

You are given the force  $F$  as function of position  $x$ , acting on a point particle of mass  $m$ .

$$F(x) = -4 e^{-0.2(x-5)}(1 - e^{-0.2(x-5)}) , \quad U(5) = 10$$

$$m = 1\text{kg}$$

1. You are expected to plot the potential energy  $U$  of the particle, as a function of its position  $x$ . (Note that  $F(x) = -\frac{dU(x)}{dx}$ . Take  $x$  as `np.arange(0, 50, 0.1)`)
2. If the total energy of the particle is constant and represented by  $E$ , from the  $U(x)$  vs  $x$  curve, identify two different values  $E_1$  and  $E_2$  (satisfying  $E_1 < E_2$ ), such that:

- $\forall E \in (E_1, E_2)$  : The phase plot is a closed curve (Well, that seems familiar xD)
- $\forall E \in (E_2, \infty)$  : The phase plot is an open curve (You might wonder why!)
- $\forall E \in (-\infty, E_1)$  : The phase plot is not real (What does that even mean? Well, in this region,  $E$  represents a classically forbidden value, which makes the kinetic energy  $K < 0 \forall x \in (-\infty, \infty)$ )

3. Plot the phase curves for the first two cases, in a single plot, taking any valid value of  $E$  in the corresponding ranges. Don't forget to plot the legend, mentioning the values of  $E$  chosen.

*Hints: For calculating  $U(x)$  from  $F(x)$ , use the Runge-Kutta method. Since the initial condition is given at  $x = 5$ , you would need to apply the method in two separate for loops: one for setting the  $U$  values for  $x \in (5, 50)$ , and other for setting those for  $x \in [0, 5)$*

*For the phase plots, you need the velocity  $v$  of the particle as a function of  $x$ . For this purpose, first find out the kinetic energy  $K(x)$ , using  $E$  and  $U(x)$ . Be careful to only use the points having a positive value in the  $K(x)$  array, to generate the  $v(x)$  array (Since negative values are classically forbidden). Further, use only the corresponding points from the  $x$  array, for plotting the phase plot.*