Reformulation of Elasticity using Peridynamic Theory

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1 Introduction

Peridynamics (PD) is a non-local extension of continuum mechanics that is compatible with the physical nature of discontinuities. It avoids the need to evaluate the partial derivatives of the deformation with respect to the spatial coordinates, instead using an integro-differential equation for the linear momentum balance. Essentially, PD replaces the local equilibrium equation with a non-local formulation as below:

$$\nabla \cdot \sigma + \mathbf{b} = \mathbf{0} \quad \rightarrow \quad \int_{\mathscr{R}} \mathbf{f}(\mathbf{u}' - \mathbf{u}, \mathbf{x}' - \mathbf{x}) dV_x + \mathbf{b} = \mathbf{0}, \tag{1}$$

where σ is the stress field, $\mathbf{b}$ is the prescribed body force and $\mathbf{f}$ is the force density (per unit volume squared) that the material point $\mathbf{x}$ exerts on $\mathbf{x}$'. $\mathscr{R}$ is the neighborhood of $\mathbf{x}$ and $\mathbf{u}$ is the displacement field of $\mathbf{x}$.

2 Methods

For the purpose of this project, we solve a bond-based simple beam vibration problem in 1D and 2D. The project will be implemented using Python and is expected to contain the following key components.

- A function that generates a meshfree grid of the model in 1D and 2D.
- A class of functions that a user may call to implement peridynamic bond-based force density formulations for different material models (Elastic, Hyperelastic, Viscoelastic).
- A solver function that numerically implements the PD theory and an explicit time-stepping algorithm that solves the peridynamic equations.
- Post-process results and compare with classical mechanics methods such finite elements.
- If possible, implement an MPI version of the code and evaluate how the code scales when solving different size (number of grids) models on compute-clusters.

References

1. Silling, S.A., Epton, M., Weckner, O, Xu, J., Askari, E., "Peridynamic States and Constitutive Modeling," Journal of Elasticity, Vol. 88, 2007, pp. 151–184.
2. Chen, B., Liu, N., Yang, G., "The application of peridynamics in predicting beam vibration and impact damage," Journal of Vibroengineering, Vol. 17, No. 5, 2015, pp. 2369–2378.
3. Galadima, Y. K., Oterkus, S., Oterkus, E., Amin, I., El-Aassar, A. H., Shawky, H, "Modelling of viscoelastic materials using non-ordinary state-based peridynamics," Engineering with Computers, 2023, pp.1-14.
4. Silling, S. A., "Reformulation of Elasticity Theory for Discontinuities and Long-range Forces," Journal of the Mechanics and Physics of Solids, Vol. 48, 2000, pp. 175-209.