Assignment on numerical simulation of Hydrogen atom

Hydrogenic atom wave function is $\psi(r,\theta,\phi) = N\frac{u}{r}\,Y_{lm}(\theta,\phi)$

1. The effective 1-D equation for u is a dimensionless form is

$$\left(\frac{d^2}{d\rho^2} - \frac{l(l+1)}{\rho^2} + \frac{2Z}{\rho} - 2\gamma\right)u = 0$$

Where $\rho = \frac{r}{a}, \gamma = -\frac{E}{\frac{e^2}{a}}, a = \frac{\hbar^2}{me^2}$= Bohr Radius

Use Shooting method to find out the eigenvalues $(\gamma)$and normalized eigenvectors in 1s,2s, 3s, 3p and 3d states (Take Z=1)

2. Repeat the above problem for 1s state with Z=1,2,4 and superpose the plots of the normalized eigenvectors

2. Instead of Coulombic potential if a Screened coulomb potential $V(r) = -\frac{e^2}{r}\exp(-\frac{r}{b})$ is used then the The effective 1-D equation for u is a dimensionless form is

$$\left(\frac{d^2}{d\rho^2} - \frac{l(l+1)}{\rho^2} + \frac{2\exp(-\frac{r}{\lambda})}{\rho} - 2\gamma\right)u = 0$$

Where $b = \lambda a$

a) Find 1s, 2s, 3s states for $\lambda = 10, 20, 50$

b) compare the eigenvalues of 3s,3p and 3d states with $\lambda = 10$