$$K_n(z,\alpha,a,b) = \int_a^b dx \frac{x^n}{z+\alpha x} \tag{4}$$

when $n = 0, 1, ...n_{max} = 10$.
a and b are numbers between 0 and 1. For simplicity you can choose $a = 0$ and $b = 1$.

  - Derive the recursion relation between K_{n+1} and K_n.
  - Then starting from K_0 you can compute all K_n up to n_{max} using the recursion. This works quite well for $|\alpha/z| >= 1$.

- Implement downword recursion for $\alpha/z < 1/2$. Make sure that you start with very accurate value for $K_{n_{max}}$. You can derive a power expansion of $K_{n_{max}}$ in powers of $(\alpha/z)^k$, and evaluate as many terms as needed to achieve desired accuracy (for example 10^{-12}).