**HW 2**
**MFE 400: Investment**
**Professor Chernov**
**Group 6**
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## Question 1

(a)

Since this is a zero coupon bond, we use the 3-year spot rate to discount back the par value to calculate the price today.

$$\frac{100}{(1+r_3)^3} = \frac{100}{(1+0.06)^3} = \$83.96$$

The bond price is \$83.96.

$$\$83.96 = \frac{0}{(1+y)^2} + \frac{0}{(1+y)^2} + \frac{100}{(1+y)^3}$$

Solving this equation YTM = 6%

(b)

This bond is paying $100 \times 0.06 = 6$ dollars coupon at year 1 and year 2. The bond is also paying back the par value at the end of year 2. Thus, the cash flow is 6 at the end of year 1 and 106 at the end of year 2.

We use 1-year spot rate to disocunt back the 6 dollars at year 1 and use 2-year spot rate to discount back the 106 dolalrs at year 2.

$$\frac{100 \times 0.06}{1+0.05} + \frac{100 + 100 \times 0.06}{(1+0.055)^2} = \$100.95$$

The bond price is \$100.95.

$$\$100.95 = \frac{6}{1+y} + \frac{106}{(1+y)^2}$$

Solving this equation YTM = 5.485568%

(c)

The cash flow at year 1, year 2, year 3 and year 4 are 8, 8, 8 and 108, respectively. We use 1-year spot rate, 2-year spot rate, 3-year spot rate and 4-year spot rate to discount cash flow at year 1, year 2, year 3 and year 4, respectively.

Coupon=$100 \times 0.08 = \$8$

Bond price=

$$\frac{8}{1+0.05} + \frac{8}{(1+0.055)^2} + \frac{8}{(1+0.06)^3} + \frac{100+8}{(1+0.063)^4} = \$106.11$$

The bond price is \$106.11.

$$\$106.11 = \frac{8}{1+y} + \frac{8}{(1+y)^2} + \frac{8}{(1+y)^3} + \frac{108}{(1+y)^4}$$

Solving this equation YTM = 6.227511%

## Question 2

Bond X:

$$\frac{100 + 100 \times (0.04/2)}{(1 + r_{0.5}/2)} = \$100.98$$

$$r_{0.5} = 0.02020202$$

The six month spot rate is 2.020202%.

Bond Y:

coupon=$100 \times 0.03 = 3$

$$\frac{3}{1 + (r_{0.5}/2)} + \frac{(100 + 3)}{1 + (r_1/2)^2} = \$103.59$$

$r_1 = 0.02351511$

The 1-year spot rate is 2.351511%.

## Question 3

Since Bond A is a one year zero coupon bond, we get calculate the 1-year spot rate:

$$r_1 = \frac{100}{95.238} - 1$$

$$r_1 = 0.05000105$$

We can also calculate the 2-year spot rate implied by information of bond B.

Bond B coupon = $100 \times 0.05 = 5$

$$\frac{5}{1 + r_1} + \frac{100 + 5}{(1 + r_2)^2} = \$98.44$$

$$r_2 = 0.05871788$$

We can use the 1-year spot rate and the 2-year calculated above to calculate the price for a bond with 7% Coupon rate.

Coupon payment = $100 \times 0.07 = 7$

Bond price =

$$\frac{7}{1 + 0.05000105} + \frac{100 + 7}{(1 + 0.05871788)^2} = \$102.13$$

The price of bond C is listed as 103.370, which is higher than the price calculated based on te spot rates implied by bond A and bond B prices. Thus, bond C is over-priced.

We can use x units of bond B and y unit of bond A to replicate bond C.

$$5x + 100y = 7$$
$$105x + 0y = 107$$

Thus,

$$x = 1.01904762$$
$$y = 0.01904762$$

Therefore, the trading strategy is buy 1.01904761904762 units of bond B and 0.019047619047619 units of bond A and sell 1 unit of bond C.