Math 116 – S2018: Assignment 1 - Due 2:00 pm Thursday 22 February 2018

Read the instructions to each question **carefully**. Provide all required reasoning and show calculations. You may discuss this with other students but use no sources other than the text, your notes, and the **pplane** and **dfield** programs. You must create your own solutions, including your own printouts from **pplane**.

For all the problems below, we are dealing with variations of the Predator-Prey Model. Measuring time t in weeks, let

$x(t)$ be the population of prey (in 1000's), and

$y(t)$ the population of predators (in 100's).

(1) Here is the standard Predator-Prey Model:

$$\frac{dx}{dt} = 0.3x - 0.09xy \tag{1}$$

$$\frac{dy}{dt} = -0.1y + .08xy \tag{2}$$

Do the Phase Plane Analysis of this model by hand, including nullclines, equilibria, graphs, direction arrows.

Run **pplane** using the bounds $x = 0, 10$, $y = 0, 10$. Select the *Show nullclines* option under **Solution**. Confirm closed trajectories for initial conditions $(1, 2.5)$ and $(6, 1)$. **Print.**

Find dy/dx at $(6, 1)$ using equations (1) and (2); compare to the slope of the trajectory at that point.

(2) Modify the model by replacing equation (1) in **(1)** with a logistic growth term for dx/dt rather than exponential growth. Use a carrying capacity $K = 9$. Keep equation (2) the same and keep the same term for the effect of the Principle of Mass Action in (1).

Do the Phase Plane Analysis of this model by hand, including nullclines, equilibria, graphs, direction arrows.

Run **pplane** using the bounds $x = 0, 10$, $y = 0, 10$. Select the *Show nullclines* option under **Solution**. Find solution trajectories for initial conditions $(0.5, 0.5)$ and $(8, 1.5)$. **Print.** What is happening to the populations $x(t)$ and $y(t)$ as $t \to \infty$ with the initial condition $(8, 1.5)$?

Check your answer as follows: select **Graph** option *both x-t and y-t.* A red bar appears at the top of **PPLANE Phase Plane** window. Place the cursor as near $(8, 1.5)$ as possible and click. Another window should appear. **Print.** Do the graphs of $x(t)$ versus t and $y(t)$ versus t confirm your guess?

(3) You have an irritating roommate who has taken a course in population biology and claims that the equation for the change in the predator population $y(t)$ is not realistic. For one thing, it takes infinite time for the predator population to die out, which is unrealistic.

The roommate suggests that it is better to assume that for, say, $y(0) = 9$ (900 hundred), it would take 30 weeks for the predators to become extinct and that if

$$\frac{dy}{dt} = -0.2\sqrt{y},\ y(0) = 9$$

is the model for $y(t)$ in the absence of prey then you get the desired result.

Your roommate can be difficult but you decide to check if this is correct. Solve for $y(t)$ and verify that $y(t)$ is decreasing and that $y(30) = 0$.

Now replace $-0.1y$ in equation (2) with $-0.2\sqrt{y}$ and run **pplane** again. Start an orbit at $(1, 3)$. What seems to be happening to $x(t)$ and $y(t)$? You can get more information by selecting **Graph** option *both x-t and y-t*, clicking on $(1, 3)$ and checking the graphs.