

# CS 101 Computer Programming and Utilization

## Practice Problems

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### Disclaimer

These are **optional** problems. As these problems are pretty involving, my advice to you would be to first solve exercises given in slides and get comfortable with the course content. The taught methods will suffice to solve these problems. (You are free to use 'other' stuff but not recommended)

### Good Programming Practices

- Clearly writing documentation explaining what the program does, how to use it, what quantities it takes as input, and what quantities it returns as output.
- Using appropriate variable/function names.
- Extensive internal comments explaining how the program works.
- Complete error handling with informative error messages.  
For example, if  $a = b = 0$ , then the  $\text{gcd}(a, b)$  routine should return the error message “ $\text{gcd}(0, 0)$  is undefined” instead of going into an infinite loop or returning a “division by zero” error.

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## §1. Practice Problems 1 - Introduction

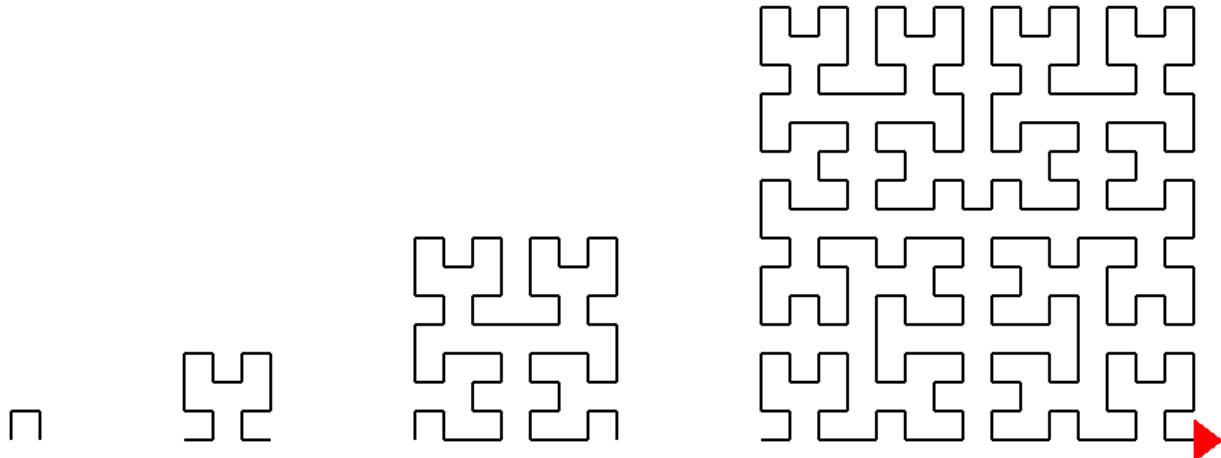


Figure 1: Hilbert Curve

### 1. Problem Statement:

Take an integer as input and draw the corresponding iteration of this fractal using turtleSim  
You may think along these lines

**Step 1** Find a simple pattern in these iterations

**Step 2** Think how can you implement this pattern in an efficient way (here think in the number of lines of code you have to write. **Word of caution:** this is just one of the possible definitions of efficient code)

**Step 3** Do you think that you need something that will implement/shorten your code?  
How will it look like? (it's a feature)

Feel free to discuss your thoughts on this.

**Note.** For people comfortable with the basics of C++, this shouldn't be difficult. You may try this

### Fun Videos

[Hilbert's Curve: Is infinite math useful?](#)

[Recursive PowerPoint Presentations \[Gone Fractal!\]](#)

### Book Chapters for Graphics

Additional chapters of the book on Simplecpp graphics demonstrating its power  
(It is just a list, you are not expected to understand/study things, CS101 is for a reason :P)

**Chapter 1** Turtle graphics

**Chapter 5** Coordinate based graphics, shapes besides turtles

**Chapter 15.2.3** Polygons

**Chapter 19** Gravitational simulation

**Chapter 20** Events, Frames, Snake game

**Chapter 24.2** Layout of math formulae

**Chapter 26** Composite class

**Chapter 28** Airport simulation

## §2. Practice Problems 2 - Loops

1. You probably heard about Fibonacci Numbers!

The Fibonacci numbers are the numbers in the integer sequence: (defined by the recurrence relation)

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_n &= F_{n-1} + F_{n-2} \quad n \in \mathbb{Z} \quad (\text{They can be extended to negative numbers}) \end{aligned} \tag{1}$$

For any integer  $n$ , the sequence of Fibonacci numbers  $F_i$  taken modulo  $n$  is periodic.

The Pisano period, denoted  $\pi(n)$ , is the length of the period of this sequence.

For example, the sequence of Fibonacci numbers modulo 3 begins:

0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, ... ([A082115](#))

This sequence has period 8, so  $\pi(3) = 8$ . (Basically, the remainder when these numbers are divided by  $n$  is a repeating sequence. You have to find the length of sequence)

### Problem Statement:

- (a) Find Pisano period of  $n$  numbers  $k_1, k_2, \dots, k_n$

**Input Format**

$n$

$k_1, k_2, \dots, k_n$

**Output Format**

$\pi(k_i)$  (each on a newline)

**Sample Input**

3

3 10 25

**Sample Output**

8

60

100

- (b) For  $n$  numbers  $k_1, k_2, \dots, k_n$ , find  $\max(\pi(i))$  for  $i = 1, 2, \dots, k$  and corresponding  $i$   
If there are 2 (or more) such  $i$ 's, output smallest of them

**Input Format**

$n$

$k_1, k_2, \dots, k_n$

**Output Format**

$k \ \pi(k)$  (each pair on a newline)

Here  $k$  is smallest possible integer satisfying  $\pi(k) = \max(\pi(i))$  for possible  $i$

**Sample Input**

5

20 40 60 80 100

**Sample Output**

10 60

30 120

50 300

50 300

98 336

2.

$$\frac{\pi}{2} = \sum_{k=0}^{\infty} \frac{k!}{(2k+1)!!} = \sum_{k=0}^{\infty} \frac{2^k k!^2}{(2k+1)!} \quad (2)$$

**Note.**  $n!!$  is called **double factorial**.  $n!! \neq (n!)!$ .

**Problem Statement:**

Calculate  $\pi$  using first  $k_i + 1$  terms <sup>1</sup> of Equation (2) for  $n$  different natural numbers  $k_1, k_2, \dots, k_n$ . Give your answers correct to 10 decimal places

**Input Format**

$n$   
 $k_1, k_2, \dots, k_n$

**Output Format**

Calculated  $\pi$  for  $k_i$  (each on a newline)

**Sample Input**

3  
10 20 30

**Sample Output**

3.1411060216  
3.1415922987  
3.1415926533

3. **Simpson's Rule:** a method for numerical integration

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(x_n)) \quad (3)$$

**Note.** Simpson's rule can only be applied when an odd number of ordinates is chosen.

**Problem Statement:**

Solve Equation (4) giving the answers correct to 7 decimal places (Use 101 ordinates)

$$\int_{0.5}^1 \frac{\sin \theta}{\theta} d\theta \quad (4)$$

**Correct Answer** = 0.4529756

---

<sup>1</sup>i.e calculate  $\pi$  till  $\frac{k_i!}{(2k_i+1)!!}$  term

### §3. Practice Problems 3 - Functions

1. Write a function that calculates the day of the week for any particular date in the past or future.  
Consider Gregorian calendar (AD)

**Task 1:** As a programming exercise, try the naive approach:

Starting from 1 Jan 0001 (Saturday) and calculate day after day till you reach the given date

Use `switch-case` statement

**Task 2:** Try to make more efficient algorithm (reduce completion time) than Task 1  
Implement it, and discuss your approach with me.

Also check for invalid dates (Write another function for this)

If dates are invalid, output **-1**

**Input Format**

$n$

Followed by  $n$  dates in **Date Month Year** format

**Output Format**

Day of the Week

**Sample Input**

5  
19 2 1627  
29 2 1700  
15 4 1707  
22 12 1887  
23 6 1912

**Sample Output**

Monday  
-1  
Friday  
Thursday  
Sunday

**Note.** Use your Task 1 program to check Task 2 implementation.

2. **Farey Sequence**

This sequence has all rational numbers in range  $[0/1 \text{ to } 1/1]$  sorted *in increasing order* such that the denominators are less than or equal to  $n$  and all numbers are in *reduced forms* i.e.,  $2/4$  does not belong to this sequence as it can be reduced to  $1/2$ .

**Input Format**

$n$

**Output Format**

Corresponding numbers in sequence in  $p/q$  format

**Sample Input**

7

**Sample Output**

$0/1 \ 1/7 \ 1/6 \ 1/5 \ 1/4 \ 2/7 \ 1/3 \ 2/5 \ 3/7 \ 1/2 \ 4/7 \ 3/5 \ 2/3 \ 5/7 \ 3/4 \ 4/5 \ 5/6 \ 6/7 \ 1/1$

Can you find efficient solution?

#### Fun Video

[Funny Fractions and Ford Circles](#)

### 3. Thue-Morse Sequence aka Fair Share Sequence

**Thue-Morse Sequence** is the infinite binary sequence obtained by starting with 0 and successively appending the Boolean complement of the sequence obtained thus far (called prefixes of the sequence).

First few steps :

- Start with 0
- Append complement of 0, we get 01
- Append complement of 01, we get 0110
- Append complement of 0110, we get 01101001

#### Problem Statement:

Consider appending complement of a prefix to itself as one iteration

Define a function to take a positive integer  $n$  as input then iterate  $n$  times to print the first  $2^n$  digits

#### Input Format

$n$

#### Output Format

Corresponding digits in sequence

#### Sample Input

6

#### Sample Output

0110100110010110100101100110010010110011010010110100110010110

Again, can you find better solution?

## Fun Video

[The Fairest Sharing Sequence Ever](#)

### 4. Collatz Conjecture

Consider the following operation on an arbitrary positive integer:

- If the number is even, divide it by two.
- If the number is odd, triple it and add one.

Collatz Conjecture states that no matter which positive integer we start with; we always end up with 1.

#### Problem Statement:

Define a function which performs this operation repeatedly on the result at each step; beginning with a given input  $n$  ( $n < 10^6$ ), returns the number of operations required to reach 1<sup>2</sup>

#### Input Format

Arbitrary number of testcases (each space separated)

Stop when input is negative

#### Output Format

Count of operations for each number (each on a newline)

#### Sample Input

1 3 7 9 27 871 77031 -1

#### Sample Output

0

7

16

19

111

178

350

<sup>2</sup>As of 2020, the conjecture has been checked by computer for all starting values up to  $2^{68} \approx 2.95 \times 10^{20}$ , so sequence from  $n$  will reach 1

## §4. Practice Problems 4 - Recursion

Practice Problem 4 are inspired from the following video do watch till 6:10 to get clear understanding of recursion  
[5 Simple Steps for Solving Any Recursive Problem](#)

- What's the simplest possible input?
- Play around with examples and visualize!
- Relate hard cases to simpler cases
- Generalize the pattern
- Write code by combining recursive pattern with base case

### 1. Ackermann function

Write a recursive function  $A()$  that takes two inputs  $n$  and  $m$  and outputs the number  $A(m, n)$  where  $A(m, n)$  is defined as

$$A(0, n) = n + 1 \quad (5)$$

$$A(m + 1, 0) = A(m, 1) \quad (6)$$

$$A(m + 1, n + 1) = A(m, A(m + 1, n)) \quad (7)$$

#### Problem Statement:

For  $k$  pairs  $(m_1, n_1), (m_2, n_2), \dots, (m_k, n_k)$ , find  $A(m, n)$  for such  $m, n$

#### Input Format

$k$

$m_i \ n_i$  (each pair on a newline)

#### Output Format

$A(m_i, n_i)$  (each result on a newline)

#### Sample Input

7

0 0

0 4

1 3

2 2

3 4

4 0

4 1

#### Sample Output

1

5

5

7

125

13

65533

### 2. GridPaths

Write a recursive function  $\text{NumberOfGridPaths}()$  that takes two inputs  $n$  and  $m$  and outputs the number of unique paths from the top left corner to bottom right corner of a  $n \times m$  grid.

Constraints:  $n, m \geq 1$  and you can only move down or right 1 unit at a time.

Examples given in Figure 2a

#### Problem Statement:

For  $k$  pairs  $(n_1, m_1), (n_2, m_2), \dots, (n_k, m_k)$ , find  $\text{NumberOfGridPaths}(n, m)$  for such  $n, m$

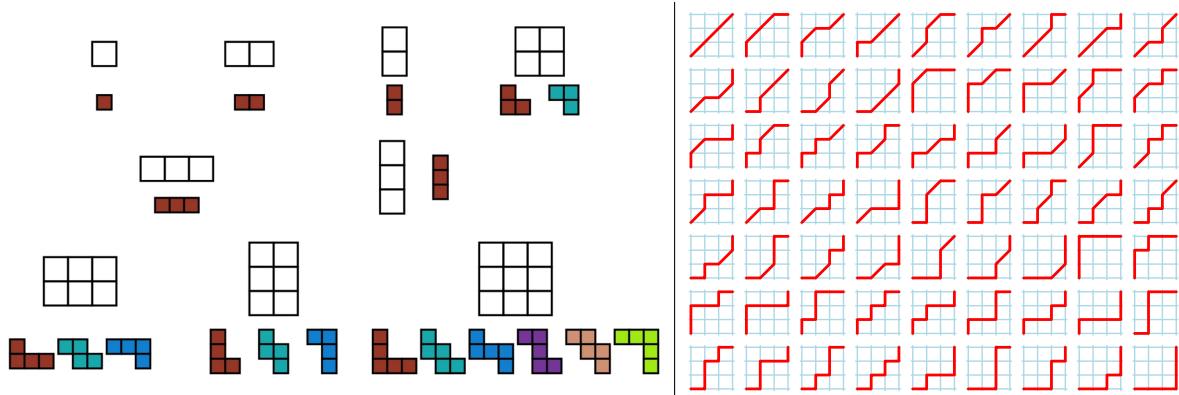


Figure 2

**Input Format**

$k$

$n_i \ m_i$  (each pair on a newline)

**Output Format**

$\text{NumberOfGridPaths}(n_i, m_i)$  (each result on a newline)

**Sample Input**

```
6
1 1
2 5
3 3
6 3
7 10
17 8
```

**Sample Output**

```
1
5
6
21
5005
245157
```

Can you find efficient solution?

### 3. Delannoy Numbers

Delannoy numbers describes the number of paths from the southwest corner  $(0, 0)$  of a rectangular grid to the northeast corner  $(m, n)$ , using only single steps north, northeast, or east.

Write a recursive function `DelannoyNumber()` to count number of these paths Constraints:  $n, m \geq 0$   
Examples given in Figure 2b

**Problem Statement:**

For  $k$  pairs  $(n_1, m_1), (n_2, m_2), \dots, (n_k, m_k)$ , find  $\text{DelannoyNumbers}(n, m)$  for such  $n, m$

**Input Format** $k$  $n_i \ m_i$  (each pair on a newline)**Output Format**DelannoyNumber( $n_i, m_i$ ) (each result on a newline)**Sample Input**

```

6
1 1
2 5
3 3
6 3
7 10
17 8

```

**Sample Output**

```

3
61
63
377
433905
245157
62390545

```

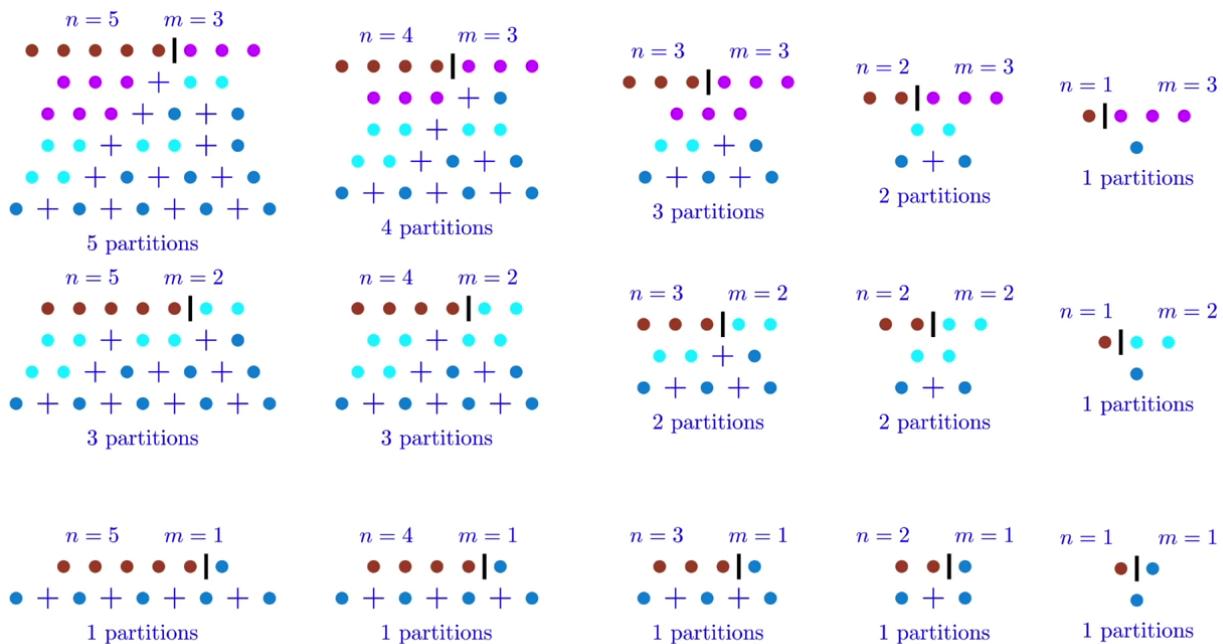
Can you find efficient solution?

**4. Partitions**

- (a) Write a recursive function NumberOfPartitions() that counts the number of ways you can partition  $n$  objects using parts up to  $m$

Constraints:  $n, m > 0$ 

Examples given in Figure 3

Figure 3: Partition  $n$  objects using parts up to  $m$

Can you find efficient solution?

**Problem Statement:**

For  $k$  pairs  $(n_1, m_1), (n_2, m_2), \dots, (n_k, m_k)$ , find  $\text{NumberOfPartitions}(n, m)$  for such  $n, m$

**Input Format**

$k$

$n_i \ m_i$  (each pair on a newline)

**Output Format**

$\text{NumberOfPartitions}(n_i, m_i)$  (each result on a newline)

**Sample Input**

7

1 1

2 5

3 3

6 3

7 10

17 8

20 12

**Sample Output**

1

2

3

7

15

230

582

- (b) Number of partitions  $P(n)$  of an integer  $n$  is same as  $\text{NumberOfPartitions}(n, n)$

**Theorem 1** (Pentagonal Number Theorem). *This theorem relates the product and series representations of the Euler function*

$$\prod_{n=1}^{\infty} (1 - x^n) = \sum_{k=-\infty}^{\infty} (-1)^k x^{k(3k-1)/2} = 1 + \sum_{k=1}^{\infty} (-1)^k \left( x^{k(3k+1)/2} + x^{k(3k-1)/2} \right) \quad (8)$$

In other words,

$$(1 - x)(1 - x^2)(1 - x^3) \cdots = 1 - x - x^2 + x^5 + x^7 - x^{12} - x^{15} + x^{22} + x^{26} - \cdots \quad (9)$$

The exponents 1, 2, 5, 7, 12, ... on the right hand side are called (generalized) pentagonal numbers ([A001318](#)) and are given by the formula  $g_k = k(3k-1)/2$  for  $k = 1, -1, 2, -2, 3, -3, \dots$

The equation (9) implies a recurrence for calculating  $P(n)$ , the number of partitions of  $n$ :

$$P(n) = P(n-1) + P(n-2) - P(n-5) - P(n-7) + \cdots \quad (10)$$

or more formally,

$$P(n) = \sum_{k \neq 0} (-1)^{k-1} P(n - g_k) \quad (11)$$

**Problem Statement:**

Write a function  $P()$  which calculates number of partitions using equation (10)

**Input Format**

Arbitrary number of testcases (each space separated)

Stop when input is negative

**Output Format**

Number of partitions for each testcase (each on a newline)

**Sample Input**

1 2 4 8 16 32 64 128 -1

**Sample Output**

```
1
2
5
22
231
8349
1741630
4351078600
```

**Crazy Video**

[The hardest What comes next \(Euler's pentagonal formula\)](#)

**Exciting Puzzle**

[Towers of Hanoi: A Complete Recursive Visualization](#)

## §5. Hints

### §§5.1. Practice Problems 1

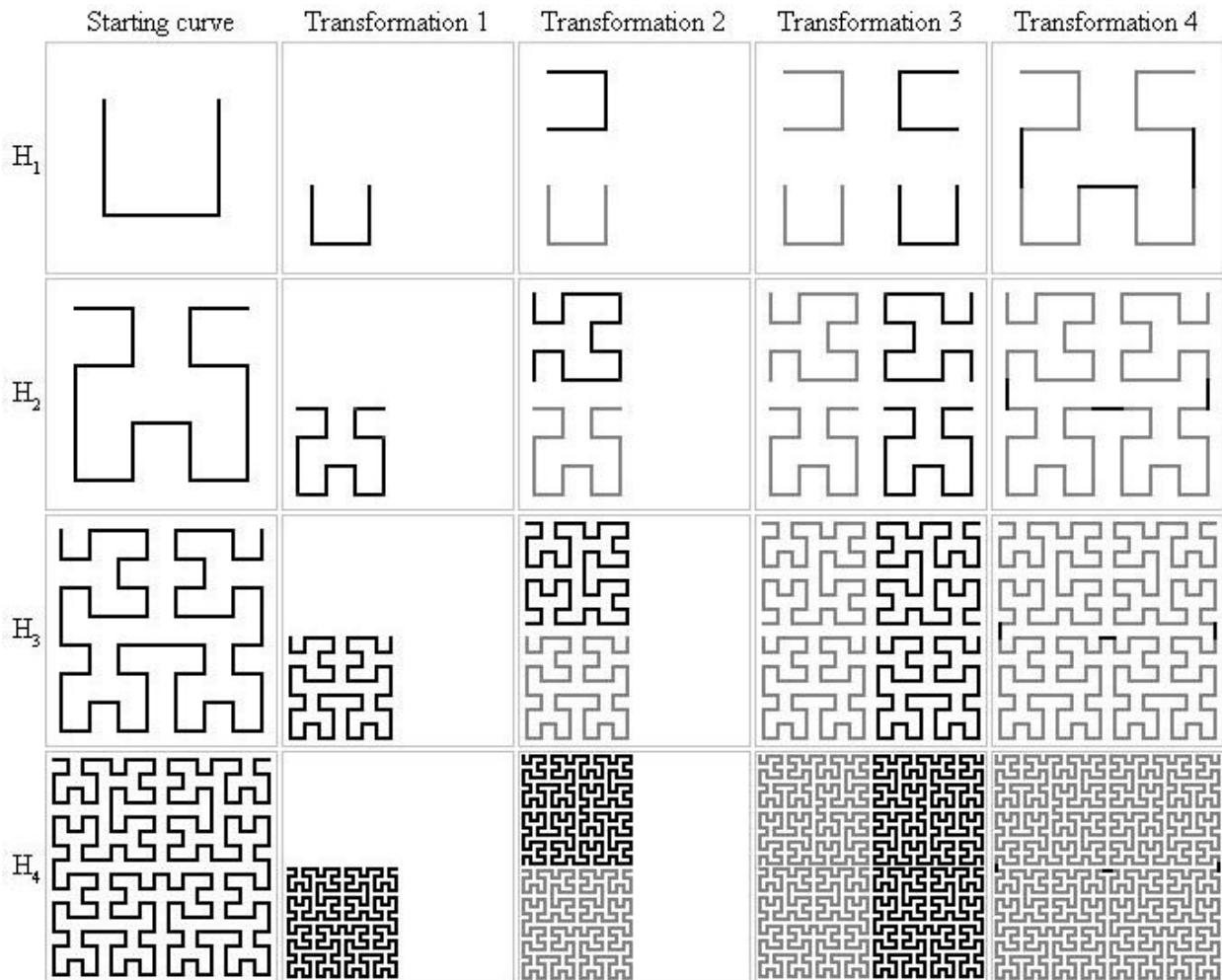


Figure 4: Hilbert Curve Transformations

### §§5.2. Practice Problems 2

1. The sequence repeats when we encounter '0' then '1'
2. Similar to calculating  $e$  (problem given in slides)

### §§5.3. Practice Problems 3

1. Try skipping days, months, years and centuries :D  
Is there a formula?
2. Similar pattern as in Stern Brocot Tree. Also, exactly same pattern between 3 consecutive terms
3. How will you calculate  $n^{\text{th}}$  term?

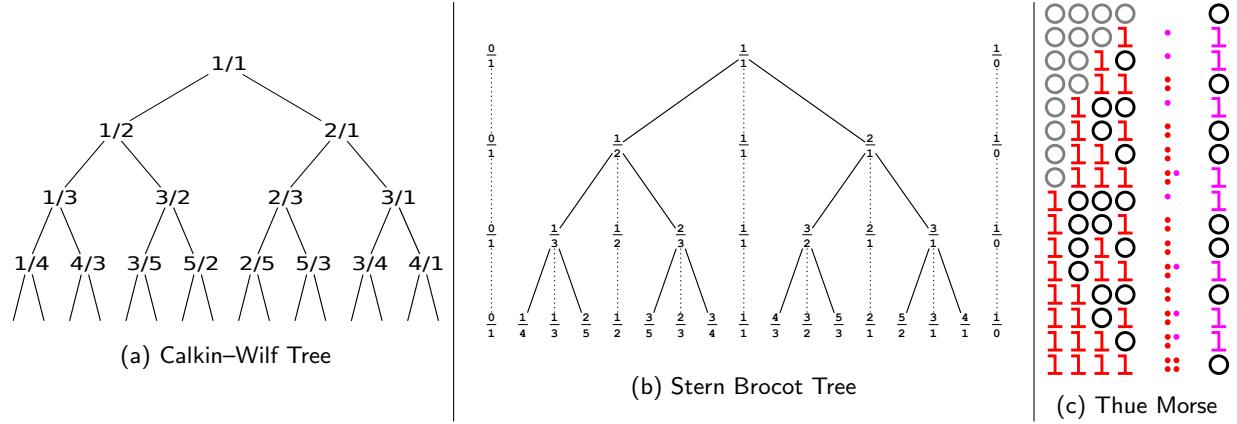


Figure 5