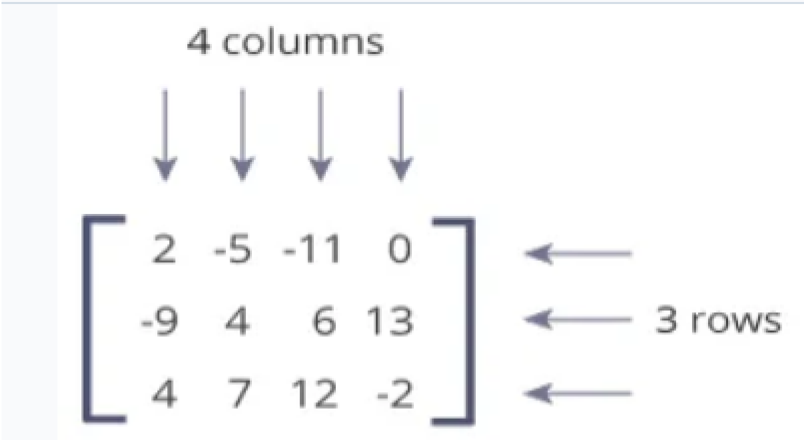
NumPy array handling for 3rd Semester B.Sc. python

Introduction

A matrix is a two-dimensional data structure where numbers are arranged into rows and columns. For example:



This matrix is a 3×4 (pronounced “three by four”) matrix because it has 3 rows and 4 columns.

All the codes, books, and documents related to this document are stored in the following GitHub repository.

https://github.com/paramphy/3rd-Semester

Python Matrix

Python doesn't have a built-in type for matrices. However, we can treat a list of a list as a matrix. For example:

```
A = [[1, 4, 5],
    [-5, 8, 9]]
```

We can treat this list of a list as a matrix having 2 rows and 3 columns.

$$\begin{bmatrix} 1 & 4 & 5 \\ -5 & 8 & 9 \end{bmatrix}$$

Let's see how to work with a nested list.

```
A = [[1, 4, 5, 12],
    [-5, 8, 9, 0],
    [-6, 7, 11, 19]]

print("A =", A)
print("A[1] =", A[1])      # 2nd row
print("A[1][2] =", A[1][2])   # 3rd element of 2nd row
print("A[0][-1] =", A[0][-1])   # Last element of 1st Row

column = [];        # empty list
for row in A:
  column.append(row[2])

print("3rd column =", column)
```

When we run the program, the output will be:

```
A = [[1, 4, 5, 12], [-5, 8, 9, 0], [-6, 7, 11, 19]]
A[1] = [-5, 8, 9, 0]
A[1][2] = 9
A[0][-1] = 12
3rd column = [5, 9, 11]
```

NumPy Array

NumPy is a package for scientific computing which has support for a powerful N-dimensional array object. Before you can use NumPy, you need to install it. For more info,

- Visit: How to install NumPy?
- If you are on Windows, download and install anaconda distribution of Python. It comes with NumPy and other several packages related to data science and machine learning.

Once NumPy is installed, you can import and use it.

NumPy provides a multidimensional array of numbers (which is actually an object). Let's take an example:

```
import numpy as np
a = np.array([1, 2, 3])
print(a)               # Output: [1, 2, 3]
print(type(a))         # Output: <class 'numpy.ndarray'>
```

As you can see, NumPy's array class is called **ndarray**.

How to create a NumPy array?

There are several ways to create NumPy arrays.

Array of integers, floats, and complex Numbers

```
import numpy as np

A = np.array([[1, 2, 3], [3, 4, 5]])
print(A)

A = np.array([[1.1, 2, 3], [3, 4, 5]]) # Array of floats
print(A)

A = np.array([[1, 2, 3], [3, 4, 5]], dtype = complex) # Array of complex numbers
print(A)
```

When you run the program, the output will be:

```
[[1 2 3]
 [3 4 5]]

[[1.1 2.  3. ]
[3.  4.  5. ]]

[[1.+0.j 2.+0.j 3.+0.j]
 [3.+0.j 4.+0.j 5.+0.j]]
```

Array of zeros and ones

```
import numpy as np

zeors_array = np.zeros( (2, 3) )
print(zeors_array)

'''
 Output:
 [[0. 0. 0.]
  [0. 0. 0.]]
'''

ones_array = np.ones( (1, 5), dtype=np.int32 ) // specifying dtype
print(ones_array)      # Output: [[1 1 1 1 1]]
```

Here, we have specified dtype to 32 bits (4 bytes). Hence, this array can take values from -2^{31} to $2^{31}-1$.

Using arange() and shape()

```
import numpy as np

A = np.arange(4)
print('A =', A)

B = np.arange(12).reshape(2, 6)
print('B =', B)
```

Output of the code

```
A = [0 1 2 3]
B = [[ 0  1  2  3  4  5]
 [ 6  7  8  9 10 11]]
```

Learn more about other ways of creating a NumPy array.

Matrix Operations

Above, we gave you 3 examples: addition of two matrices, multiplication of two matrices and transpose of a matrix. We used nested lists before to write those programs. Let's see how we can do the same task using NumPy array.

Addition of Two Matrices

We use **+** operator to add corresponding elements of two NumPy matrices.

```
import numpy as np

A = np.array([[2, 4], [5, -6]])
B = np.array([[9, -3], [3, 6]])
C = A + B      # element wise addition
print(C)
```

Output of the code

```
[[11  1]
 [ 8  0]]
```

Multiplication of Two Matrices

To multiply two matrices, we use dot() method. Learn more about how numpy.dot works.

Note: is used for array multiplication (multiplication of corresponding elements of two arrays) not matrix multiplication.

```
import numpy as np

A = np.array([[3, 6, 7], [5, -3, 0]])
B = np.array([[1, 1], [2, 1], [3, -3]])
C = A.dot(B)
print(C)
```

Output of the code

```
[[ 36 -12]
 [ -1   2]]
```

Transpose of a Matrix

We use numpy.transpose to compute transpose of a matrix.

```
import numpy as np

A = np.array([[1, 1], [2, 1], [3, -3]])
print(A.transpose())
```

Output of the code

```
[[ 1  2  3]
 [ 1  1 -3]]
```

As you can see, NumPy made our task much easier.

Access matrix elements, rows, and columns

Access matrix elements

Similar like lists, we can access matrix elements using index. Let's start with a one-dimensional NumPy array.

```
import numpy as np
A = np.array([2, 4, 6, 8, 10])

print("A[0] =", A[0])     # First element
print("A[2] =", A[2])     # Third element
print("A[-1] =", A[-1])   # Last element
```

When you run the program, the output will be:

```
A[0] = 2
A[2] = 6
A[-1] = 10
```

Now, let's see how we can access elements of a two-dimensional array (which is basically a matrix).

```
import numpy as np

A = np.array([[1, 4, 5, 12],
    [-5, 8, 9, 0],
    [-6, 7, 11, 19]])

#  First element of first row
print("A[0][0] =", A[0][0])

# Third element of second row
print("A[1][2] =", A[1][2])

# Last element of last row
print("A[-1][-1] =", A[-1][-1])
```

When we run the program, the output will be:

```
A[0][0] = 1
A[1][2] = 9
A[-1][-1] = 19
```

Access rows of a Matrix

```
import numpy as np

A = np.array([[1, 4, 5, 12],
    [-5, 8, 9, 0],
    [-6, 7, 11, 19]])
```

```
print("A[0] =", A[0]) # First Row
print("A[2] =", A[2]) # Third Row
print("A[-1] =", A[-1]) # Last Row (3rd row in this case)
```

When we run the program, the output will be:

```
A[0] = [1, 4, 5, 12]
A[2] = [-6, 7, 11, 19]
A[-1] = [-6, 7, 11, 19]
```

Access columns of a Matrix

```
import numpy as np

A = np.array([[1, 4, 5, 12],
    [-5, 8, 9, 0],
    [-6, 7, 11, 19]])

print("A[:,0] =",A[:,0]) # First Column
print("A[:,3] =", A[:,3]) # Fourth Column
print("A[:,-1] =", A[:,-1]) # Last Column (4th column in this case)
```

When we run the program, the output will be:

```
A[:,0] = [ 1 -5 -6]
A[:,3] = [12  0 19]
A[:,-1] = [12  0 19]
```

If you don't know how this above code works, read slicing of a matrix section of this article.

Slicing of a Matrix

Slicing of a one-dimensional NumPy array is similar to a list. If you don't know how slicing for a list works, visit Understanding Python's slice notation.

Let's take an example:

```
import numpy as np
letters = np.array([1, 3, 5, 7, 9, 7, 5])

# 3rd to 5th elements
print(letters[2:5])        # Output: [5, 7, 9]
```

```
# 1st to 4th elements
print(letters[:-5])        # Output: [1, 3]

# 6th to last elements
print(letters[5:])         # Output:[7, 5]

# 1st to last elements
print(letters[:])          # Output:[1, 3, 5, 7, 9, 7, 5]

# reversing a list
print(letters[::-1])          # Output:[5, 7, 9, 7, 5, 3, 1]
```

Now, let's see how we can slice a matrix.

```
import numpy as np

A = np.array([[1, 4, 5, 12, 14],
    [-5, 8, 9, 0, 17],
    [-6, 7, 11, 19, 21]])

print(A[:2, :4])  # two rows, four columns

''' Output:
[[ 1  4  5 12]
 [-5  8  9  0]]
'''


print(A[:1,])  # first row, all columns

''' Output:
[[ 1  4  5 12 14]]
'''

print(A[:,2])  # all rows, second column

''' Output:
[ 5  9 11]
'''

print(A[:, 2:5])  # all rows, third to the fifth column

'''Output:
[[ 5 12 14]
 [ 9  0 17]
 [11 19 21]]
'''
```

As you can see, using NumPy (instead of nested lists) makes it a lot easier to work with matrices, and we haven't even scratched the basics. We suggest you to explore NumPy package in detail, especially if you are trying to use Python for data science/analytics.

NumPy Resources you might find helpful:

- NumPy Tutorial
- NumPy Reference

Gauss Elimination

Solve

$$
\begin{aligned}
3x + 2y + z = 11 \\
2x + 3y + z = 13 \\
x + y + 4z = 12
\end{aligned}
$$

The determinant of the inhomogeneous linear equations is 18, so a solution exists. For convenience and for the optimum numerical accuracy, the equations are rearranged so that the largest coefficients run along the main diagonal (upper left to lower right). This has already been done in the preceding set.
The Gauss technique is to use the first equation to eliminate the first unknown, x, from the remaining equations. Then the (new) second equation is used to eliminate y from the last equation. In general, we work down through the set of equations, and then, with one unknown determined, we work back up to solve for each of the other unknowns in succession.

Dividing each row by its initial coefficient, we see that Eqs. (3.19) become

$$
\begin{aligned}
x + \frac{2}{3}y + \frac{1}{3}z = \frac{11}{3} \\
x + \frac{3}{2}y + \frac{1}{2}z = \frac{13}{2} \\
x + y + 4z = 12
\end{aligned}
$$

Now, using the first equation, we eliminate x from the second and third equations:

$$
\begin{aligned}
x + \frac{2}{3}y + \frac{1}{3}z = \frac{11}{3} \\
\frac{5}{6}y + \frac{1}{6}z = \frac{17}{6} \\
\frac{1}{3}y + \frac{11}{3}z = \frac{25}{3}
\end{aligned}
$$

and

$$x + \frac{2}{3}y + \frac{1}{3}z = \frac{11}{3}$$
$$y + \frac{1}{5}z = \frac{17}{5}$$
$$y + 11z = 25.$$

Repeating the technique, we use the new second equation to eliminate y from the third equation:

$$x + \frac{2}{3}y + \frac{1}{3}z = \frac{11}{3}$$
$$y + \frac{1}{5}15z = \frac{17}{5}$$
$$54z = 108$$
$$or,$$
$$z = 2$$

Finally, working back up, we get

$$y + 15 \times 2 = \frac{17}{5},$$
$$or$$
$$y = 3.$$

Then with z and y determined,

$$x + \frac{2}{3} \times 3 + \frac{1}{3} \times 2 = \frac{11}{3},$$
$$and$$
$$x = 1.$$

Python code for the above problem

```
# Importing NumPy Library
import numpy as np
import sys
n = 3
# Making numpy array of n x n+1 size and initializing
# to zero for storing augmented matrix
a = np.array([[3.0,2.0,1.0,11.0],[2.0,3.0,1.0,13.0],[1.0,1.0,4.0,12.0]])

# Making numpy array of n size and initializing
# to zero for storing solution vector
```

```
x = np.zeros(n)

print(a)
print(x)
# Applying Gauss Elimination
for i in range(n):
    if a[i][i] == 0.0:
        sys.exit('Divide by zero detected!')

    for j in range(i+1, n):
        ratio = a[j][i]/a[i][i]

        for k in range(n+1):
            a[j][k] = a[j][k] - ratio * a[i][k]

# Back Substitution
x[n-1] = a[n-1][n]/a[n-1][n-1]

for i in range(n-2,-1,-1):
    x[i] = a[i][n]

    for j in range(i+1,n):
        x[i] = x[i] - a[i][j]*x[j]

    x[i] = x[i]/a[i][i]

# Displaying solution
print('\nRequired solution is: ')
for i in range(n):
    print('X%d = %0.2f' %(i,x[i]), end = '\t')
```

```
[[ 3.  2.  1. 11.]
 [ 2.  3.  1. 13.]
 [ 1.  1.  4. 12.]]
[0. 0. 0.]

Required solution is:
X0 = 1.00       X1 = 3.00       X2 = 2.00
```

Code for an arbitrary matrix

```
# Importing NumPy Library
import numpy as np
import sys

# Reading number of unknowns
n = int(input('Enter number of unknowns: '))

# Making numpy array of n x n+1 size and initializing
# to zero for storing augmented matrix
a = np.zeros((n,n+1))
```

```
# Making numpy array of n size and initializing
# to zero for storing solution vector
x = np.zeros(n)

# Reading augmented matrix coefficients
print('Enter Augmented Matrix Coefficients:')
for i in range(n):
    for j in range(n+1):
        a[i][j] = float(input( 'a['+str(i)+']['+ str(j)+']='))
print(a)
# Applying Gauss Elimination
for i in range(n):
    if a[i][i] == 0.0:
        sys.exit('Divide by zero detected!')

    for j in range(i+1, n):
        ratio = a[j][i]/a[i][i]

        for k in range(n+1):
            a[j][k] = a[j][k] - ratio * a[i][k]

# Back Substitution
x[n-1] = a[n-1][n]/a[n-1][n-1]

for i in range(n-2,-1,-1):
    x[i] = a[i][n]

    for j in range(i+1,n):
        x[i] = x[i] - a[i][j]*x[j]

    x[i] = x[i]/a[i][i]

# Displaying solution
print('\nRequired solution is: ')
for i in range(n):
    print('X%d = %0.2f' %(i,x[i]), end = '\t')
```