Legislative Decision Making

- Distributive bargaining
- **Logrolling**

Logrolling

- The term ‘logrolling’ refers to the *exchange of votes* in legislative decision making.

Example (Stratman 1997)
Suppose that the payoffs from two projects A and B are summarized in the following table:

	Voter		
Project	1	2	3
A	5	-1	-1
B	-1	5	-1

- If the group uses majority rule and all voters vote sincerely, neither project will pass.
- Voters 1 and 2 could trade votes:
  - 1 votes for B and 2 votes for A.
  - Then both projects pass.
- This "deal" benefits voters 1 and 2, but voter 3 is harmed.

Is logrolling good or bad?

- Historically, logrolling has been regarded with suspicion.
- Some commentators find the trading of votes intrinsically repugnant, perhaps because it involves "insincere" voting.
- From an *economic* perspective, the relevant issue is that logrolling is associated with *externalities*.
    - $\Rightarrow$ Logrolling may produce *inefficient* outcomes.

Example (Riker and Brams 1973)

	Voter		
Project	1	2	3
A	3	2	-4
B	3	-4	2
C	2	-4	3
D	-4	2	3
E	-4	3	2
F	2	3	-4

Majority rule

- With sincere voting, all projects pass and each voter's total payoff is 2.
- With logrolling, voters 2 and 3 might agree to block projects A and B, and indeed all projects could be blocked by such deals.
- Thus, logrolling might lead to all projects failing, which is *Pareto dominated* by the sincere outcome.

A **common critereon** to evaluate the effects of logrolling is to look at "aggregate" benefits. (Alternatively, *expected* benefits from behind a veil of uncertainty.)

Example 2 (Stratman 1997)

	Voter		
Project	1	2	3
A	5	-1	-c
B	-1	5	-c

- If voters 1 and 2 trade votes, the effect on aggregate / expected benefits depends on the size of the externality c.
- If $c < 2$, ‘aggregate benefits’ *increase* as a result of logrolling.
- If $c > 2$, the fall.

Logrolling and 'bundling'

- Many logrolling deals could also be organized by *bundling* two projects and then voting sincerely on the bundle.
- A 'constructive' logroll is a bundle of two or more projects that are not majority preferred in isolation but are majority preferred as a bundle.
- A 'destructive' logroll is a bundle of two or more majority preferred projects which is *not* majority preferred as a bundle.

Example (Charroin and Vanberg 2019)

	Voter			
Project	1	2	3	Net benefits
A	-2	3	2	3
B	3	-1	2	4
C	1	1	-3	-1

- Projects A and C are majority preferred in isolation.
- The *bundle* A&C is not majority preferred.
  - Voters 1 and 3 could agree to vote N on both. (A destructive logroll.)

‘Mixed’ logrolls

- Logrolling can also be used to simulataneously pass some projects and block others.

Example (Charroin and Vanberg 2019)

	Voter			
Project	1	2	3	Net benefits
A	3	-1	-1	1
B	1	-3	1	-1

- Under sincere voting, only project B would pass.
- Voters 1 and 2 could agree as follows:
  - 2 votes *for* project A
  - 1 votes *against* project B
- Note that this ‘mixed’ logroll cannot be organized by *bundling* the projects.

Logrolling and instability

Example (previous)

	Voter			
Project	1	2	3	Net benefits
A	3	-1	-1	1
B	1	-3	1	-1

- We saw that voters 1 and 2 can agree: 2 votes for A and 1 votes against B.
- Then only project A passes. Look at the payoffs that result!
- But then voters 2 and 3 could make a new deal:
  - 'Let's both vote *against* both A and B'
  - Then nothing would pass.
- But then voters 1 and 3 could make yet another deal:
  - Let's both vote for project B
- etc. ad infinitum...
- Bernholz (1973) shows: Whenever preferences are such that logrolling could occur, there will be *cycles*.

Charroin and Vanberg (2019): Logrolling and q-majortiy rules

The issue

- A group of N voters is faced with multiple binary choices of whether or not to undertake ‘projects’
- Each project yields a vector of payoffs $(v_1, ..., v_N)$
- Question: What voting rule should the group use when voting on these projects?
- Guttman (1998) argues that (simple) majority rule maximizes the *expected* payoff in this context (we will see how).
- We argue:
  - Guttman’s argument is valid only if voters cannot (or do not) engage in log-rolling agreements.
  - If this is permitted (and actually done), higher majority requirements become relatively more attractive.

Our methods

- Theory and simulations
  - Develop algorithms to predict log-rolling agreements and outcomes for a given set of binary choices using different q-majority rules.
  - Apply these algorithms to a large number of randomly generated ‘situations’ (sets of potential projects) and compare outcomes under alternative q-majority rules (simple majority vs. unanimity).
- Laboratory Experiments
  - Identify ‘interesting’ situations - i.e. those in which logrolling should occur and have different effects depending on the decision rule.
  - Implement selected situations, allowing subjects to form log-rolling agreements via unstructured (public) communication.
  - Test predictions regarding the relative performance of decision rules and log-rolling agreements reached.

Example

	Voter			
Project	1	2	3	Net benefits
1	1	-1	1	1
2	-3	1	1	-1

- Assume: Behind a veil of uncertainty, our goal is to maximize aggregate net benefits. (Equivalent: *expected* utility)
- Project 1 produces net benefits. We'd like it to pass
  - Under majority rule, it will pass (good)
  - Under unanimity rule, it won't (bad)
- Project 2 produces net losses. We'd like this to fail.
  - Under majority rule, it will pass (bad)
  - Under unanimity rule, it won't (good)
- In this example, both rules produce a net benefit of zero
  - But notice that costs and benefits are not symmetric.

Example

	Voter			
Project	1	2	3	Net benefits
1	-2	3	2	3
2	3	-1	2	4
3	1	1	-3	-1

Symmetric intensities

- Suppose that *on average*, the individual preference intensities in favor are similar to those opposed.
- Then whenever a *majority* benefits, I should expect (statistically) *aggregate* net benefits to be positive.
- With sincere voting, majority rule maximizes expected aggregate benefits.

Note: This result is actually a variation on Condorcet's Jury Theorem: In Guttman's analysis, the 'correct' decision on any given project is to pass it if and only if aggregate payoffs are positive. Since payoffs are drawn from a symmetric distribution, the probability that any individual has a positive payoff, *conditional* on aggregate payoffs being positive, is greater than $1/2$ (and vice versa if aggregate payoffs are negative). Thus, the individual valuations are analogous to the signals received in Condorcet's analysis.

Guttman's (1998) argument

Example

	Voter			
Project	1	2	3	Net benefits
1	-2	3	2	3
2	3	-1	2	4
3	1	1	-3	-1

Objection: Guttman excludes log-rolling

- Guttman assumes that voters vote *sincerely* on each separate issue.
- In reality, voters may form log-rolling agreements.
- This is likely to alter the relative merits of alternative decision rules.

Possible effects of logrolling

Example

	Voter			
Project	1	2	3	Net benefits
1	-2	3	2	3
2	3	-1	2	4
3	1	1	-3	-1

Unanimity rule with log-rolling

- Mr. 1 to Mr. 2: ‘I’ll vote *for* project 1 if you vote *for* project 2’
- Outcome: projects 1 and 2 pass, 3 fails (maximum aggregate benefit)
- This is a ‘constructive’ logroll (projects that fail in isolation pass as a bundle)

Majority rule with log-rolling

- Mr. 1 to Mr. 3: ‘I’ll vote *against* project 3 if you vote against project 1’
- Outcome: Only project 2 passes (worse than sincere voting)
- This is a ‘destructive’ logroll (projects that pass in isolation fail as a bundle)

General point

Log-rolling and unanimity rule

- A possible *constitutional* argument for majority rule is that the *set* of projects which pass is preferred *by all voters* to the set that would be passed under alternative rules.
- If so, then it should be possible *in principle* to construct log-rolling agreements such that the set passes even under unanimity rule.
- Whether this works in practice will depend on the number of projects available, and on the ability of voters to form the necessary agreements.

Our main conjecture

- As the number of potential projects increases, and as the ability of voters to construct log-rolls increases, so does the relative performance of rules that require larger majorities.

Framework

- N voters, L projects
- $Z =$ a matrix of payoffs z_{li}
- $q =$ number of votes required to pass a project

Random payoff matrices

- z_{li} are independently drawn from a distributionthat is symmetric around zero.
- Without loss of generality, we normalize expected positive and negative payoffs to $+1$ and -1, respectively.

Theory

Sincere voting benchmark
Under any q-majority rule, a project passes under sincere voting if there are $s \geq q$ voters in favor. For each such s, the probability of exactly that many supporters is $(1/2)^N$ times N choose s. In each such case, the expected payoffs to individual supporters and opponents are $+1$ and -1, respectively. Thus the expected total payoff is $s - (N - s) = 2s - N$, and so the *ex ante* expected utility of an individual voter is

$$EU_q(N, L) = L \cdot (1/2)^N \sum_{s=q}^{N} \binom{N}{s} \cdot \left(\frac{2s}{N} - 1\right)$$

Theory

Theorem: *Under any q-majority rule, the ex ante expected utility of an individual voter under sincere voting is given by*

$$EU_q(N, L) = L \cdot (1/2)^N \frac{q}{N} \binom{N}{q}$$

Relative to unanimity rule, payoffs under q-majority rule are $\frac{q}{N}\binom{N}{q}$ times as high. This expression is maximized for $q = \frac{N+1}{2}$ (N odd) or $q = N/2$ (N even).

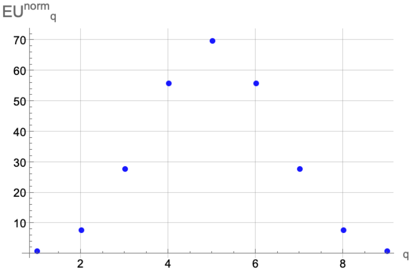


Figure: Expected payoffs relative to unanimity rule ($N = 9$)

Log-rolling algorithm

- Beginning with sincere voting, voters sequentially propose *binding agreements* to change their votes on any *subset* of projects containing at most $K \leq L$ elements. (If a proposer could engage in multiple deals, he myopically chooses the one that generates the largest immediate gain relative to sincere voting.) Following such an agreement, a vote on those issues is immediately conducted. Proposal rights follow a predefined order of 'turns'. All possible orders are equally likely. The process continues until noone wishes to make a further deal, at which point voters vote sincerely on any remaining issues.

Simulations

Example

	Voter			
Project	1	2	3	Net benefits
1	-2	3	2	3
2	3	-1	2	4
3	1	1	-3	-1

Majority rule logrolling

- When it's voter 1 or voter 3's turn, they agree to block projects 1 and 3.
- The vote is conducted and no further deals are possible.
- Result: only project 2 passes, utilities$=(3,-1,2)$, average 1.33.

Unanimity rule logrolling

- When it's voter 1 or voter 2's turn, they agree to pass projects 1 and 2
- The vote is conducted and no further deals are possible.
- Result: projects 1 and 2 pass, utilities$=(1,2,4)$, average 2.33

Simulations

(1) 'Situations' are randomly constructed

- We construct NxL payoff matrices Z for different N and L.
- I will focus on results for $N = 3$ voters and $L = \{3, 5, 9, 12, 15, 18\}$. projects.
- Individual payoffs are independently drawn from $U[-2, +2]$.
- We draw 10000 matrices for each combination of N and L.

Note:

- Since the distribution of payoffs is symmetric around zero, the argument above applies.
- For $N = 3$, expected payoffs under majority rule are *twice* as large as under unanimity rule if voting is sincere.

Simulations

(2) Log-rolling algorithms are applied to all payoff matrices

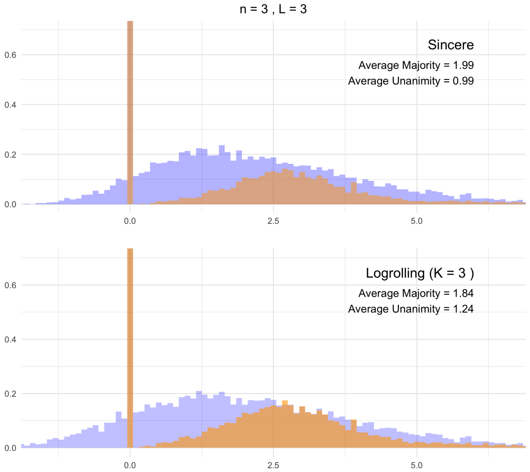
- Computerized (Mathematica)
- (Main) output: Payoffs achieved for each matrix
  - Vector of individual payoffs (total over all projects passed)
  - Average / expected payoff (over projects and individuals)

(3) Inspect distributions of individual and average payoffs achieved under different rules

- Main Hypothesis: The relative performance of unanimity rule improves as L and K get larger.
  - Average / expected payoff increases and eventually becomes larger than under majority rule

Simulation Results: Comparative statics with respect to L

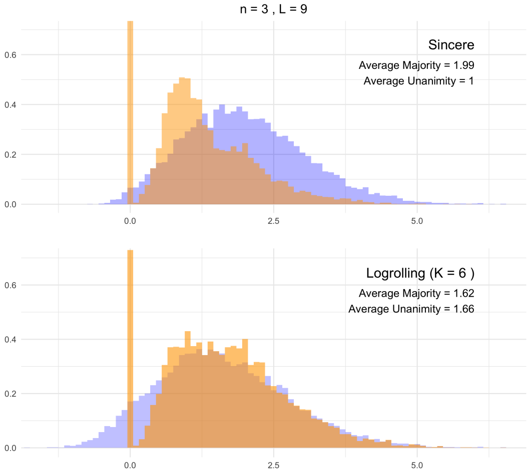
Figure: Average payoffs from simulations with $N = 3$ voters



The blue distribution is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Simulation Results: Comparative statics with respect to L

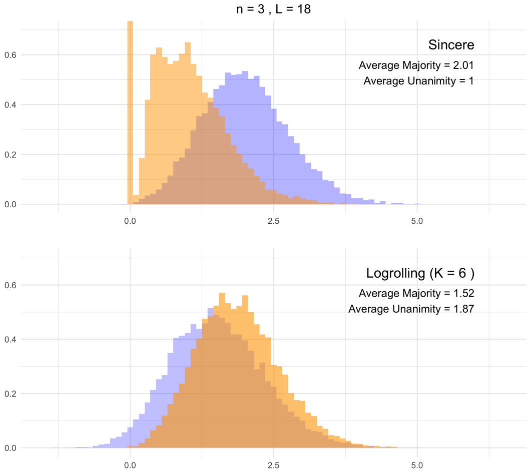
Figure: Average payoffs from simulations with $N = 3$ voters



The blue distribution is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Simulation Results: Comparative statics with respect to L

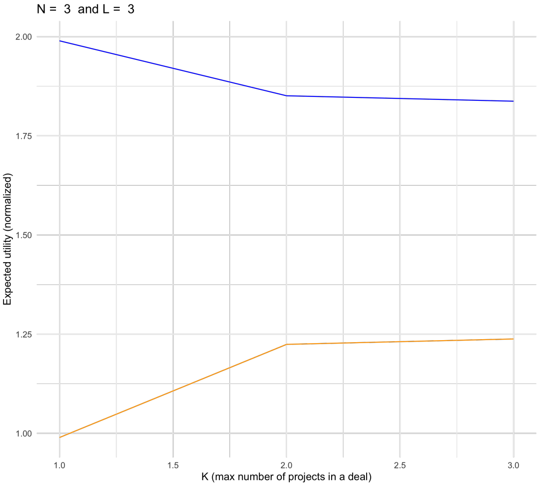
Figure: Average payoffs from simulations with $N = 3$ voters



The blue distribution is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Simulation Results: Comparative statics with respect to K

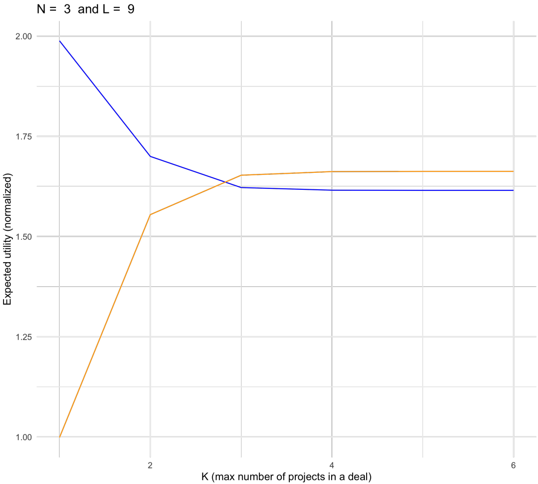
Figure: Average payoffs from simulations with $N = 3$ voters



The blue curve is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Simulation Results: Comparative statics with respect to K

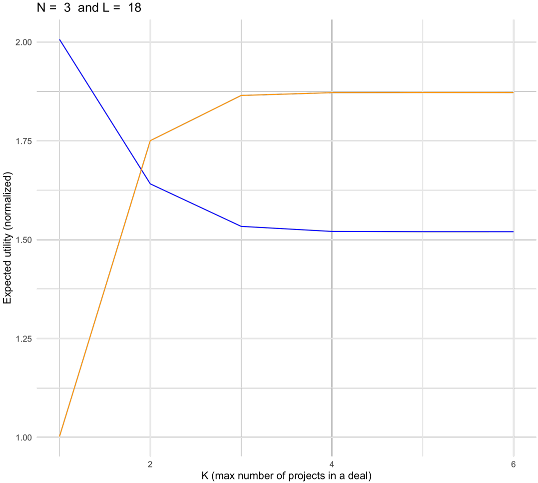
Figure: Average payoffs from simulations with $N = 3$ voters



The blue curve is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Simulation Results: Comparative statics with respect to K

Figure: Average payoffs from simulations with $N = 3$ voters



The blue curve is for majority rule, the orange for unanimity rule. Payoffs are normalized such the EU is 1 under sincere voting with unanimity.

Main takeaways from simulation exercise

- Logrolling always *improves* the distribution of payoffs under unanimity rule. This is, of course, trivially true.
- Logrolling consistently *worsens* the distribution of payoffs under majority rule. This was not obvious ex ante.
- When the number of potential projects is large (> 9), unanimity rule outperforms majority rule once logrolling is allowed.

Substantive Implication: *The larger is the number of decisions that a group (e.g. parliament) is making, and the greater their ability to make 'deals' as to how they will vote on various issues, the larger should be the majority requirement.*

Experiments

- Games involving 3 student subjects deciding on 3 projects.
  - 18 different payoff matrices.
  - Opportunities for different types of logrolls (contructive, destructive, ‘efficient’, ‘inefficient).
  - Largely unstructured, with opportunity to "chat" and to visibly lock in votes project by project.
- Questions
  - Do subjects engage in all types of logrolls?
  - Is the relative performance of unanimity rule better than under sincere voting?
- Results
  - Logrolls are less likely to occur if they impose significant negative externalities and reduce the aggregate payoff.
  - ‘Complex’ logrolls (involving all three projects or voters) are also less likely.

Conclusion: Our student subjects do engage in logrolling. However, the theoretically predicted reversal of performance appears to be mitigated by efficiency concerns and cognitive constraints.

Concluding remarks on legislative decision making

- The literature on legislative decision making deals with decisions made in (relatively) small groups.
- For purely distributional issues (divide-a-dollar), simple majority rule by itself does not yield a stable outcome (cycling). This highlights the importance of *procedural rules* in addition to voting rules ('structure induced equilibrium').
- When comparing alternative voting rules, there is a tradeoff between efficiency (e.g. speed) and fairness (size of winning coalition, distribution within coalition).
- In a world with zero transactions costs (i.e. where decision-making process is efficient), all Pareto improvements are achievable using unanimity rule, perhaps with logrolling.
- Therefore, constitutional arguments in favor of less-than-unanimity rules must necessarily (but perhaps implicitly) assume some source of transactions costs (limited ability to logroll, strategic posturing,...).

Literature

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