Peer Grading Rubric PHSX ~~516~~ 616

Date   Reviewer   Author Theresa Kelly   Title of paper Departures of Linearity in Driven Torsional Oscillations

	Good	Fair	Poor
Quality of data	Cross checked data and worked to improve results.	Completed all aspects of experiment but no attempt to cross check	Followed manual, no attempt to fix issues
Title	Evokes interest in physics	Plain statement of what was done	Misleading
Abstract	Clearly states what was done, provides final result & explains significance	States what was done but with no physics context significance of this study	Does not include final result nor put the result into context
Introduction	Sets current experiment in physics context of the time. Explains theory or theories that are relevant. $N_{citations} > 2$	Explains theory but does not highlight significance of experiment $N_{citations} = 1$	Poor explanation of theory $N_{citations} = 0$
Method	Reader feels that they really understand how to repeat the experiment. Diagram of apparatus is clear.	Most steps are explained but some missing. Figure may be poor.	Explanation is garbled Figure is missing or very poor.
Results	This section begins with "Figure 2 shows stopping voltage ….. "Figures are clear and uncluttered. Legend font at least 12pt Each fig cited. Systematic & statistical uncertainties estimated.	Figure are cited but for some Figures no conclusions are made about the data. Captions are incomplete. Legends smaller than 12pt text.	Figures are not cited in text. Captions are unclear, Figures are cluttered with legends smaller than 12pt text in paper.
Uncertainties	X^2 is reasonable	X^2 unreasonable	No uncertainty analysis
Summary	Recapitulates what was measured and why it matters. Compares to other results	States final result but does not put it in context why does it matter? eg any applications or uses in other physics context?	Missing or makes claims not supported by data.
Writing	Spelling & grammar correct. Paper gradually builds up an argument-based on data.	A few grammar mistakes. Text doesn't clarify what can be learned from data.	Many grammar mistakes. Text does not make a coherent argument.
Improvements from draft	Final report addresses all concerns	Some corrections applied.	Final report essentially the same as draft

67/70

Departures of linearity in driven torsional oscillations

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We inspect the non-linear properties of magnetic and sliding damping in a driven torsional oscillator. We find that the resonant frequency of the apparatus occurs at a driving frequency of 862±2 mHz 864±2 mHz for a driving amplitude of 2.500±0.005 V and 1.125±0.005 V, respectively. We operate the device at these resonant frequencies when testing the magnetic and sliding damping. For magnetic damping, the resultant amplitude linearly increases (v^1) with the waveform amplitude up until around 0.18±0.02 V before leveling off; at higher angular velocities, there is less time for Eddy currents to form to oppose motion. For sliding damping, the oscillator does not move until a threshold, which is inconsistent between trials, where it overcomes the friction from the cable. It then sharply increases before leveling off. The sliding damping is not clearly correlated with velocity (v^0).

I. INTRODUCTION

Oscillations occur in a a wide variety of physical systems. In the ideal case, the amplitude of the oscillation is constant. The simplest case of simple harmonic motion is Hooke's law

$$F(x(t)) = -k \cdot x(t) \tag{1}$$

where F is force, x is position, and k is the spring constant [1]. Newton's second law ($F = m\ddot{x}$; [2]) determines the acceleration ($\ddot{x}$) of the system

$$\ddot{x}(t) = -\frac{k}{m}x(t) = -\omega_0^2 x(t) \tag{2}$$

where m is mass. The differential equation can be solved to get the oscillating position

$$x(t) = A_0 \cos(\omega_0 t - \delta_0) \tag{3}$$

in which A is the amplitude, ω is the oscillating frequency, and δ is the phase shift due to the initial condition.

However, the physical world is not under ideal conditions. The oscillating system is often damped by outside forces that dissipate energy from the system. Using Newton's second law with a resistance force (2β) gives

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \tag{4}$$

whose solution is

$$\begin{aligned} x(t) &= e^{-\beta t}\left(C_1 e^{\omega_1 t} + C_2 e^{-\omega_1 t}\right) \\ \omega_1 &= \sqrt{\beta^2 - \omega_0^2} \end{aligned} \tag{5}$$

The effects of damping can be opposed by adding a driving force $f(t)$ to the system. Using Newton's second law yields the following expression:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t) \tag{6}$$

We use a sinusoidal driving force in this work, given by

$$f(t) = A_f \cos(\omega t + \delta) \tag{7}$$

Solving Equation 6 using the driving force in Equation 7 gives the solution

$$\begin{aligned} x(t) &= A\cos(\omega t - \delta) + A_{tr}\cos(\omega_1 t - \delta_{tr}) \\ A &= \sqrt{\frac{A_f^2}{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}} \\ \delta &= \tan^{-1}\left(\frac{2\beta\omega}{\omega_0^2 - \omega^2}\right) \end{aligned} \tag{8}$$

where the transient (tr) term results from the initial conditions.

In an oscillating system that is driven and damped, resonance occurs when the driving frequency ω equals the natural frequency ω_0. The amplitude is maximized at resonance.

Equation 8 shows that the driving amplitude A_f is proportional to the resultant amplitude A and are thus linearly related. However, this relation is based on ideal assumptions about damping and driving forces. Mechanical devices do not operate under physically ideal conditions. In this research, we will be inspecting sources of non-linearity in torsional oscillations. We vary the amplitude of the driving signal and record the amplitude of the response for magnetic and sliding damping.

II. METHOD

Figure 1 shows a diagram of the torsional oscillator used in this research. The oscillations of the apparatus can be damped or driven using magnets and inductors. A pair of magnets generate a magnetic field from the North to South magnet. Moving metal between the magnets causes a changing magnetic field in the metal, which creates a counterclockwise electric field. The electric field induces circular currents, called Eddy currents, that an opposing magnetic field; this causes the moving metal to decelerate. In the apparatus, there are two pairs of magnets surrounding the oscillating disk that dampen the torsional oscillating motion. These magnets can be adjusted to cover more or less area on the oscillator to

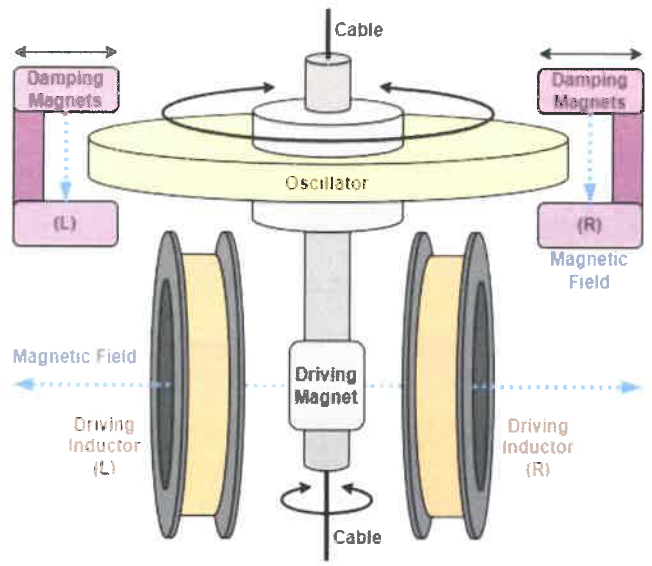

FIG. 1. Torsional oscillator apparatus [3]. A large central copper disk (yellow) is suspended by a taut cable and can twist freely. There are two sets of magnets (purple) on the left and right sides of the disc which can be moved closer and farther from the oscillating disk. These magnets create Eddy currents in the disk which oppose the changing magnetic field, which dampen the oscillations. Below the disc are a two inductors (orange) whose current is controlled by an external wave generator. The inductors create a magnetic field that generates a force on a central magnet, which drives the oscillations. The torsional motions of the disc is measured with an oscilloscope.

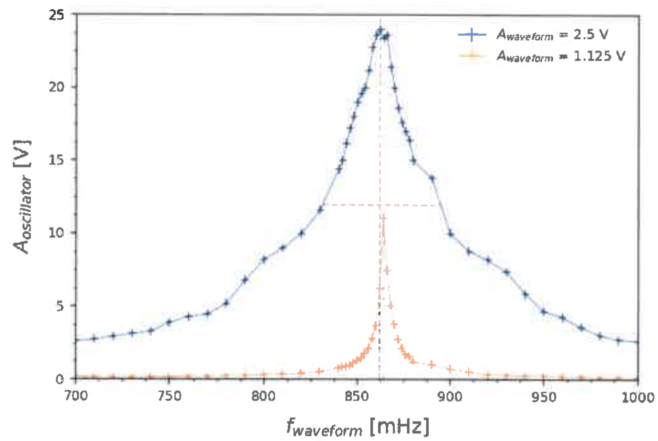

FIG. 2. Amplitude of torsional oscillations (A) as a function of the driving waveform frequency (f). The horizontal lines show the full-width half max. The vertical dotted lines show the resonant frequency. For the blue curve, we set the source amplitude ($A_{waveform}$) to 2.500±0.005 V, which has resonance at 862±2 mHz. The orange curve has an $A_{waveform}$ of 1.125±0.005 V is resonant at 864±2 mHz. Data is taken at higher resolution near the resonant peak.

increase or decrease the damping. Magnetic fields can also be used to increase oscillations. An inductor, or coil of current carrying wire, generates a strong magnetic field through the core. The field applies forces on external charges. In the apparatus, there is a magnet connected to the oscillating disk placed between two inductors. Powering the inductors attracts or repulses the central magnet, depending on the direction of the current. Oscillating the inductor currents drives torsional oscillations in the apparatus.

In this research, we will be inspecting the non-linear properties of the torsional oscillator apparatus. Under ideal conditions, the driving inductors should cause linear changes in the torsional oscillations: increasing the amplitude of the source waveform increases the amplitude of rotational motion. To test the extent of the linear range, we will increase the amplitude of the source waveform at its resonant frequency and record the resultant amplitudes of the oscillations. After testing the v^1 magnetic damping, we attached a string across the oscillating column of the apparatus to introduce friction. We then observe how the driving amplitude changes with respect to the response amplitude to quantify the v^0 sliding damping effects.

III. RESULTS AND DISCUSSION

In this work, we test the non-linearity of v^1 and v^0 damping in driven oscillations. We claim the following instrumental uncertainties: the driving amplitude is ±0.005 V, the driving frequency is ±2 mHz, the response amplitude is ±0.5 V

We determine the resonant frequency of the apparatus by varying the frequency of the driving waveform and measuring the amplitude of the response. Resonance occurs when the amplitude is maximized. The results are shown in Figure 2. We find the resonance for two values of the waveform amplitude: 2.500±0.005 V and 1.125±0.005 V; we find that the apparatus is resonant at a driving frequency of 862±2 mHz and 864±2 mHz, respectively. The amplitude drops off rapidly at lower and higher frequencies.

Next, we inspect the v^1 damping from the magnets located above and below the copper oscillator disk. These magnets are adjusted to the furthest distance from the disk to provide weak damping. We vary the driving amplitude from 0.005±0.005 V to 0.500±0.005 V at the resonant frequency and measure the resulting torsional amplitude; the results are shown in Figure 3. The apparatus behaves linearly up to around a source amplitude of 0.18±0.02 V. Beyond this, the response begins to drop off towards an asymptote. The disk is oscillating quickly at these high frequencies, which leaves less time for the Eddy currents to form in the disk and oppose the velocity.

To investigate v^0 sliding damping, we place two cables across the central oscillator column. We add 150±2 g weights to the end of each cable. These cables provide

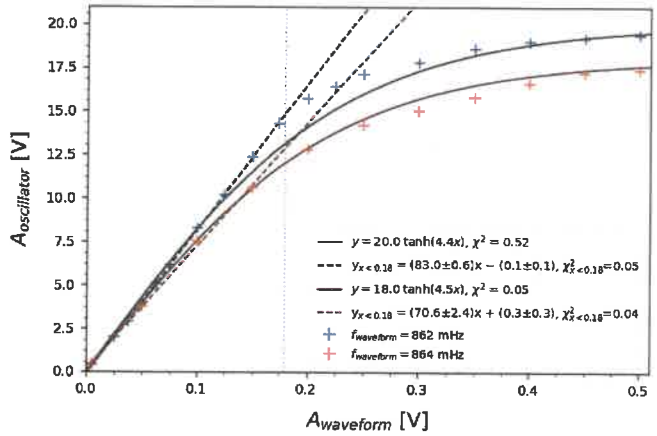

FIG. 3. Response amplitude for varying driving amplitude voltages with magnetic v^0 damping. The response is linear up until a $A_{waveform} < 0.18 \pm 0.02$ V, where the slope begins to decrease. The crosses mark the data points with error bars, the solid line is a hyperbolic tangent model, and the dashed line is a linear model. The blue points mark a waveform frequency of 862±2 mHz and the orange had 864±2 mHz.

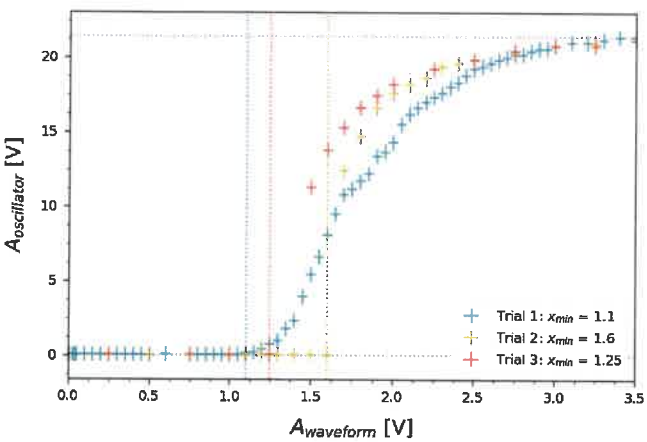

FIG. 4. Response amplitude for varying driving amplitude voltages with sliding v^1 damping and magnetic v^0 damping. All trials used a driving frequency of 864±2 mHz. The oscillator does not move until it is able to overcome the friction, then sharply increases before leveling off around 21.4±0.2 V. Each trial overcomes friction at different waveform amplitude.

some frictional forces to oppose the oscillations. Figure 4 shows the oscillating amplitude with a variable driving amplitude. All trials do not oscillate until they overcome a threshold waveform amplitude; this threshold is significantly different for each trial. The oscillator then sharply increases. The slope begins to decrease until the amplitude levels off to 21.4±0.2 V for a waveform amplitude 3.500±0.005 V. The system is very sensitive to external vibrations, which can cause the apparatus to overcome the friction threshold and begin oscillating. The sliding damping is not linearly related to the oscillator velocity.

IV. CONCLUSION

The goal of this research was to define the non-linear properties of driven torsional oscillations for two different sources of damping, each having a different relationship with the rotational velocity (v). Magnetic damping (v^1) occurs by placing the oscillating copper disk between magnets and generating Eddy currents. Sliding damping (v^0) results from friction in the cable. To perform this research, we use a torsional oscillator apparatus, which consists of a copper disk suspended between tight cables, two driving inductor coils, and two sets of damping magnets.

First, we find the resonant frequency of the apparatus for two amplitudes of the driving waveform, 2.500±0.005 V and 1.125±0.005 V, to be 862±2 mHz and 862±2 mHz, respectively. To test the non-linear limits of the apparatus, we vary the driving waveform amplitude for constant damping. First, we test just the magnetic damping. The resulting amplitude of the oscillator is linear (v^1) for waveform amplitudes less than 0.8 V, before leveling off around 0.5 V; a hyperbolic tangent provides a good fit for the full range of source amplitudes. At higher amplitudes, the disc is moving too quickly to be effectively slowed by the damping Eddy currents from the magnets. Next, we add cables to provide sliding damping on the oscillator. We observe that the oscillator cannot move until it has enough energy to overcome the starting friction. Then, the resultant amplitude sharply increases with driving amplitude before leveling off to 21.4±0.2 V. The starting waveform amplitude is highly variable, showing how sensitive the apparatus is to external vibrations. The damping is not related to the velocity of the oscillator (v^0).

V. IMPROVEMENTS

Here is a list of the improvements from the previous draft of this research paper: (1) completed the abstract and introduction, (2) improved the data presentation, (3) modeled the waveform and result amplitudes for v^1 magnetic damping, (4) expanded the analysis to include v^0 sliding damping, and (5) added new results to the conclusion and abstract.

[1] R. Hooke, *Lectures de potentia restitutiva or of spring explaining the power of Springing Bodies* (1678).

[2] I. Newton and A. Liechti, *Philosophiae Naturalis Principia Mathematica* (Apud Guil. &; Joh. Innys, 1726).

[3] TeachSpin, Torsional oscillator (2008).