From Autoencoders to Variational Autoencoders: The Encoder

Valerio Velardo

Issues with vanilla autoencoders

- The latent space plot isn’t symmetrical around the origin
- Some labels are represented over small areas, others over large ones
- There are discontinuities in the latent space

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- Modify encoder component
- Modify loss function

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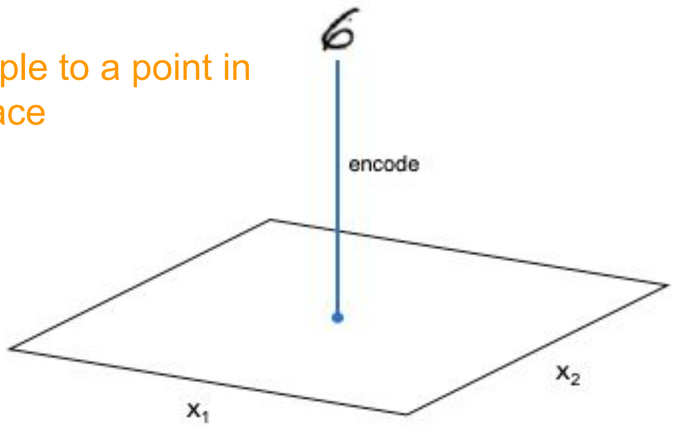

autoencoder

*Images taken from *Generative Deep Learning* by David Foster

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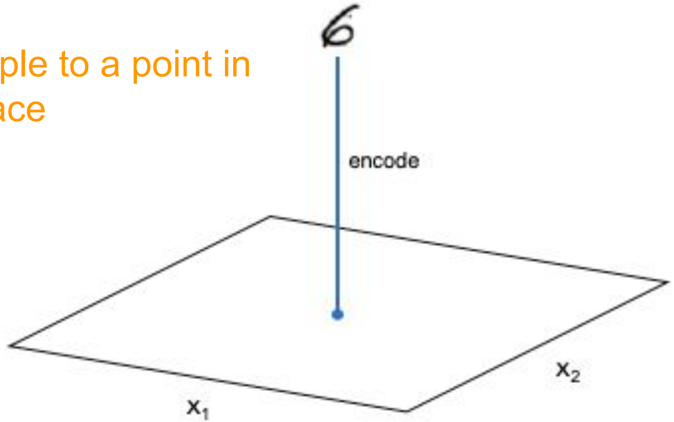

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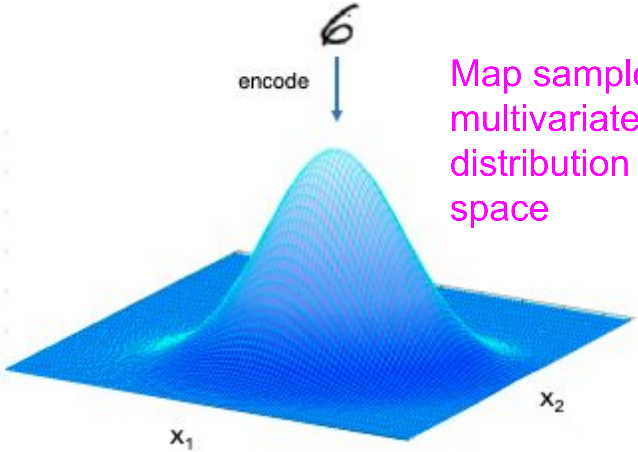

Map sample to a multivariate normal distribution in latent space

variational autoencoder

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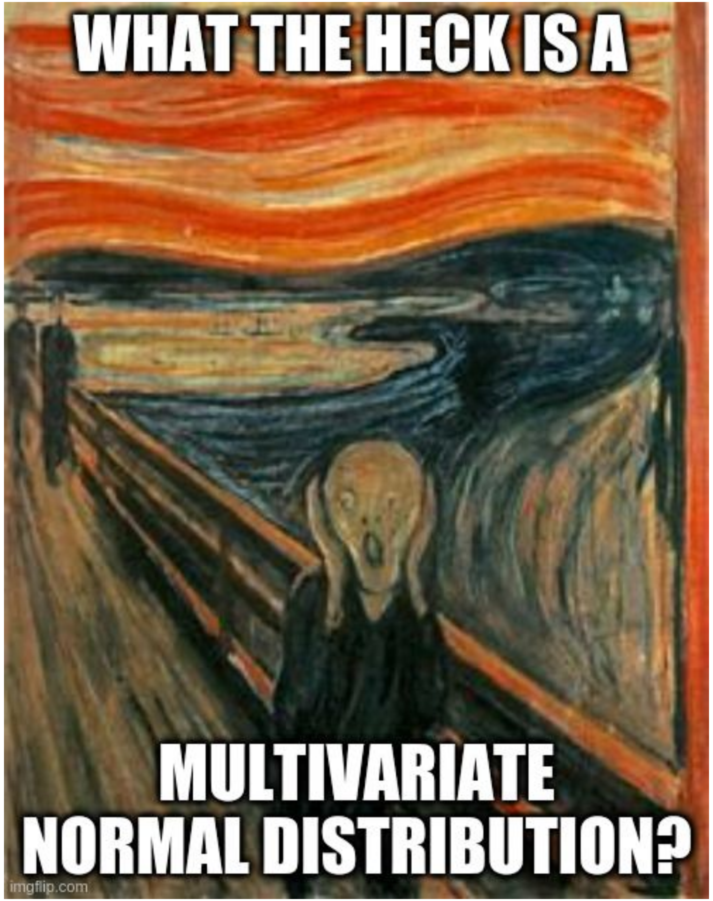
WHAT THE HECK IS A
MULTIVARIATE
NORMAL DISTRIBUTION?
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Defining the normal distribution in 1d

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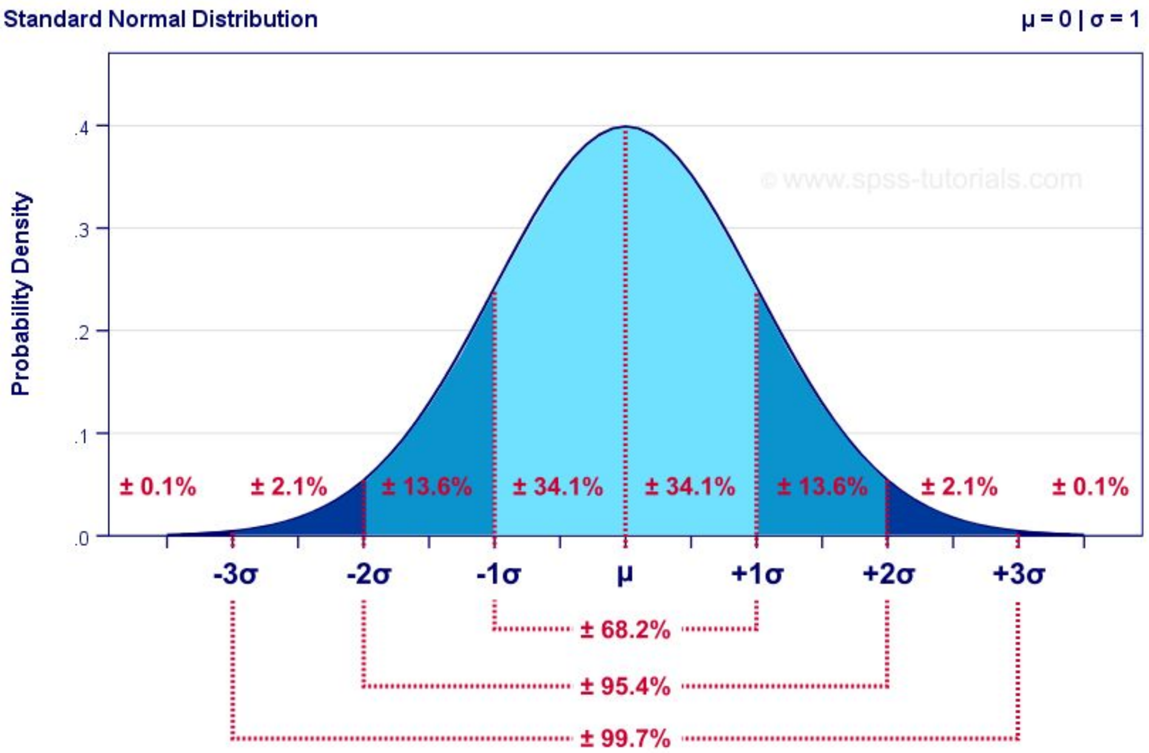
- Describe real-valued random variables (e.g., population height, weight)
- Defined by 2 variables:
    - μ - mean value -> centre of the distribution
    - σ - standard deviation -> variability of the distribution

Visualising the (standard) normal distribution



Probability density function

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Defining the normal distribution in 1d

- The greater σ the flatter the curve
- Changing μ shifts the curve right and left
- Let’s play around with the normal distro!

Sampling a point from a normal distribution

$$z = \mu + \sigma\varepsilon$$

Sampling a point from a normal distribution

$$z = \mu + \sigma \boxed{\varepsilon}$$

Sampled point from
standard normal distribution


LET'S GENERALISE A NORMAL DISTRIBUTION TO MORE THAN 1 DIMENSION
MULTIVARIATE NORMAL DISTRIBUTION
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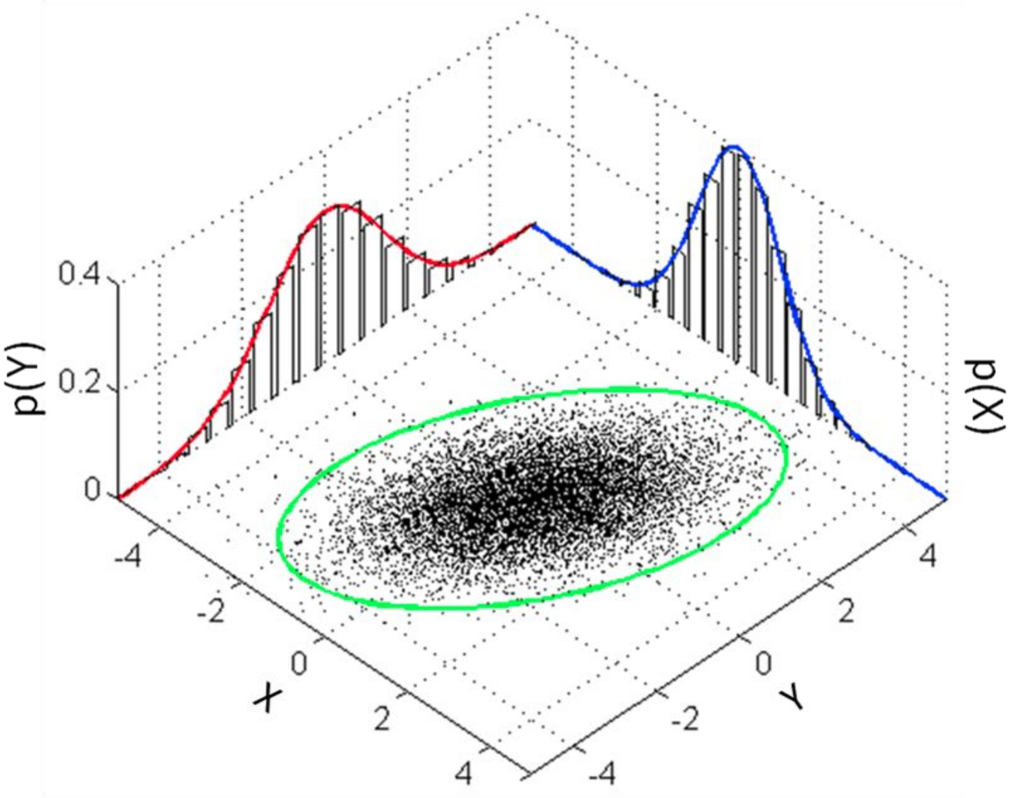
Visualising the multivariate normal distribution



Formalising the multivariate normal distribution

$$f(x_1, \dots, x_k) = \frac{e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T \Sigma^{-1} (\vec{x}-\vec{\mu})}}{\sqrt{(2\pi)^k |\Sigma|}}$$

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$\propto$ Mahalanobis distance:

Distance between **x** and **μ**

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Covariance matrix

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Correlation between x, y

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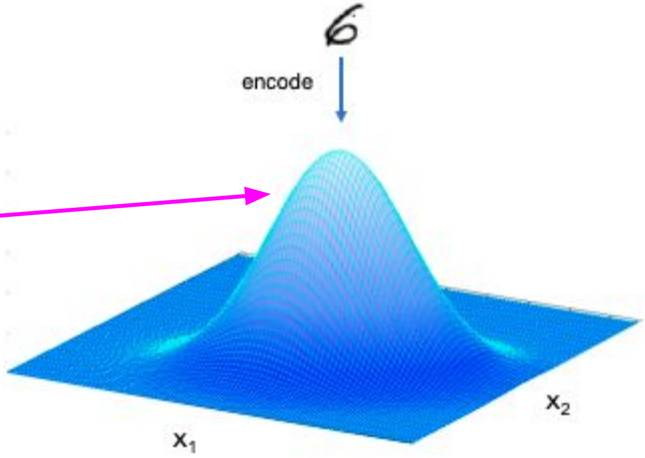

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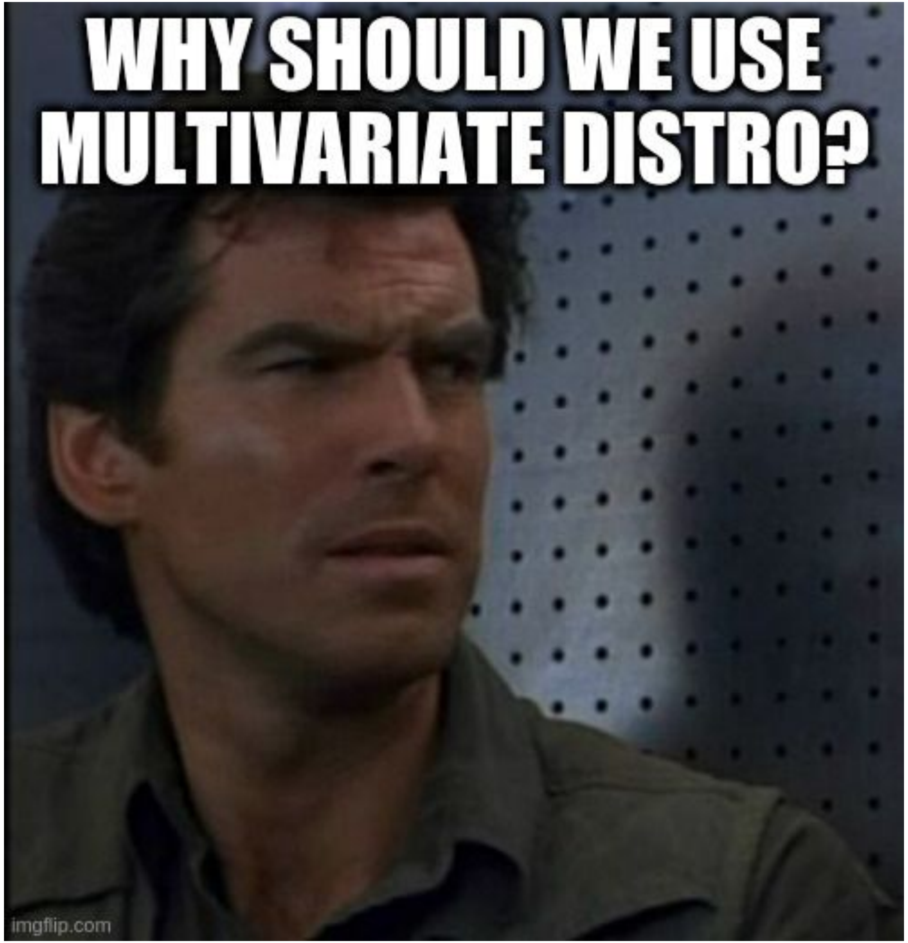
Log can take any value in - / + infinity


WHY SHOULD WE USE
MULTIVARIATE DISTRO?
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A major autoencoder drawback

There are discontinuities
in the latent space

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Some generated images
will be poor

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- We’re sampling a random point around the mean value
- Points in the same area produce very similar images when decoded
- Reconstruction loss is small
- VAE ensures a quasi-continuous latent space
- We can sample any point in the latent space, expecting the decoder to create a well formed image

What next?

- Discuss techniques to fix ulterior AEs' issues
- Focus on loss function for VAEs