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Ans1) Normal distribution function for mean $=\theta_1$ and variance $=\theta_2$

$$f(x;\theta_1,\theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x-\theta_1)^2}{2\theta_2}\right)$$

~~[illegible]~~ Likelihood function:

$$L(x_1,x_2,\dots;\theta_1,\theta_2) = \prod_{i=1}^{n} f(x_i,\theta_1,\theta_2)$$

$$L = \frac{1}{(2\pi\theta_2)^{n/2}} \exp\left[\frac{-1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\theta_1}{\theta_2}\right)^2\right]$$

taking natural log on both sides

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\theta_2 - \frac{1}{2}\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2}$$

taking derivative of above equation with respect to θ_1, and the for θ_2

$$\frac{1}{L}\frac{dL}{d\theta_1} = \frac{-1}{2}\sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2}\times 2(-1)$$

$$\frac{1}{L}\frac{dL}{d\theta_1} = \sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2}$$

$$\frac{1}{L}\frac{dL}{d\theta_1} = -\frac{n}{2\theta_2} + \left(\frac{-1}{2}\right)\sum_{i=1}^{n}(x_i-\theta_1)^2\times\left(\frac{-1}{(\theta_2)^2}\right)$$

$$\frac{1}{L}\frac{dL}{d\theta_1} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Now, $\quad \frac{dL}{d\theta_1} = \frac{dL}{d\theta_2} = 0$

$$\frac{dL}{d\theta_1} = L\left(\sum_{i=1}^{n}\frac{(x_i-\theta_1)}{\theta_2}\right) = 0$$

$$\sum_{i=1}^{n}(x_i-\theta_1) = 0$$

$$\sum_{i=1}^{n}x_i - n\theta_1 = 0$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n} \Bigg] \text{MLE of mean}$$

$$\frac{dL}{d\theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$= -n + \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\frac{1}{\theta_2} \left(\sum_{i=1}^{n} (x_i - \theta_1)^2 \right) = n$$

$$\theta_2 = \frac{\sum_{i=1}^{n} (x_i - \theta_1)}{n} \Bigg] \text{MLE of variance, } \theta_1 = \text{mean of population}$$

Ans 2) Binomial distribution ~~function~~ formula:

$$B(m,\theta) = {}^nC_m \, \theta^n (1-\theta)^{n-m}$$

Likelihood function:

$$L(m,\theta) = B(m,\theta)$$

$$L = {}^nC_m \, \theta^m (1-\theta)^{n-m}$$

taking ~~~~ natural log on both ~~~~ side.

$$\ln L = \ln({}^nC_m) + m \ln\theta + (n-m) \ln(1-\theta)$$

taking ~~~~ derivative with respect to θ

$$\frac{1}{L}\frac{dL}{d\theta} = 0 + \frac{m}{\theta} + \frac{(n-m)(-1)}{(1-\theta)}$$

$$\frac{1}{L}\frac{dL}{d\theta} = \frac{m}{\theta} - \frac{(n-m)}{1-\theta}$$

$$\frac{1}{L}\frac{dL}{d\theta} - \frac{m(1-\theta) - (n-m)\theta}{\theta(1-\theta)}$$

$$\frac{dL}{d\theta} = 0$$

$$\frac{m(1-\theta) - (n-m)\theta}{\theta(1-\theta)} = 0$$

$$m(1-\theta) - (n-m)\theta = 0$$

~~~~ $m(1-\theta) = (n-m)\theta$

$$\frac{m}{n} = \theta \Bigg] \text{MLE of } \theta.$$
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