UCLA Anderson **Fall 2018**
MGMTMFE 400 **Prof. Mikhail Chernov**

Problems for Lecture 3

1. You are holding a 3-year bond with coupon rate 10%. Coupon payments are annual and par values are 100. Spot rates are: $r_1 = 5\%$, $r_2 = 5.5\%$, $r_3 = 6.5\%$.

   (a) Determine the bond's price and YTM.

   (b) Determine as many forward rates as you can, based on the spot rates above.

   (c) You would like to get a guaranteed 3-year return on your coupon bond. Explain how this can be achieved using forward rates. Which forward rates should you use? What is your guaranteed 3-year return?

2. You will receive \$1M one year from now, but will not need to use it until three years from now. You thus need to invest the \$1M for two years.

   (a) Suppose that you want to guarantee a rate today for your investment. What is the relevant forward rate?

   (b) Suppose that the one-year spot rate is 6% and the three-year spot rate is 7%. Compute the forward rate in (a).

   (c) Suppose that you are unable to contact a bank that offers forward rates. You can contact, however, a dealer who trades zero-coupon bonds. Explain how you can use zero-coupon bonds to lock in the forward rate in (a).

3. Consider the following 4 US Treasury bonds (the par value is \$100):

   Bond A is a 5-year bond with a 1% annual coupon. Bond B is a 10-year bond with a 1% annual coupon. Bond C is a 5-year bond with a 4% annual coupon. Bond D is a 10-year bond with a 4% annual coupon.

   Assume all of the bonds are currently trading at a 3.5% yield and, as their maturity is over 1 year, they pay semi-annual coupons. Recall if a bond has a 4% annual coupon, it pays 2% of par every 6-months.

   (a) Write down the formula for pricing bonds.

   (b) Compute the current price for each of the bonds.

   (c) Suppose the yield increases to 3.8% from 3.5%. Compute the new prices and compute the change in the bond's price.

(d) Suppose the yield decreases to 3.2% from 3.5%. Compute the new prices and compute the change in the bond's price.

(e) Compute durations of each of these bonds and answer (c) and (d) using duration.

(f) What can you conclude about reaction of bond prices to changes in yields for bonds with different coupons and maturities?

4. You just sold to a client a custom-made 31-year bond which pays a single coupon of \$ 1M 30 years from now and \$ 2M at maturity. You would like to hedge this liability. Assume that the term structure is currently flat at 6%.

(a) Your first idea is to synthetically replicate the liability. Construct the hedging portfolio consisting of the following bonds paying annual coupons:

Bond	Coupon rate (%)	Maturity
A	0	31 years
B	4	30 years
C	6	30 years

The par values of the three bonds are \$ 100.

(b) You consider instead duration-based hedging. Assume that you have 10- and 15-year zero-coupon bonds available, with par values \$ 100. Construct the hedging portfolio of the two bonds that has the same market value and the same interest-rate sensitivity (measured using the duration model) as your liability.

(c) Suppose the interest rate fell to 4.8%. Would your hedging portfolio computed in part (b) remain the same? If not, explain why not and comment on the composition of the new hedging portfolio relative to the old one. (No computations are necessary.)

(d) What are the advantages of synthetic replication (option described in part (i)) relative to approximate hedging (option described in part (ii))? Why might one use approximate hedging instead?