Practice Problems

CS 101 Autumn 2020

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December 17, 2020

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§1. Practice Problems 1

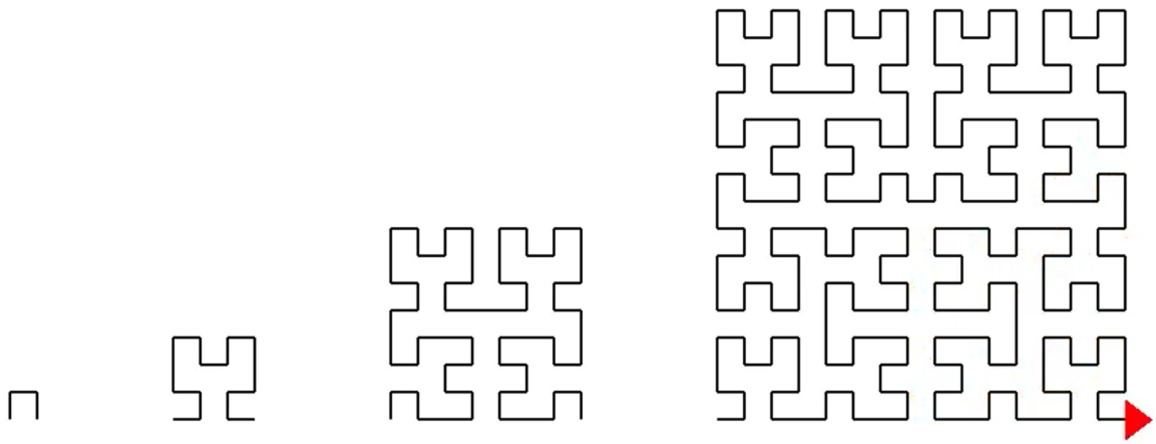

Figure 1: Hilbert Curve

1. **Problem Statement:**
   Take an integer as input and draw the corresponding iteration of this fractal using turtleSim
   You may think along these lines

   Step 1 Find a simple pattern in these iterations

   Step 2 Think how can you implement this pattern in an efficient way (here think in the number of lines of code you have to write. **Word of caution**: this is just one of the possible definitions of efficient code)

   Step 3 Do you think that you need something that will implement/shorten your code?
   How will it look like? (it's a feature)

   Feel free to discuss your thoughts on this.

   Note. *For people comfortable with the basics of C++, this shouldn't be difficult. You may try this*

Fun Videos

Hilbert's Curve: Is infinite math useful?
Recursive PowerPoint Presentations [Gone Fractal!]

Book Chapters for Graphics

Additional chapters of the book on Simplecpp graphics demonstrating its power
(It is just a list, you are not expected to understand/study things, CS101 is for a reason :P)

Chapter 5 Coordinate based graphics, shapes besides turtles

Chapter 15.2.3 Polygons

Chapter 19 Gravitational simulation

Chapter 20 Events, Frames, Snake game

Chapter 24.2 Layout of math formulae

Chapter 26 Composite class

Chapter 28 Airport simulation

§2. Practice Problems 2

1. You probably heard about Fibonacci Numbers!

   The Fibonacci numbers are the numbers in the integer sequence: (defined by the recurrence relation)

$$
\begin{aligned}
&F_0 = 0 \\
&F_1 = 1 \\
&F_n = F_{n-1} + F_{n-2} \quad n \in \mathbb{Z} \quad \text{(They can be extended to negative numbers)}
\end{aligned} \tag{1}
$$

   For any integer n, the sequence of Fibonacci numbers F_i taken modulo n is periodic.

   The Pisano period, denoted $\pi(n)$, is the length of the period of this sequence.

   For example, the sequence of Fibonacci numbers modulo 3 begins:

$$0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, ...\text{(A082115)}$$

   This sequence has period 8, so $\pi(3) = 8$. (Basically, the remainder when these numbers are divided by n is a repeating sequence. You have to find the length of sequence)

   Problem Statement:

   (a) Find Pisano period of n numbers $k_1\ k_2\ \ldots\ k_n$
   Input Format
   n
   $k_1\ k_2\ \ldots\ k_n$
   Output Format
   $(\pi(k_i))$ (each on a newline)
   Sample Input
   3
   3 10 25
   Sample Output
   8
   60
   100

   (b) For n numbers $k_1\ k_2\ \ldots\ k_n$, find $\max(\pi(i))$ for $i = 1, 2, \ldots, k_1$ and corresponding i
   If there are 2 (or more) such i's, output smallest of them
   Input Format
   n
   $k_1\ k_2\ \ldots\ k_n$
   Output Format
   $k\ \pi(k)$ (each pair on a newline)
   Sample Input
   4
   20 40 60 100
   Sample Output
   10 60
   30 120
   50 300
   98 336

2.
$$\frac{\pi}{2} = \sum_{k=0}^{\infty} \frac{2^k k!^2}{(2k+1)!} \tag{2}$$

   Problem Statement:
   Take a natural number k as input and calculate π till k^{th} iteration using Equation (2)

3. **Simpson's Rule** a method for numerical integration

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 4f(x_{n-1}) + f(x_n)\right) \tag{3}$$

Note. *Simpson's rule can only be applied when an odd number of ordinates is chosen.*

Problem Statement:
Solve Equation (4) giving the answers correct to 5 decimal places (Use 101 ordinates)

$$\int_{0.5}^{1} \frac{\sin\theta}{\theta}\mathrm{d}x \tag{4}$$