Effects of variable high voltage and emission current on X-ray energy spectrum of molybdenum.

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In the atomic current model, electrons orbit the nucleus in discrete energy levels. Electrons can jump to higher or lower energy levels by absorbing or emitting photons, respectively. Thus, scientists can examine the atomic structure of a molecule by analyzing its spectra. In this research, I will be inspecting the effects of voltage and current of an X-ray tube with a molybdenum anode. I find that the emission spectra consists of a continuous Bremsstrahlung radiation spectrum with two emission lines at wavelengths of 61.9 pm and 69.7 pm. These emission lines are only visible at voltages greater than 20 kV. Increasing the voltage causes a decrease of the minimum wavelength, decrease in the wavelength of maximum Bremsstrahlung radiation, an increase in the Bremsstrahlung photon frequency, and an increase in the emission line photon frequency. The voltage creates photons with higher energies that are able to penetrate deeper into the molybdenum and scatter larger amounts of electrons. Increasing the current does not affect the minimum or peak Bremsstrahlung wavelengths, but shows a moderate increase in the photon frequency across the spectrum. The current creates a larger number of high energy photons to bombard the molybdenum.

I. INTRODUCTION

The atomic modal has gone through several major changes throughout history. John Dalton proposed the "solid sphere" model of the atom in 1803. He claimed that atoms are indivisible particles characterized by their mass that can be combined in whole-number ratios to make compounds. Next, in 1897, Joseph John Thomson discovered a negatively-charged particle–the electron. Thus, the atom was not indivisible; instead, it had a complex internal structure. This supported Lord Kelvin's model, which claimed that the atom was a sphere of positive charge with pockets of negatively-charged electrons. The atomic model was further refined in 1911 by Ernest Rutherford. From his famous gold-foil experiment, Rutherford concluded that atoms must have a small, dense, positively charged core–the nucleus–with the electrons orbit randomly around it. Soon after, Niels Bohr proposed his quantized shell model. Here, the electrons orbit the nucleus of fixed energy. Electrons can absorb energy from a photon and jump to a higher orbital with a higher energy level, or emit energy and jump to a lower level.

Bremsstrahlung radiation occurs when highly energetic, free electrons decelerate, moving to a lower energy level, when they contact matter. This produces a continuous spectrum of X-ray photons. Electrons penetrating deep into the atomic shell produce characteristic emission lines that peak above the continuous spectrum. These eject the innermost orbital electrons via collisions, which create gaps that the outer-orbital electrons fall into. Energetic, free electrons can be produced by an electrical voltage or current. Electrons emitted from the cathode are incident on the anode, and produce X-ray emission lines unique to the anode material.

One method of observing an X-ray spectra is by reflecting the X-rays off of a crystal. Bragg's law of refraction (Equation 1) relates the scattering angle (θ) to the wavelength (λ) of first order diffraction [1]. In this research, I use a NaCl crystal with a lattice plane spacing (d) of 282.01 pm.

$$\lambda = 2d\sin\theta \tag{1}$$

The energy of the X-ray radiation can be calculated using electromagnetic frequency equations (Equation 2) where ν is the frequency, and c is the speed of light, and h is Planck's constant [2].

$$\nu = \frac{c}{\lambda}\,,\ E = h\nu \tag{2}$$

In this research, I investigate the x-ray energy spectrum of molybdenum X-ray tube. I observe the effects of varying the high voltage and emission current on the resulting spectrum.

II. METHOD

For this research, I use the 554 800 Rontgengerat X-ray apparatus [3]. The apparatus is controlled by a microprocessor, which controls the X-ray tube, goniometer, and Geiger-Muller counter tube [3]. The X-ray tube is a hot cathode tube with a molybdenum (Mo) anode. The anode material is $K_\alpha = 17.4$ keV and $K_\beta = 19.6$ keV with a voltage range of $0.0\text{ kV} < U < 35\text{ kV}$ and current range of $0.0\text{ mA} < I < 1\text{ mA}$. The goniometer consists of two independent stepper motors to control the position of the target stage and sensor arm. It has an angular resolution of $1°$ across a range of $-10°$ to $170°$. The NaCl crystal is placed on the target stage. The target and sensor arm can be coupled to maintain a 2:1 angle. The Geiger-Muller counter tube is a sensor that can count the number of incident X-rays, up to 9999 s^{-1}. The control panel allows the user to set the X-ray tube high voltage (U), emission current (I), measuring time per angular step (δt), goniometer angular step width ($\delta\beta$), and

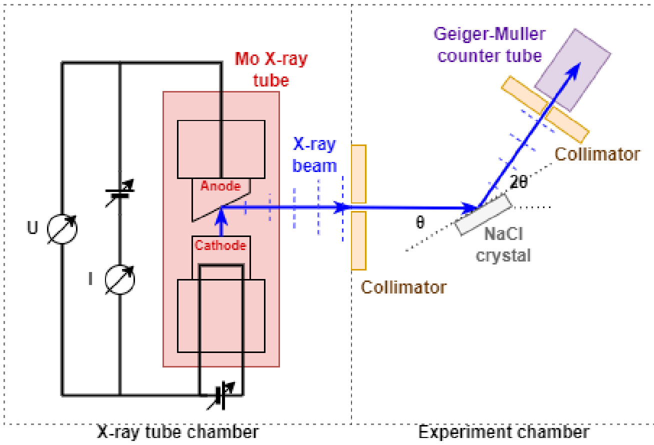

FIG. 1. Experiment setup. The X-ray beam (shown in blue) is generated by the X-ray tube with molybdenum (Mo) anode (red). The diffracted X-ray beam is collimated (orange) as it enters the experiment chamber. The beam reflects off the NaCl (gray) at an angle θ before passing through a second collimator (orange) and being absorbed by the Geiger-Muller counter tube. The counter tube is coupled to be at an angle of 2θ with reference to the horizontal X-ray beam. The emission current (I) and high voltage (V) are variable.

goniometer upper and lower limits (β) [3]. The NaCl crystal in Bragg configuration in combination with the counter tube comprise a spectrometer.

In this research, I am investigating the effects of the high voltage and emission current on the X-ray energy spectrum by first order Bragg reflection on an NaCl crystal. To do this, I conduct two experiments: (a) varying the high voltage at a constant current, and (b) varying the emission current for a constant voltage. The setup for these experiments is shown in Figure 1. For both experiments, the slit opening of the X-ray tube collimator is set to be 5 cm from the center of the NaCl crystal on the target stage; the Geiger-Muller sensor is then adjusted to be 6 cm from the crystal. The measuring time is set to $\delta t = 10$ s, angular step width $\delta\beta = 0.1°$, and goniometer upper limit of $\beta = 2.5°$ and lower limit $\beta = 12.5°$. The apparatus is also set to 2θ coupling between the target and sensor on the goniometer. For experiment (a), I take measurements between the goniometer limits for each $U = 15$, 20, 25, 30, and 35 keV while the maintaining a constant $I = 1.00$ mA. For experiment (b), I take measurements for each $I = 0.40$, 0.60, 0.80, and 1.00 mA with a constant $U = 35$ kV. I perform two trials for each experiment to cross-check the data and improve results.

III. RESULTS AND DISCUSSION

In this research, I investigate the effects of varying voltage and current on the X-ray spectrum for a molybdenum (Mo) anode. I claim the following instrumental uncertainties: I is ± 0.01 mA, U is ± 0.1, β is ± 0.1, and t is

Voltage	[± 0.1 kV]	Current	[± 0.01 mA]
U_0	15.0	I_0	0.40
U_1	20.0	I_1	0.60
U_2	25.0	I_2	0.80
U_3	30.0	I_3	1.00
U_4	35.0		

TABLE I. Five different voltages for the constant $I = (1.00 \pm 0.01)$ mA (left) and four different currents for the constant $U = (35.0 \pm 0.1)$ kV (right).

± 1. Table I shows the different voltages and currents used at a constant current of $I = (1.00 \pm 0.01)$ mA and $U = (35.0 \pm 0.1)$ kV, respectively. For each experiment, I complete two independent trials where I measure the X-ray photon counts from $(2.5 \pm 0.1)° \leq \beta \leq (12.5 \pm 0.1)°$ at $(0.1 \pm 0.1)°$ resolution.

The results for the constant current is shown in Figure 2 and for the constant voltage in Figure 3. Both trials show good agreement. The shapes of the X-ray spectra change more significantly for the variable voltage; increasing the voltage shifts the minimum wavelength to lower wavelengths and increases the peak photon frequency. For variable current, the minimum wavelength does not change while the radiation increases across the spectrum. Most of X-ray spectra show two emission lines at 61.9 pm and 69.7 pm. These are clearly visible for all spectra where $U \geq 25$ kV. For $U = 20$ kV, the emission 69.7 pm line is weakly present. The minimum wavelength is larger than the two emission line wavelengths for $U = 15$ kV.

To better investigate the effects of current and voltages on the X-ray spectra, I extract the minimum wavelength, maximum Bremsstrahlung radiation, and the peak at the two emission lines. Figure 4 shows that the minimum wavelength and wavelength of peak Bremsstrahlung radiation have a strong negative correlation with voltage and no correlation with current. Thus, higher voltages generate photons with higher energies which are able to excite a larger range and number of electrons in the Mo anode. For all four lines in Figure 4 have small χ^2 values, which suggest that the linear fits are strong. Next, Figure 5 shows that the photon frequency increases with both voltage and current across all parameters. The peak photon frequency for both emission lines shows a very strong positive correlation (small χ^2) with voltage; the best-fit line predicts a very small number of emission line photons at 61.9 pm and 69.7 pm for $U \approx 20$ kV, which can be observed in Figure 2 upon close inspection. The peak Bremsstrahlung radiation shows a moderate positive correlation with current, which can be observed in Figure 3. The remaining plots (including peak Bremsstrahlung versus voltage and the emission line peak versus current) have larger χ^2, which suggest that the linear fit is not sufficient. For variable current, increases in the emission line peak R are most likely due to additional Bremsstrahlung photons.

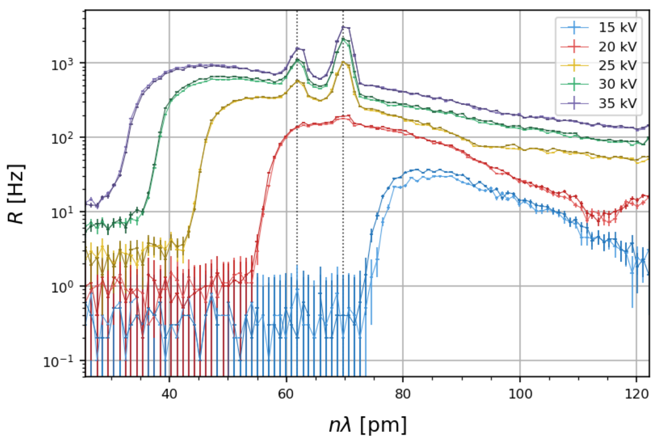

FIG. 2. Photon count frequency (R) versus wavelength ($n\lambda$) of the Mo X-ray tube for variable voltage. The data for trial 2 are darker than trial 1. The error bars are small and difficult to resolve. The two emission lines are denoted by vertical dotted gray lines at 61.9 pm and 69.7 pm; these features are clearly observed in $U > 20$ kV. The minimum wavelength decreases and the photon frequency increases with increasing voltage.

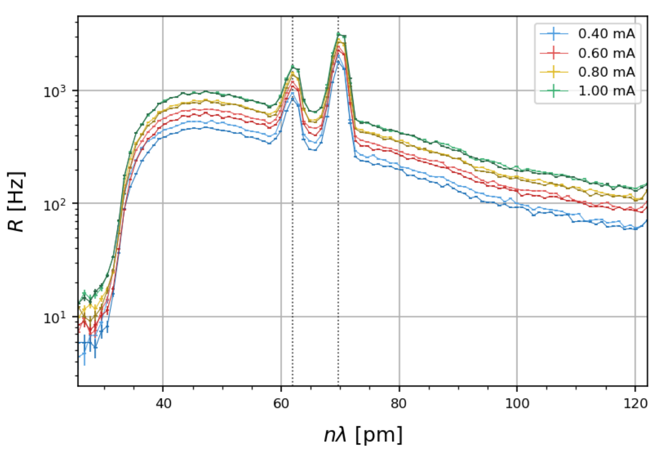

FIG. 3. Photon count frequency (R) versus wavelength ($n\lambda$) of the Mo X-ray tube for variable current. The data for trial 2 are darker than trial 1. The two emission lines are denoted by vertical dotted gray lines at 61.9 pm and 69.7 pm. The error bars are small and difficult to resolve. The photon frequency increases slightly with current.

IV. CONCLUSION

The goal of this research was to investigate how the high voltage and emission current effect the X-ray spectrum of a molybdenum (Mo) X-ray tube. We expect to observe two main features of the spectrum: Bremsstrahlung radiation and emission lines. Bremsstrahlung radiation is a continuous spectrum caused by free electrons decelerating in the Mo anode. Emission lines appear as sharp peaks in the spectrum, which originate from electrons penetrating deep into the inner orbitals. We observe the X-ray spectra by reflecting the emitted photons off of a NaCl crystal lattice and counting the number of photons incident on a detector.

Our main results are as follows:

1. The minimum wavelength of the X-ray spectrum has a strong negative correlation with voltage; higher voltages result in smaller minimum wavelengths. Thus, large voltages generate higher energy photons which can penetrate deeper into the atomic orbitals, which causes a larger range of emission.

2. The Mo anode shows two emission lines at 61.9 pm and 69.7 pm. These emission lines are clearly observable for high voltages $U > 20$ kV; at smaller voltages, the minimum wavelength is higher than the emission lines. The peak photon frequency of these emission lines have a strong positive correlation with voltage.

3. The wavelength of Bremsstrahlung radiation decreases linearly with voltage and is unaffected by the current. Its photon frequency increases nonlinearly with voltage and moderately linear with current. The energy of the photons incident on the Mo has a greater affect than the number of photons. This is consistent with Einstein's photoelectric effect.

V. IMPROVEMENTS

Here is a list of the improvements from the previous draft of this research paper: (1) removed redundant diagram of the apparatus, (2) addressed grammatical errors, (3) improved the presentation of X-ray spectrum data, (4) added additional analysis by plotting parameters of the spectra against voltage and current, (5) performed statistical analysis on the data models, and (5) completed the results, conclusion and abstract sections.

[1] W. H. S. Bragg and W. L. S. Bragg, The reflection of x-rays by crystals, Nature **91**, 477 (1913).

[2] M. Planck, Ueber das gesetz der energieverteilung im normalspectrum, Annalen der Physik **309**, 553–563 (1901).

[3] L. D. GmbH, Instruction sheet 554 800 (2014).

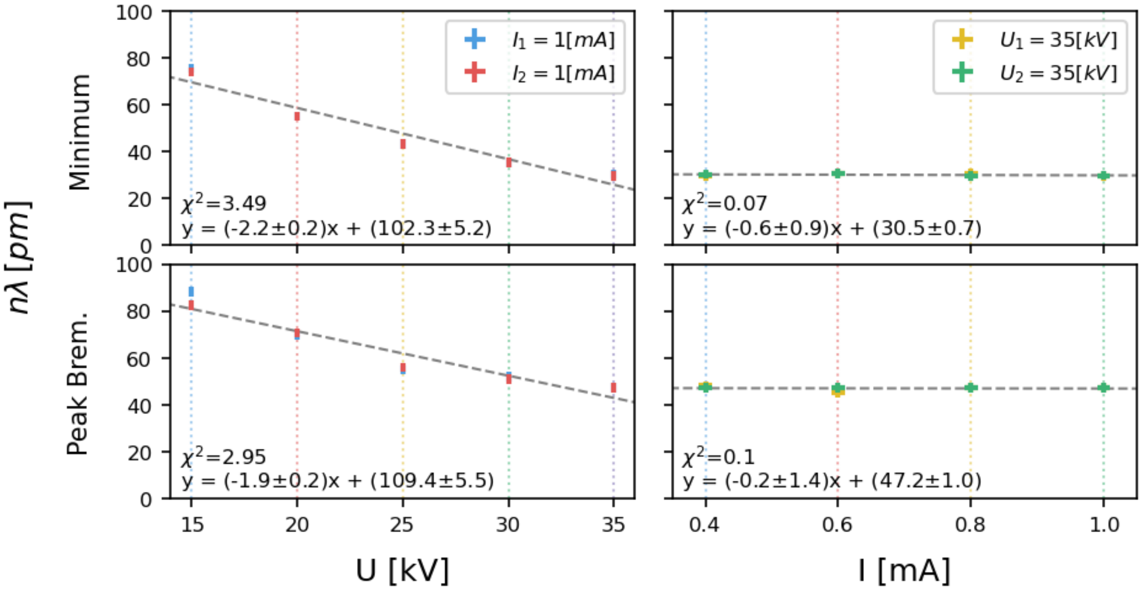

FIG. 4. Minimum and peak Bremsstrahlung wavelength ($n\lambda$) versus voltage (U) and current (I). The error bars are small. The best fit lines (gray) were generated using a least-squares algorithm. The vertical colored dotted lines refer to the data in Figure 2 (voltage) and Figure 3 (current). Both wavelength parameters have negative correlation with voltage, and no correlation with current.

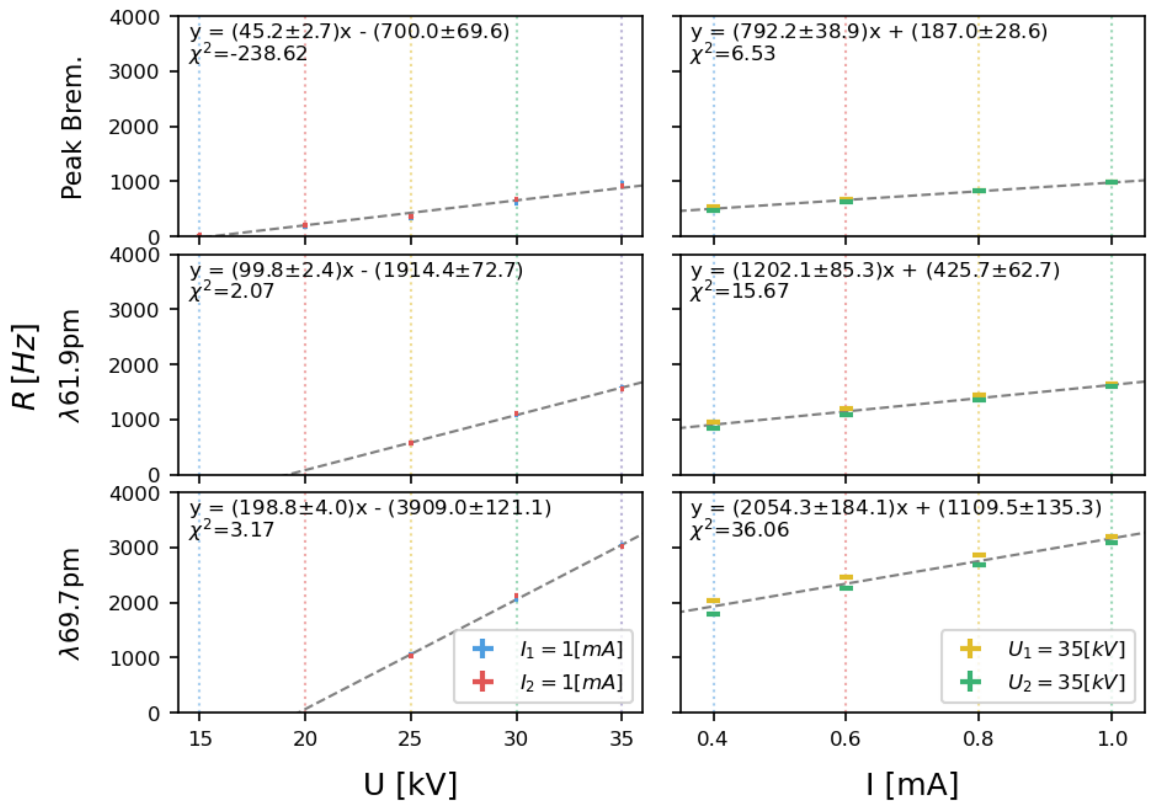

FIG. 5. Peak Bremsstrahlung and emission lines 61.9 pm and 69.7 pm photon count frequency (R) versus voltage (U) and current (I). The error bars are small. The best fit lines (gray) were generated using a least-squares algorithm. The vertical colored dotted lines refer to the data in Figure 2 (voltage) and Figure 3 (current). All parameters appear to increase with both voltage and current.