 10% (as original) and 2% (as proposed).

Answer to question #4

Fig. 6 – Cumulative Return 10% vs 2% Margin

The result is somehow not as surprising. Our strategy (under 10% capital) produces negative returns in a long run and higher leverage basically magnifies return (in this case loss). As per shown in the diagram above, the 2% capital margin return drops to -100% after 9 years.

IV Conclusion

Our strategy is getting negative return of -13.2% as of 6/30/2018 as yields fluctuated and the widening spreads between 2yr ZCB and 10yr ZCB are observed, which is not favorable to our "*Flattener Strategy*". This model suffers from volatilities contributed mostly by the spread return and time return. Over a long run, a diversified portfolio is possibly to achieve a better return with lower volatility. To sum up, if the yield spread between rates are decreasing, a "steepener strategy" will generate profits.