Simulink Project

System Modelling For Magneto Active Elastomers

- Parag Pathak

What are MAEs? (Recap)

- MAEs consist of magnetic particles, such as micron-size iron particles, dispersed in an elastomeric matrix.
- They can undergo large deformations when excited by a magnetic field.
- Uses include tunable vibration absorbers, damping components , noise barrier system and sensors.
- Instabilities can cause failure of these components.

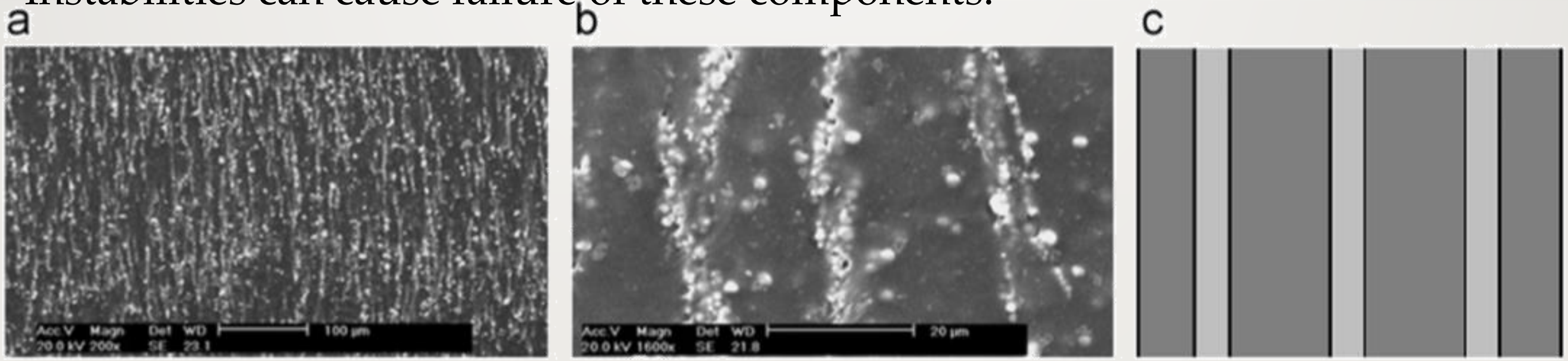
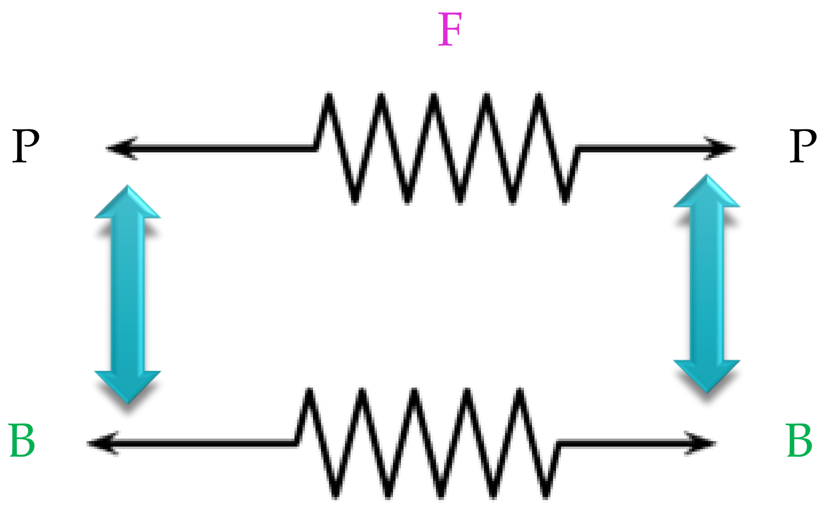


Fig. 1. (a) SEM image with 200 times magnification of MRE prepared in 800 mT (Chen et al., 2007); (b) SEM image with 1600 times magnification of MRE prepared in 800 mT (Chen et al., 2007); (c) schematic representation of the idealized layered microstructure considered in this work. (a) MRE (800 mT) X200. (b) MRE (800 mT) X1600. (c) Idealized MRE.

Theoretical model of Magneto elastomers

$$P = \bar{G}A\left(\left[\lambda^2 - \frac{1}{\lambda^2}\right] - \frac{B^2}{\lambda^2\hat{\mu}\bar{G}}\right)$$



$$H = \mathcal{H}B$$

- The deformation gradient (F) is a function of first Piola stress (P) with proportionality constant ($\mathcal{A}$).
- The magnetic intensity (H) is a function of magnetic field (B) with proportionality constant ($\mathcal{H}$).
- Note : This is a simplified format. Refer (1,2) for full details

Instability Limit

- While the heterogeneity provides access to the tailored and enhanced coupled behaviour, it is also a source for the development of microstructural instabilities.
- **The instability phenomenon historically has been considered as a failure mode,** which is to be predicted and avoided.
- The magnetic field values have a certain limit which depends on the deformation of the MAE.

$$B < \left[\left(\lambda^4 - 1 + \frac{\check{G}}{\overline{G}}\right)\left(1 - \frac{\check{\mu}}{\overline{\mu}}\right)^{-1} \check{\mu}\overline{G}\right]^{1/2}.$$

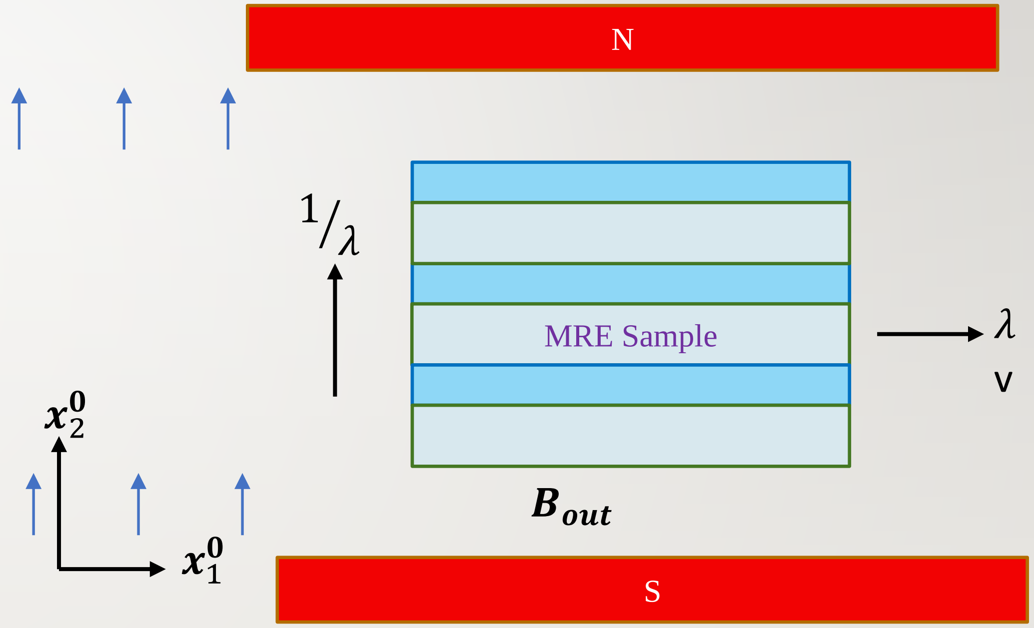
Loading condition

- Displacement field

$$x_1 = \lambda x_1^0 \quad , \quad x_2 = \frac{x_2^0}{\lambda} \quad , \quad x_3 = x_3^0$$

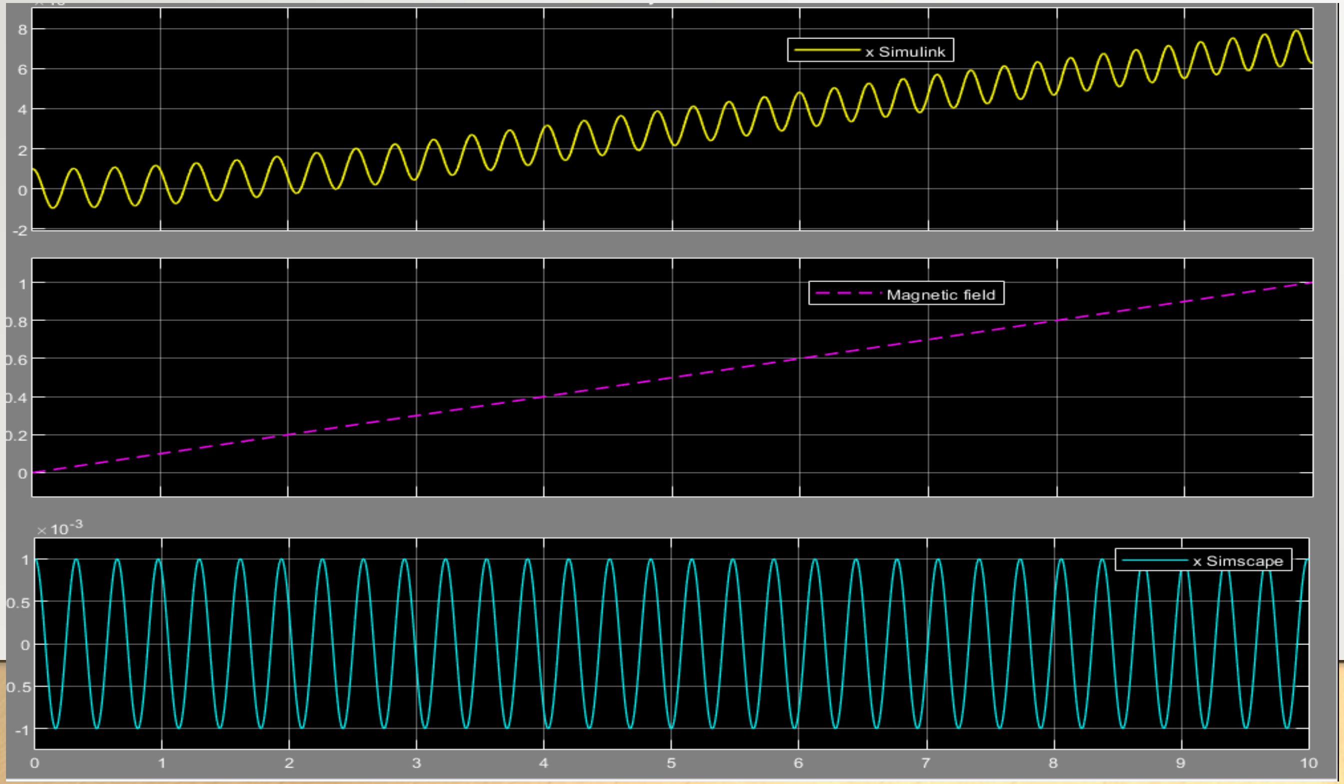
- Deformation gradient , Magnetic field

$$\boldsymbol{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^{-1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad \boldsymbol{B}_{out}^{\boldsymbol{0}} = \begin{bmatrix} 0 & B^0 & 0 \end{bmatrix}$$

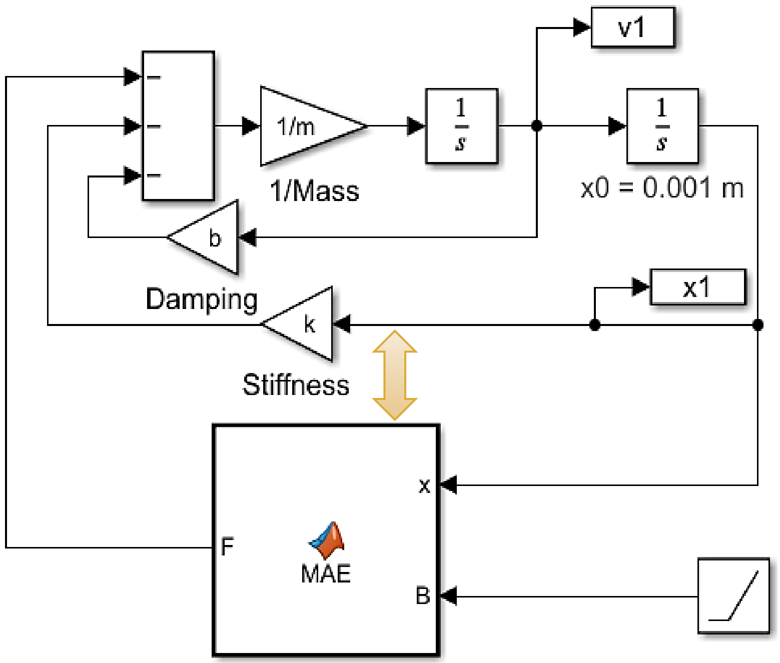


Free vibrations

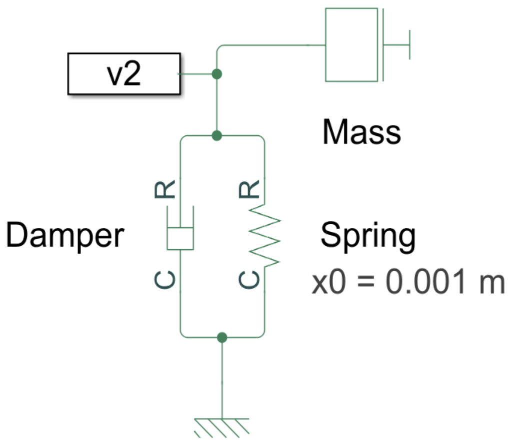
- The behaviour of an MAE was studied under free oscillations. The system was allowed to oscillate freely. This was compared to a simscape model system.



Simulink Model

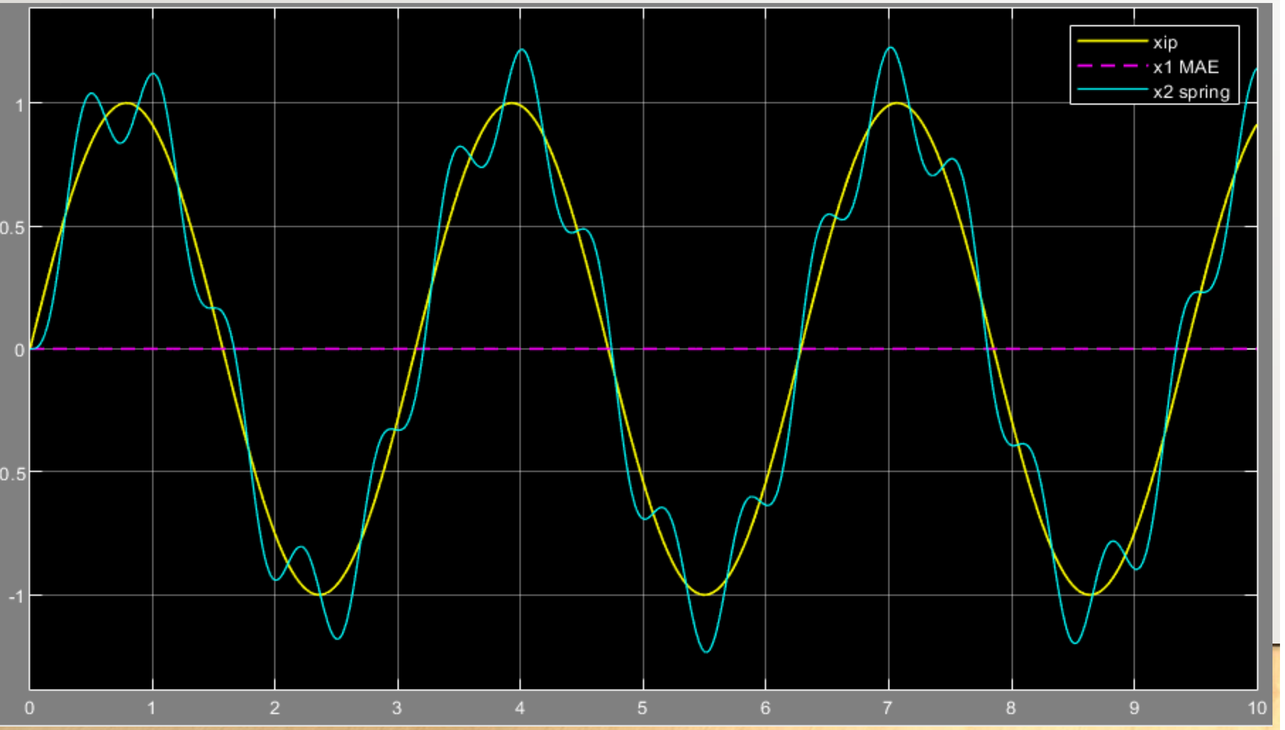


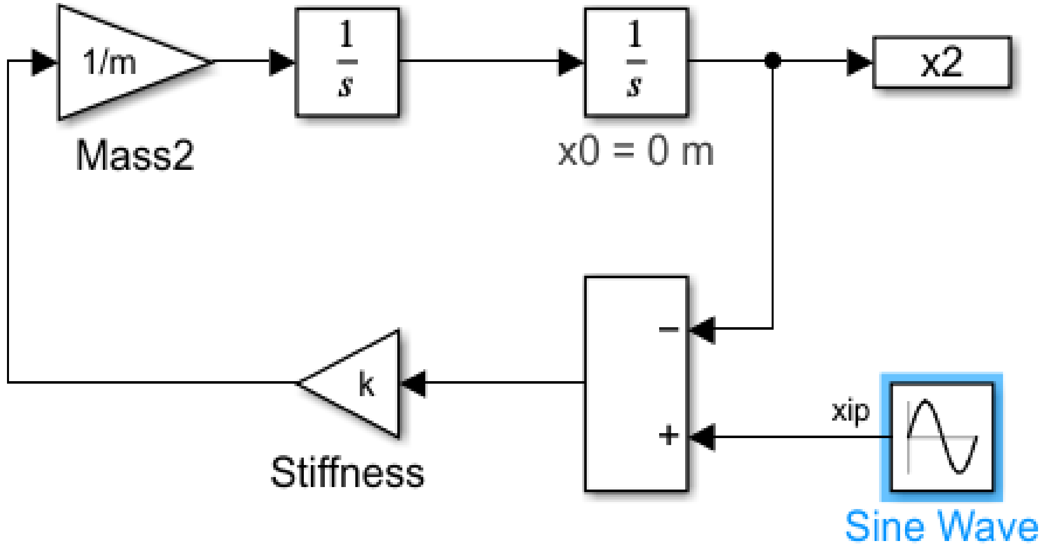
Simscape Model

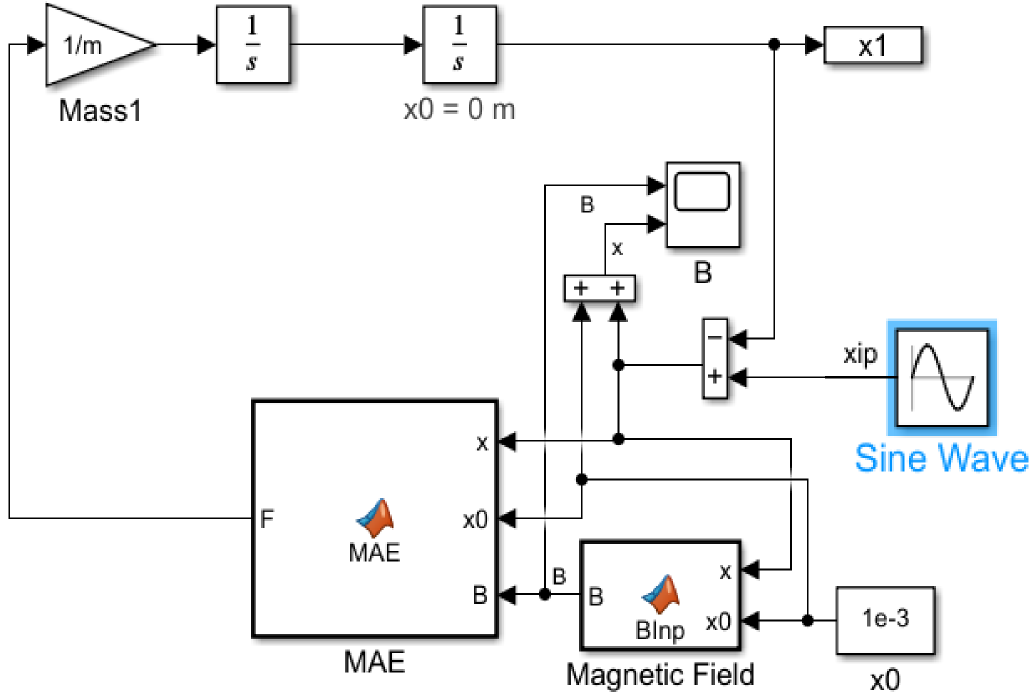


Forced vibrations

- Output signal was isolated from the input signals







Conclusions

1. The MAEs could be tuned to remove noise and isolate the output from the input.
2. The equilibrium point can be adjusted by applying a magnetic field.
3. The only limitations are saturation and the stability limits for MAEs which need to be tuned for the desired output range.

References

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- Rudykh, S., & Debotton, G. (2011). Stability of anisotropic electroactive polymers with application to layered media. Zeitschrift für angewandte Mathematik und Physik, 62(6), 1131-1142.
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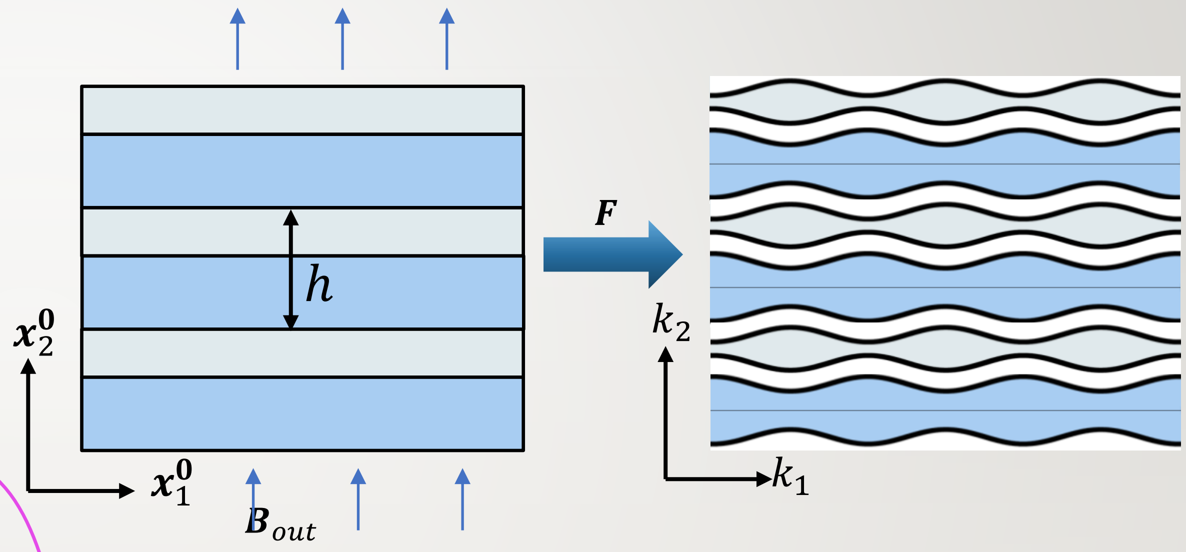
Backup slides

General Solution form

- $\dot{u}_1(x_1, x_2) = v_1(x_2)e^{ik_1x_1}$
- $\dot{u}_2(x_1, x_2) = v_2(x_2)e^{ik_1x_1}$
- $\dot{B}_1(x_1, x_2) = \mathcal{B}_1(x_2)e^{ik_1x_1}$
- $\dot{B}_2(x_1, x_2) = \mathcal{B}_2(x_2)e^{ik_1x_1}$
- $\dot{p}(x_1, x_2) = q(x_2)e^{ik_1x_1}$



Substitute into Incremental Constitutive relations

$$\mathcal{A}_{iklj}\frac{\partial v_k}{\partial x_j \partial x_l} + \mathcal{M}_{ijk}\frac{\partial \dot{B}_k}{\partial x_j} - \frac{\partial \dot{p}}{\partial x_i} = 0$$

$$\epsilon_{isp}\left(\mathcal{M}_{i,jk}\frac{\partial v_j}{\partial x_k \partial x_p} + \mathcal{H}_{ij}\frac{\partial \dot{B}_j}{\partial x_p}\right) = 0$$

Add Governing relations

$$\boldsymbol{\nabla}\cdot\dot{\boldsymbol{B}} = 0 \quad , \quad \boldsymbol{\nabla}\times\dot{\boldsymbol{H}} = 0 \quad , \quad \boldsymbol{\nabla}\cdot\dot{\boldsymbol{\tau}} = \boldsymbol{0}$$

Incompressibility

$$\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0$$

Second order System

$$\Rightarrow \boldsymbol{v_1' - w_1 = 0}$$

$$-ik_1q + k_1^2(C_{1122} + C_{1221} - C_{1111})v1 + C_{1212}v_1'' + ik_1B_{112}\mathrm{B}_2 + B_{121}\mathrm{B}_1' = 0$$

$$ik_1v_1 + v_2' = 0$$

$$iq' + ik_1^2C_{2121}v_2 + (C_{1221} + C_{1122} - C_{2222})k_1v_1' + \ k_1(B_{121} - B_{222})\Delta_1 = \ 0$$

$$ik_1\mathcal{B}_1 + \mathcal{B}_2' = 0$$

$$B_{121}v_1'' + (B_{112} + B_{121} - B_{222})k_1^2v_1 + A_{11}\Delta_1' - iA_{22}k_1\Delta_2 = 0$$

First order system

$$\Rightarrow \boldsymbol{v_1' - w_1 = 0}$$

$$+C_{1212}w_1' + B_{121}\Delta_1' + k_1^2(C_{1122} + C_{1221} - C_{1111})v1 + ik_1B_{112}\Delta_2 - ik_1q = 0$$

$$v_2' + ik_1v_1 = 0$$

$$+B_{121}w_1' + A_{11}\Delta_1' + (B_{112} + B_{121} - B_{222})k_1^2v_1 - iA_{22}k_1\Delta_2 = 0$$

$$\mathit{\Delta}_2' + ik_1\mathit{\Delta}_1 = 0$$

$$+iq' + (C_{1221} + C_{1122} - C_{2222})k_1w_1 + ik_1^2C_{2121}v_2 + \ k_1(B_{121} - B_{222})\Delta_1 = \ 0$$

General solution for single medium

$$\boldsymbol{y} = [v_1 \quad w_1 \quad v_2 \quad B_1 \quad B_2 \quad q]^T$$

- First Order system

$$\boldsymbol{A}\boldsymbol{y}' + \boldsymbol{B}\boldsymbol{y} = \boldsymbol{0}$$

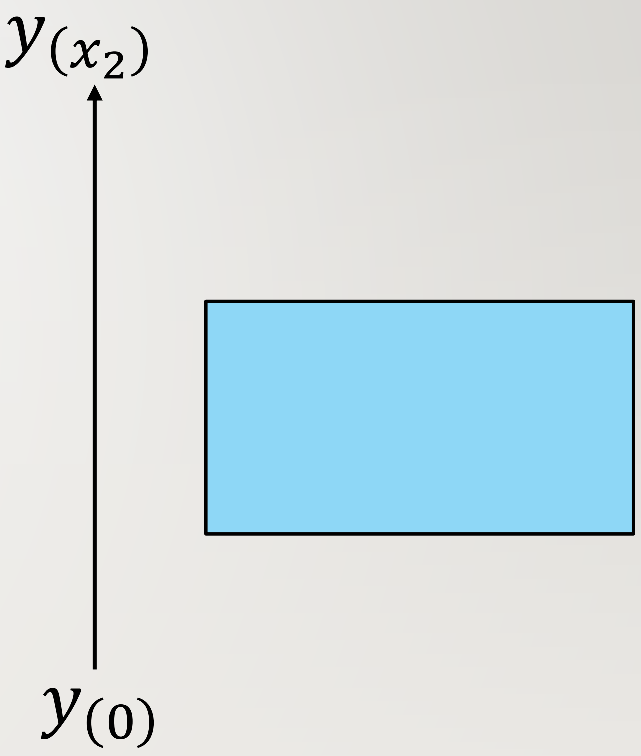
$$\boldsymbol{y}' = -\left(\boldsymbol{A}^{-1}\boldsymbol{B}\right)\boldsymbol{y} = \boldsymbol{V}\boldsymbol{y}$$

- General Solution

$$\boldsymbol{y} = \boldsymbol{W} e^{\boldsymbol{Z} x_2} \boldsymbol{W}^{-1} \boldsymbol{y_0}$$

$\boldsymbol{W} \rightarrow$ Eigen vector matrix (V),

$\boldsymbol{Z} \rightarrow$ Eigenvalue matrix (V)



Add Interface conditions for change in medium

- B-H Interface Conditions (2 eqns)

$$\boldsymbol{N} \cdot [\![\boldsymbol{B}]\!] = 0 \quad , \quad \boldsymbol{N} \times [\![\boldsymbol{H}]\!] = \boldsymbol{0}$$

- Displacement Continuity (2 eqns)

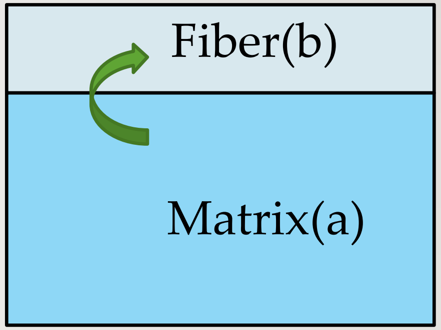
$$[\![\boldsymbol{v}]\!] = \boldsymbol{0}$$

- Traction Continuity (2 eqns)

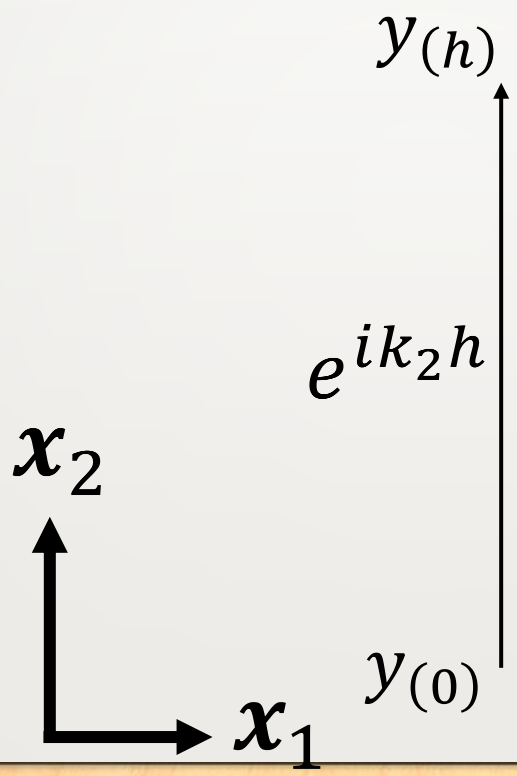
$$[\![\mathbf{T}]\!] \cdot \boldsymbol{N} = \boldsymbol{0}$$

- General form

$$\boldsymbol{Q}_a \boldsymbol{y}_a = \boldsymbol{Q}_b \boldsymbol{y}_b$$
$$\Rightarrow \boldsymbol{y}_b = \left(\boldsymbol{Q}_b^{-1} \boldsymbol{Q}_a\right) \boldsymbol{y}_a$$

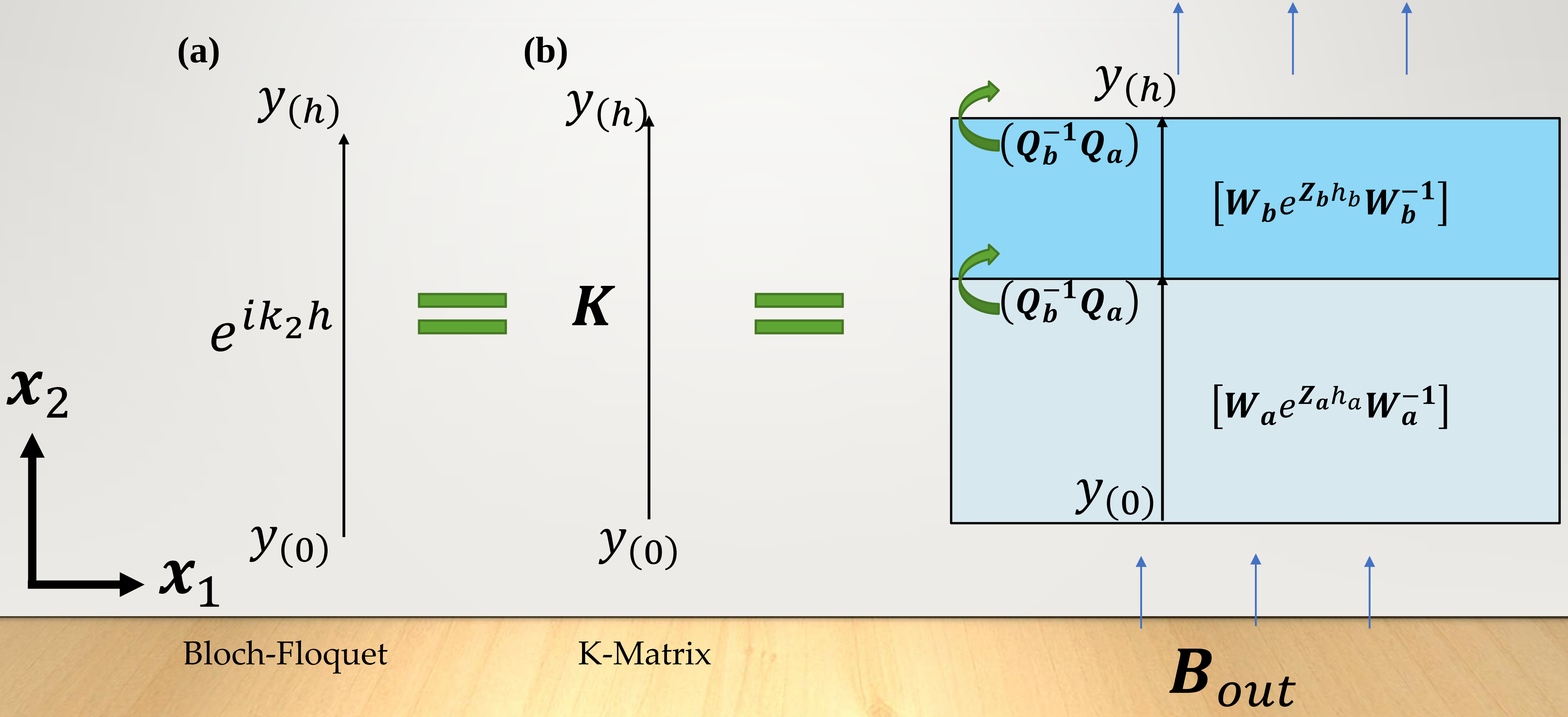


Apply Bloch-Floquet condition to the K-Matrix



Bloch-Floquet

K-Matrix expressions



Eigen value constraint

- Eigen value problem

$$\boldsymbol{y_h} = \boldsymbol{K}\boldsymbol{y_0} = e^{ik_2h}\boldsymbol{y_0}$$
$$\Longrightarrow \boldsymbol{K}\boldsymbol{y_0} = \zeta_i\boldsymbol{y_0}$$

- Eigen value Constraint

$$\zeta_i = e^{ik_2h}$$
$$\Longrightarrow |\zeta_i| = 1$$

Instability condition

Substitute Eigen value constraint in expression

- Eigen value problem

$$\boldsymbol{y_h} = \boldsymbol{K}\boldsymbol{y_0} = e^{i(\phi)}\boldsymbol{y_0}$$
$$\Rightarrow \boldsymbol{K}\boldsymbol{y_0} = \zeta_i \boldsymbol{y_0}$$

$\phi = k_2 h$
(k_2 Periodicity)
$\phi = 0°, 180°, 360°$

- Eigen value Constraint

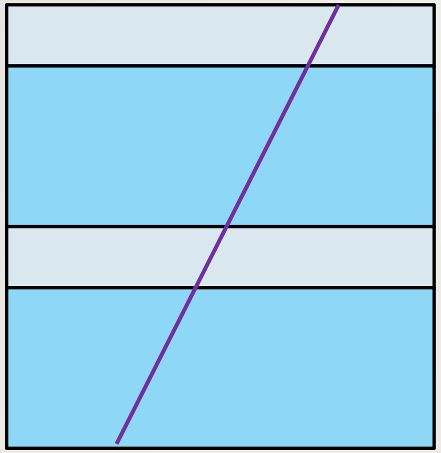
$$f_{B_m}(\lambda, k_1) = |K - \zeta I|$$
$$\Rightarrow f_{B_m}\left(\lambda_{cr}, k_{1_{cr}}\right) = 0$$
$$\forall k_1, \zeta = 1, -1$$

New Instability condition

Three cases of instability

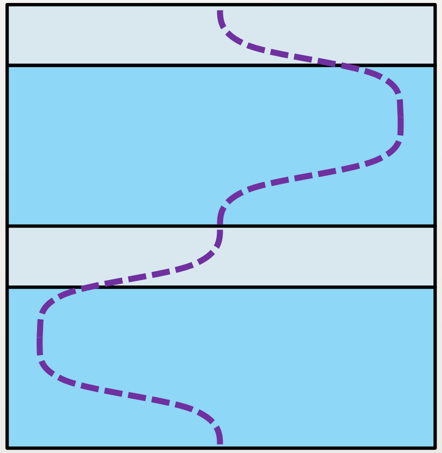
- Macroscopic long wave
  Instability: $\phi = 0°$

$$k_2 h = \frac{k_2 h^0}{\lambda} = 0$$



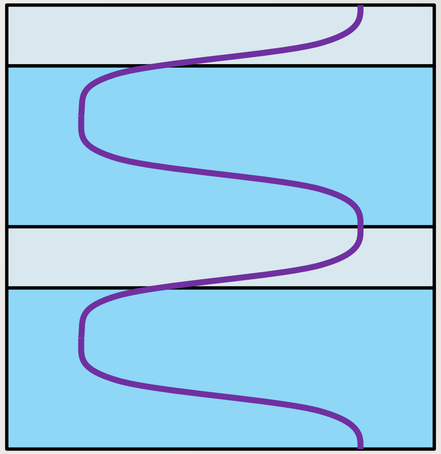
- Microscopic anti-symmetric
  Instability: $\phi = 180°$

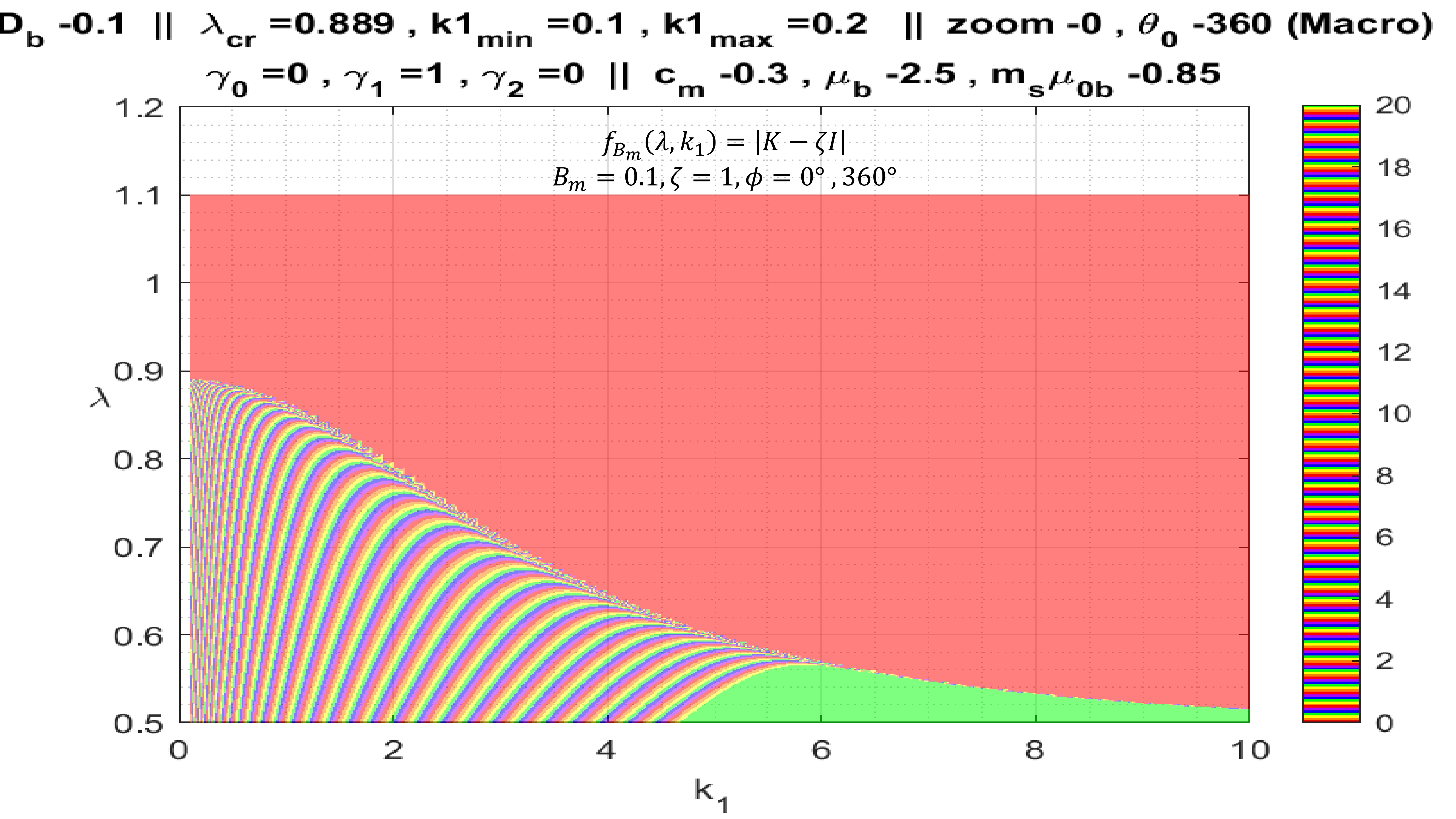
$$k_2 h^0 = \lambda \pi$$



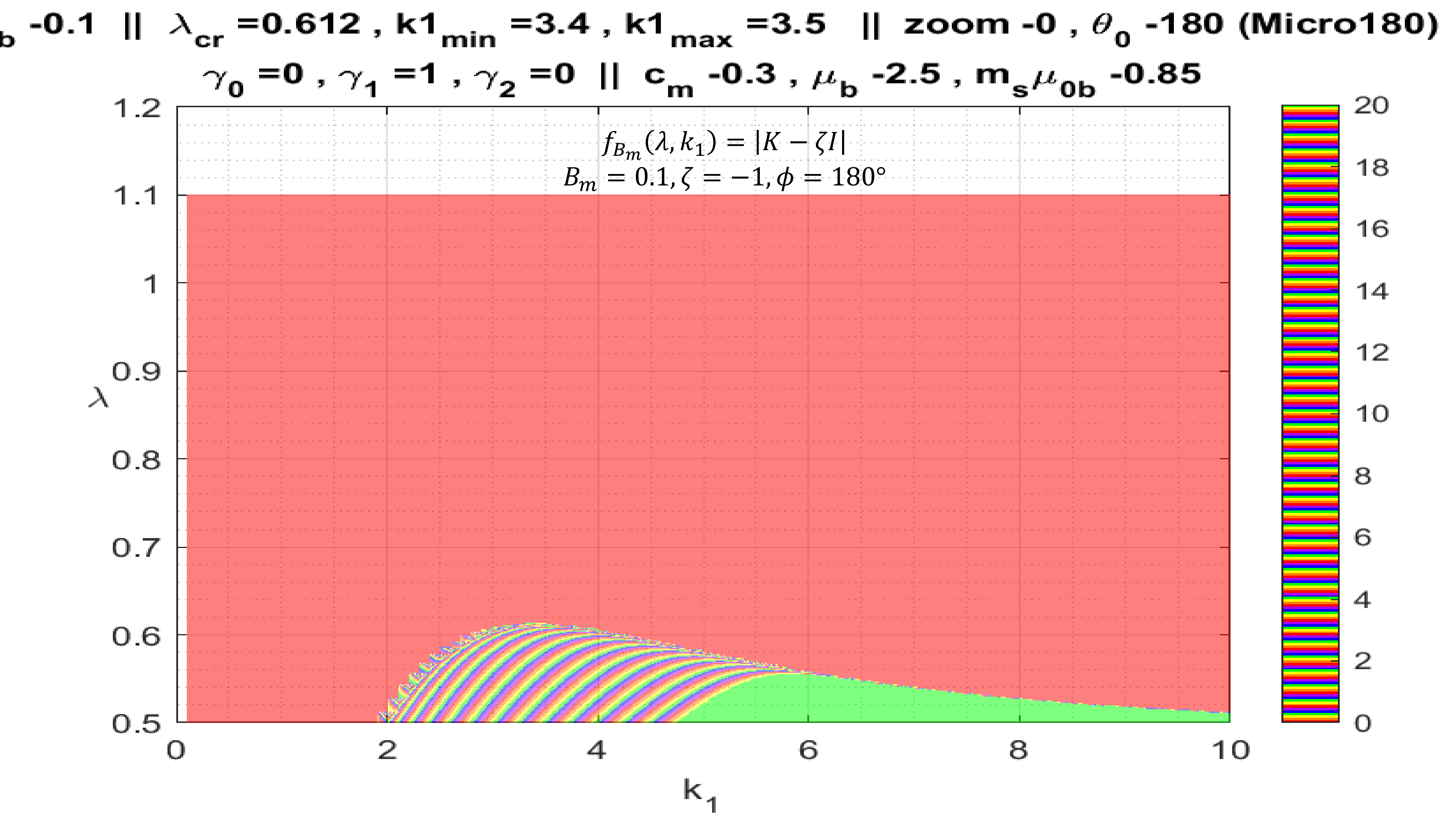
- Microscopic periodic
  Instability: $\phi = 360°$

$$k_2 h^0 = \lambda 2\pi$$

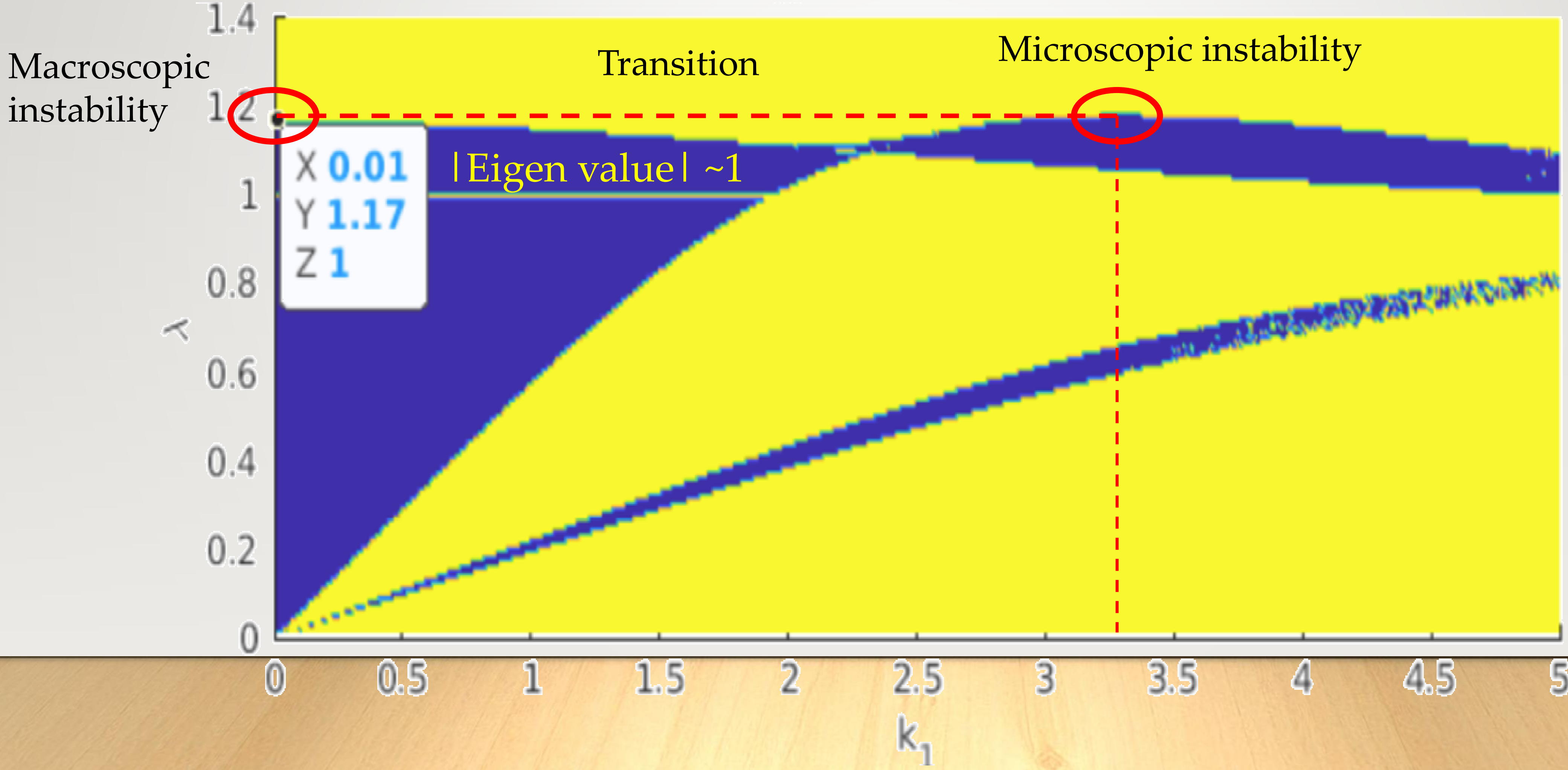




D_b -0.1 || λ_{cr} =0.889 , $k1_{min}$ =0.1 , $k1_{max}$ =0.2 || zoom -0 , θ_0 -360 (Macro)
γ_0 =0 , γ_1 =1 , γ_2 =0 || c_m -0.3 , μ_b -2.5 , $m_s\mu_{0b}$ -0.85
$f_{B_m}(\lambda, k_1) = |K - \zeta I|$
$B_m = 0.1, \zeta = 1, \phi = 0°, 360°$
λ
k_1
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
2
4
6
8
10
20
18
16
14
12
10
8
6
4
2
0

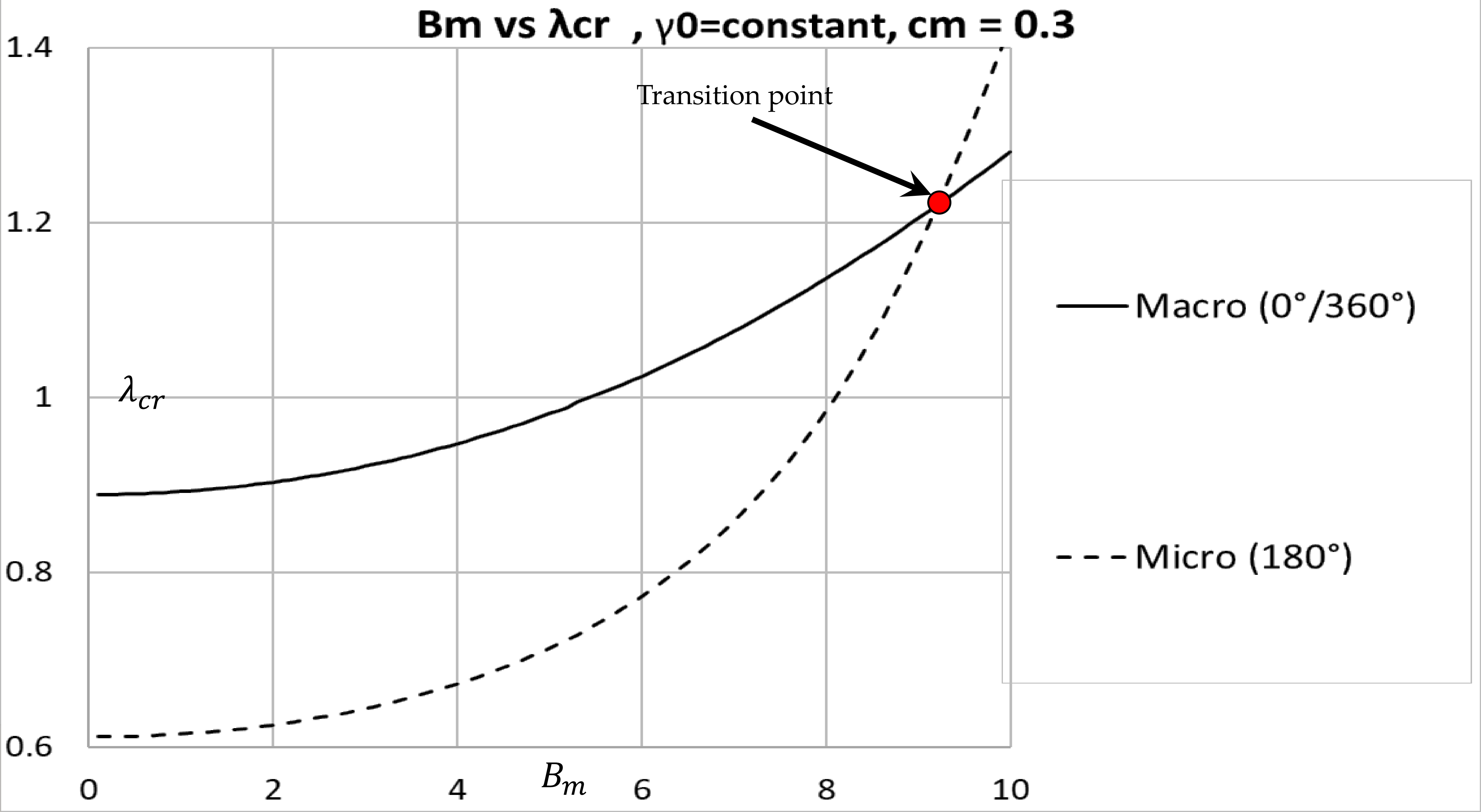

b -0.1 || λ_{cr} =0.612 , $k1_{min}$ =3.4 , $k1_{max}$ =3.5 || zoom -0 , θ_0 -180 (Micro180)
γ_0 =0 , γ_1 =1 , γ_2 =0 || c_m -0.3 , μ_b -2.5 , $m_s\mu_{0b}$ -0.85
$f_{B_m}(\lambda, k_1) = |K - \zeta I|$
$B_m = 0.1, \zeta = -1, \phi = 180°$
λ
k_1

Sample Plot : |Eigen value| vs λ, k_1

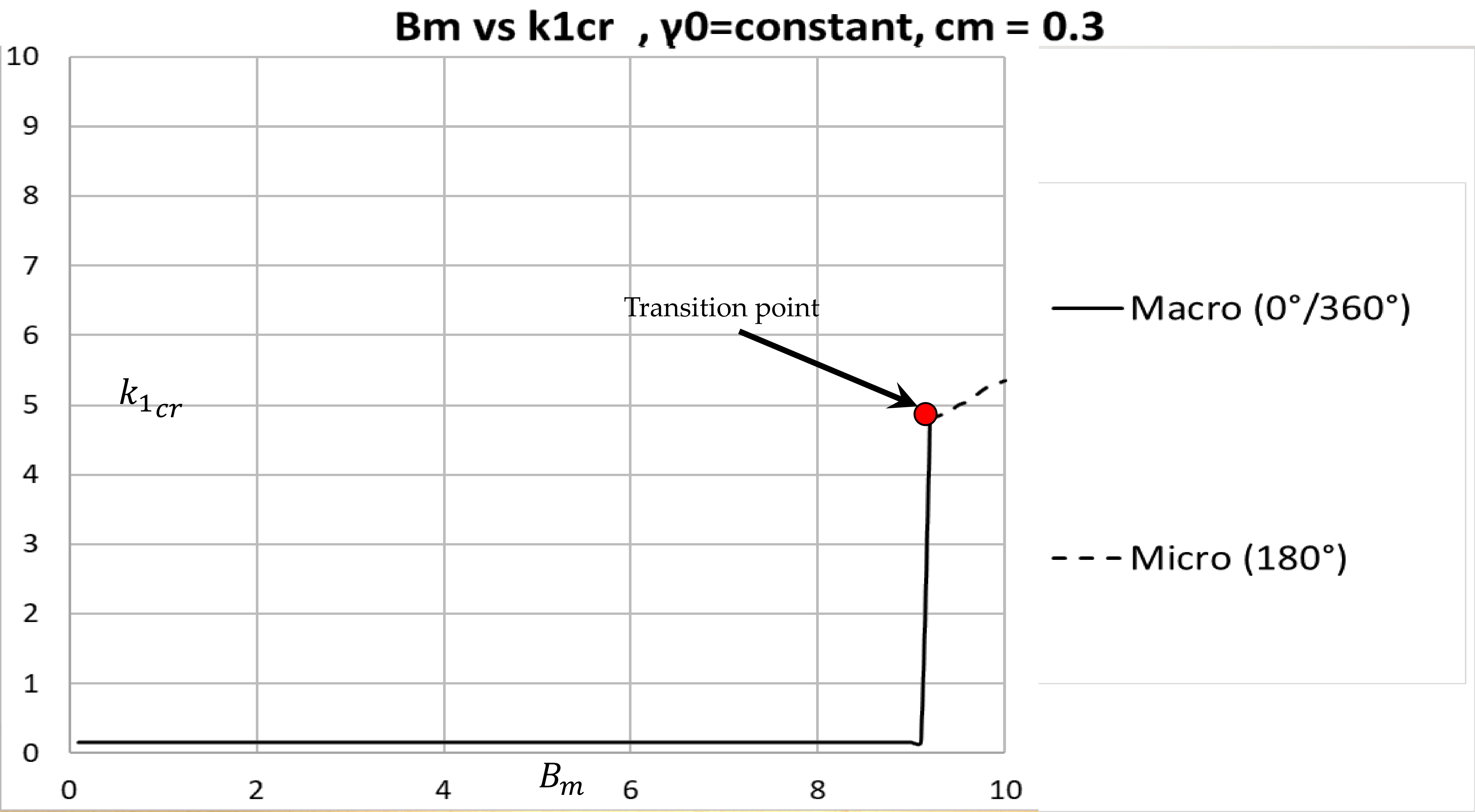


$\mathrm{B}_m\ vs\ \lambda_{cr}$

Transition point from macroscopic to microscopic



Bm vs λcr , γ0=constant, cm = 0.3
1.4
1.2
1
0.8
0.6
0
2
4
6
8
10
λ_{cr}
B_m
Transition point
Macro (0°/360°)
Micro (180°)



Bm vs k1cr , γ0=constant, cm = 0.3
Transition point
$k_{1_{cr}}$
B_m
Macro (0°/360°)
Micro (180°)
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10

Transition point Analysis

- The transition from macroscopic instability to microscopic depends on where the two curves (0° and 180°) intersect.

$$\boxed{\Longrightarrow G_m = 1 \quad , \quad \mu_0 = 1 \quad , \quad \mu_\mathrm{m} = 1 \quad , \quad h^0 = 1}$$

Non-dimentional numbers

- Magnetic field

$$\bar{\mathcal{B}}_s = \frac{B}{m_s \mu_0} \quad , \quad \bar{\mathcal{B}}_m = \frac{B}{\sqrt{G_m \mu_0}} = B$$

- Wavenumber normalization

$$\bar{k}_1 = k_1 h^0 \quad , \quad \bar{k}_2 = k_2 h^0$$

- Shear ratio

$$\Gamma = \frac{G_f}{G_m} = 10$$

- Permeability ratio

$$\mu = \frac{\mu_f}{\mu_m} = 2.5$$

- Saturation co-efficient

$$\eta = \frac{m_s \mu_0}{\sqrt{G_m \mu_0}}$$

- Volume fractions

$$c_m = \frac{h_m}{h} \quad , \quad c_f = \frac{h_f}{h}$$

- Initial Susceptibilities

$$\chi = \frac{\mu_0 M}{B} = \frac{\mu - 1}{\mu}$$

Energy model

- **Linear Magnetic** : Neo-Hookean + Magnetic Energy.

$$\Psi(\boldsymbol{F},\boldsymbol{B}^0) = \frac{G_m}{2}(I_1 - 3) + \frac{1}{2\mu_0\mu J}(\gamma_0 I_4 + \gamma_1 I_5 + \gamma_2 I_6)$$

$$\Longrightarrow \gamma_0 + \gamma_1 + \gamma_2 = 1$$

- **Additional Invariants** :$\Psi'(I_4, I_5, I_6)$

$$I_4 = \boldsymbol{B^0} \cdot \boldsymbol{B^0}$$

$$I_5 = \boldsymbol{FB^0} \cdot \boldsymbol{FB^0}$$

$$I_6 = \boldsymbol{CB^0} \cdot \boldsymbol{CB^0}$$

$$at\ \boldsymbol{F} = \boldsymbol{I}, I_4 = I_5 = I_6 = I_m$$

$$\Psi'(I_4, I_5, I_6) = \Psi(I_m)$$

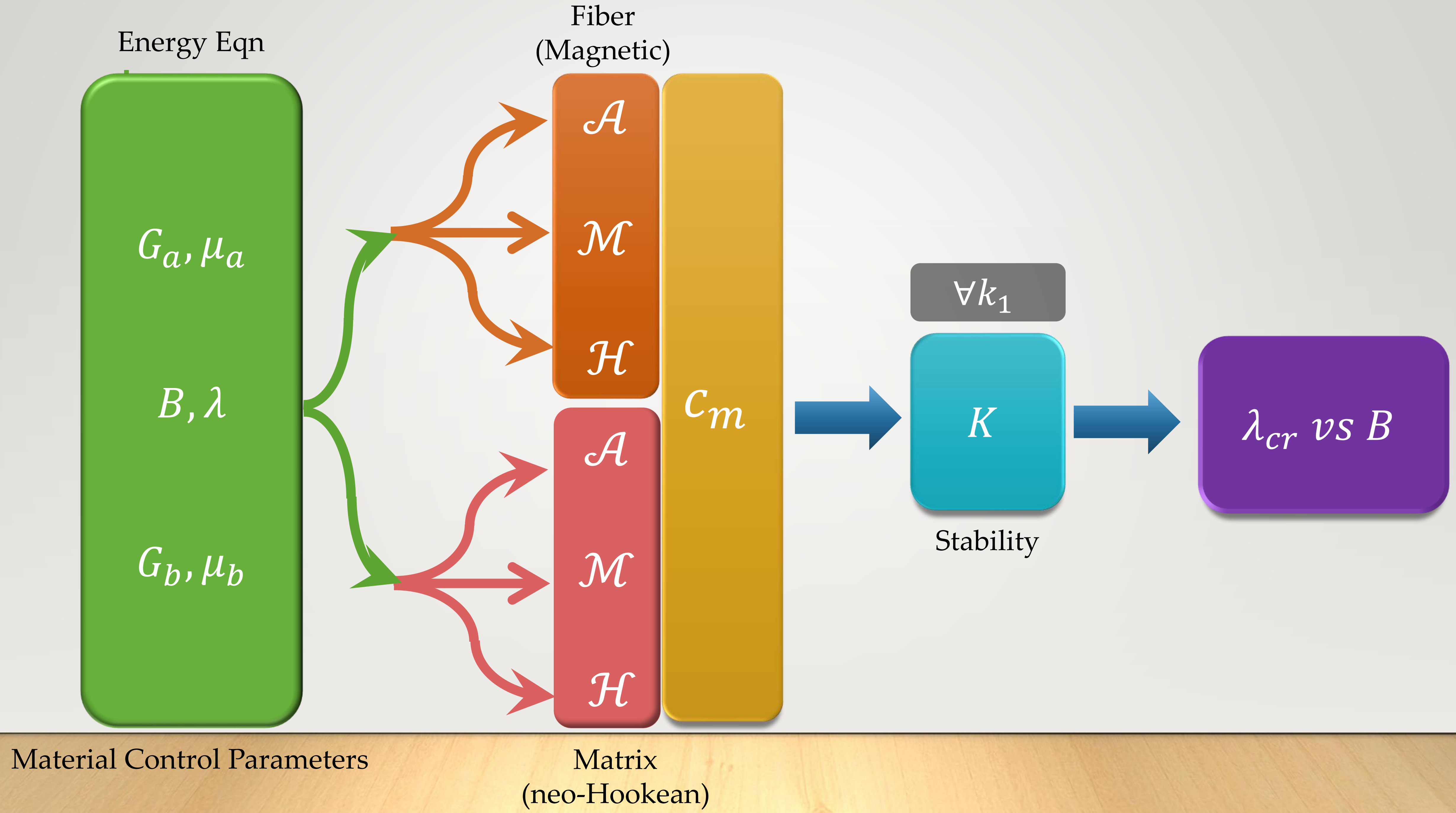
$$(\gamma_0 I_4 + \gamma_1 I_5 + \gamma_2 I_6) = I_m$$

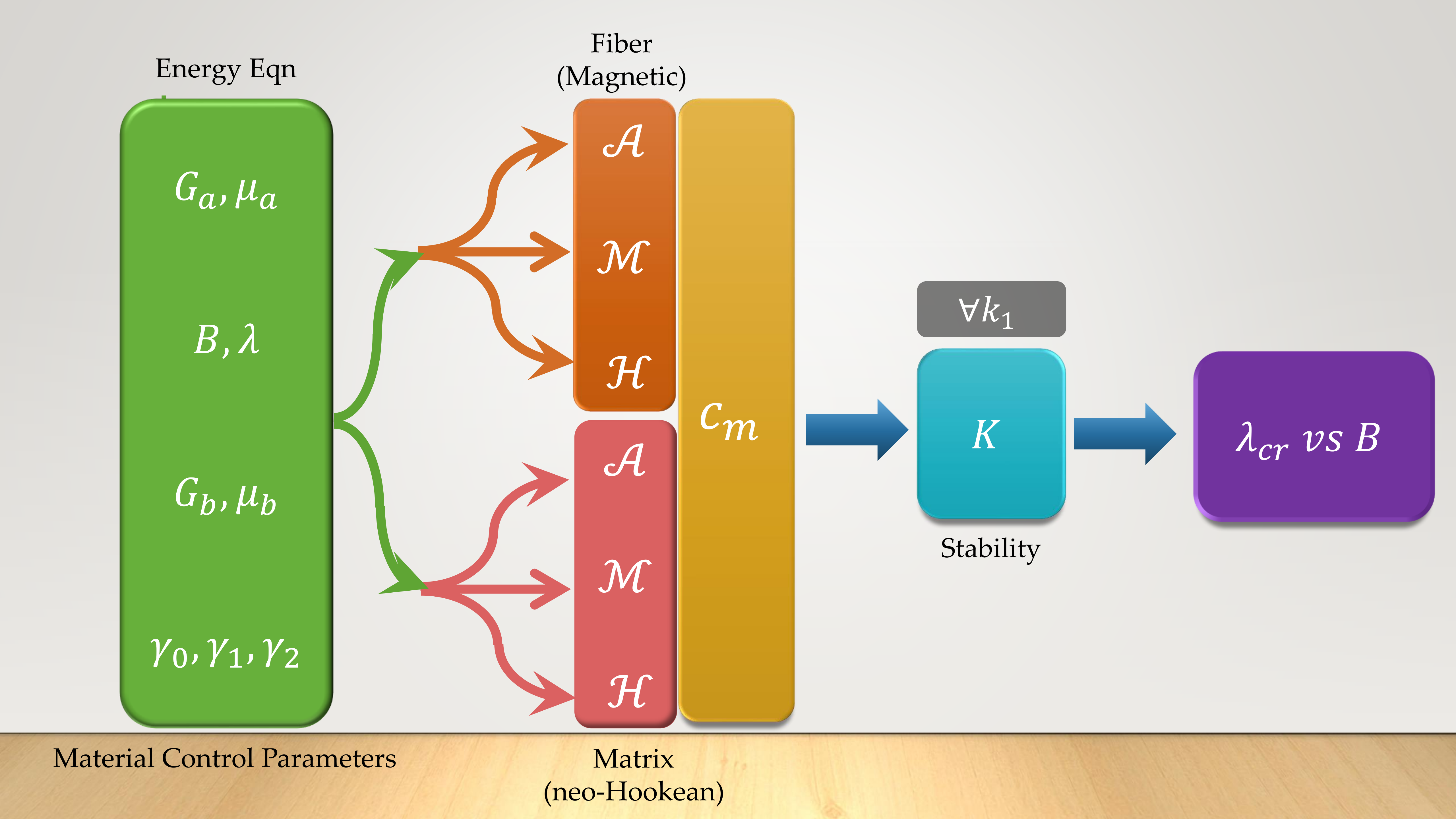
Eigen value expressions

Sample Γ Coefficients (Dielectric case)

Table 1
Material constants of DE model (13).

Reference	γ_0	γ_1	γ_2
Ideal DE (Zhao et al., 2007)	0	1	0
Wissler and Mazza (2007)	0.00104	1.14904	−0.15008
Li et al. (2011)	0.00458	1.3298	−0.33438


Energy Eqn
G_a, μ_a
B, λ
G_b, μ_b
Material Control Parameters
Fiber
(Magnetic)
$\mathcal{A}$
$\mathcal{M}$
$\mathcal{H}$
$\mathcal{A}$
$\mathcal{M}$
$\mathcal{H}$
Matrix
(neo-Hookean)
c_m
$\forall k_1$
K
Stability
$\lambda_{cr}\ vs\ B$



Energy Eqn
G_a, μ_a
B, λ
G_b, μ_b
$\gamma_0, \gamma_1, \gamma_2$
Material Control Parameters
Fiber
(Magnetic)
$\mathcal{A}$
$\mathcal{M}$
$\mathcal{H}$
$\mathcal{A}$
$\mathcal{M}$
$\mathcal{H}$
Matrix
(neo-Hookean)
c_m
$\forall k_1$
K
Stability
$\lambda_{cr}\ vs\ B$

Linear Model for Fiber Phase:

$$\mathcal{A}_{ijkl} = J^{-1} F_{j\alpha} F_{l\beta} \left(\frac{\partial^2 \Psi(F, B^0)}{\partial F_{ij} \partial F_{kl}} \right)$$

$$\mathcal{M}_{ijk} = F_{j\alpha} F_{k\beta}^{-1} \left(\frac{\partial^2 \Psi(F, B^0)}{\partial F_{ij} \partial B_k^0} \right)$$

$$\mathcal{H}_{ij} = J F_{i\alpha}^{-1} F_{j\beta}^{-1} \left(\frac{\delta^2 \Psi(F, B^0)}{\partial B_i^0 \partial B_k^0} \right)$$

$$\mathcal{H}_{11} = \mathcal{H}_{22} = \frac{1}{\mu \mu_0}$$

$$\mathcal{M}_{121}(B) = \mathcal{M}_{211}(B) = \frac{1}{\mu \mu_0} B$$

$$\mathcal{M}_{222}(B) = \frac{2}{\mu \mu_0} B$$

$$\mathcal{A}_{1111}(\lambda) = \mathcal{A}_{2121}(\lambda) = G_i \lambda^2$$

$$\mathcal{A}_{1212}(\lambda, B) = \mathcal{A}_{2222}(\lambda, B) = \frac{G_i}{\lambda^2} + \frac{B^2}{\mu \mu_0}$$

Linear Model for Fiber Phase: Γ Coefficients

$$\mathcal{H}_{11} = \mathcal{H}_{22} = \frac{1}{\mu\mu_0}\left(\frac{\gamma_0}{\lambda^2} + \gamma_1 + \gamma_2\lambda^2\right)$$

$$\mathcal{M}_{121}(B) = \mathcal{M}_{211}(B) = \frac{1}{\mu\mu_0} B\left(\frac{\gamma_0}{\lambda^2} + \gamma_1 + \gamma_2\lambda^2\right)$$

$$\mathcal{M}_{222}(B) = \frac{2}{\mu\mu_0} B\left(\gamma_1 + 2\frac{\gamma_2}{\lambda^2}\right)$$

$$\mathcal{A}_{1111}(\lambda) = G_i\lambda^2$$

$$\mathcal{A}_{1221}(\lambda) = \mathcal{A}_{2112}(\lambda) = \frac{B^2}{\mu\mu_0}(\gamma_2\lambda^2)$$

$$\mathcal{A}_{2121}(\lambda) = G_i\lambda^2 + \frac{B^2}{\mu\mu_0}(\gamma_2\lambda^2)$$

$$\mathcal{A}_{1212}(\lambda, B) = \frac{G_i}{\lambda^2} + \frac{B^2}{\mu\mu_0}\left(\gamma_1 + \gamma_2\left(\frac{2}{\lambda^2} + \lambda^2\right)\right)$$

$$\mathcal{A}_{2222}(\lambda, B) = \frac{G_i}{\lambda^2} + \frac{B^2}{\mu\mu_0}\left(\gamma_1 + \frac{6\gamma_2}{\lambda^2}\right)$$

B-H Relationships

- Stress-Magnetization-Energy relationship

$$\boldsymbol{P} = \frac{\partial \Psi(\boldsymbol{F}, \boldsymbol{B}^0)}{\partial \boldsymbol{F}} - p\boldsymbol{F}^{-T}$$

$$\boldsymbol{H}^0 = \frac{\partial \Psi(\boldsymbol{F}, \boldsymbol{B}^0)}{\partial \boldsymbol{B}^0}$$

B-H Relationships

	Linear Magnetic model	With Γ Coefficients
Magnetization	$\mu_0\boldsymbol{M} = \boldsymbol{B}\chi = \boldsymbol{B}\frac{(\mu-1)}{\mu}$	$\mu_0\boldsymbol{M} = \boldsymbol{B}\chi' = \boldsymbol{B}\frac{(\mu'-1)}{\mu'}$
Magnetic intensity	$\mu_0\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \boldsymbol{B}(1-\chi)$	$\mu_0\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu'} = \frac{\boldsymbol{B}}{\mu}\left(\frac{\gamma_2}{\lambda^2} + \gamma_1 + \gamma_0\lambda^2\right)$

Conclusions

- Instability can be of three types long wave microscopic , microscopic periodic and anti symmetric periodic. Transitions usually happen from long wave to anti-symmetric microscopic.
- Higher γ_2 values lowers the λ_{cr}. Higher γ_0 values lowers the λ_{cr} for low magnetic fields, but this trend reverses for higher magnetic fields.
- Transitions happen at lower magnetic fields for higher γ_0 values and lower volume fraction of matrix c_m.