Stolper–Samuelson, Rybczynski

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Theorem: Stolper and Samuelson

In a two-good, two-factor Heckscher–Ohlin model under perfect competition, an **increase in the relative price** of one good **raises the real return to the factor used intensively** in its production and lowers the real return to the other factor.

Assumptions Two goods (**A and B**) and two factors (**capital K and labor L**). PF $F(K, L)$ is C^2, **linearly homogeneous (CRS)**, strictly concave, and has positive marginal products ($\mathbf{F_K > 0}$, $\mathbf{F_L > 0}$, $\mathbf{F_{KK} < 0}$, $\mathbf{F_{LL} < 0}$). Factors are mobile domestically but immobile internationally. **Good A is capital-intensive** ($\mathbf{K_A/L_A > K_B/L_B}$), and good B is labor-intensive.

Proof(Sketch) Suppose P_B/P_A increases. Under perfect competition, unit costs satisfy:

$$P_A = c_A(r, w)$$
$$P_B = c_B(r, w),$$

where $c_i(r, w)$ is the unit-cost function in sector i. Since B is more labor-intensive, $c_B(r, w)$ increases with w/r more rapidly than $c_A(r, w)$. Hence there is a one-to-one relation

$$\frac{P_B}{P_A} \uparrow \Longleftrightarrow \frac{w}{r} \uparrow .$$

An increase in P_B/P_A raises w/r, inducing firms in both sectors to substitute capital for labor and thus to increase $k = K/L$. Expressing $F(K, L) = L\, f(k)$ ($\because F$: CRS). Then

$$\mathrm{MPL}(K, L) = g(k) \equiv f(k) - k\, f'(k),$$
$$\mathrm{MPK}(K, L) = f'(k).$$

From concavity, $f''(k) < 0$ implies:

$$g'(k) = -k\, f''(k) > 0, \quad \text{so}\, \mathrm{MPL} = w/P \,\text{rises with}\, k,$$
$$f''(k) < 0, \quad \text{so}\, \mathrm{MPK} = r/P \,\text{falls with}\, k.$$

Thus, the real wage (w/P) increases and the real rental rate (r/P) falls in both sectors. □

Role in the Heckscher–Ohlin Model

- In a country, links relative goods prices to factor returns
- Serves as a stepping stone to the *Factor Price Equalization Theorem.*

Difference between Theorems

- *Stolper–Samuelson theorem*: Price fluctuations of goods → Explains **fluctuations** in **domestic** factor prices (real wages & interest rates)
- *Factor price equalization theorem*: When the relative prices of two goods are unified across countries through **international trade**, the relative prices of each factor (e.g. wages/interest rates) are **equalized internationally**

Theorem: Rybczynski

Assume the **relative price** of two goods is **fixed**. In a two-good, two-factor Heckscher–Ohlin model under perfect competition, an **increase in the endowment of one factor** (holding the other factor and goods prices constant) **raises the output** of the good which uses that **factor intensively** and **lowers the output** of the other good.

Assumptions Same as above, except that P_A, P_B are taken as given (**small open economy**).

Proof (Sketch) Let total factor endowments be K, L with

$$K = K_A + K_B, \quad L = L_A + L_B.$$

Define the economy-wide capital-labor ratio

$$\frac{K}{L} = \underbrace{\frac{L_A}{L}}_{\alpha_A} \frac{K_A}{L_A} + \underbrace{\frac{L_B}{L}}_{\alpha_B} \frac{K_B}{L_B}, \quad \alpha_A + \alpha_B = 1.$$

Since there is a one-to-one relation as before

$$\frac{P_B}{P_A} \text{ const.} \iff \frac{w}{r} \text{ const..}$$

meaning goods prices fix factor price ratio w/r. Thus the sectoral intensities K_i/L_i are determined by cost minimization and remain constant when endowments change. Now increase K. Because each K_i/L_i is constant, the weighted average must satisfy

$$\frac{K}{L} = \alpha_A \frac{K_A}{L_A} + \alpha_B \frac{K_B}{L_B} \quad \text{where } K_A/L_A > K_B/L_B.$$

This implies that K/L is written as increasing linear function of α_A (decreasing in α_B)

$$\frac{K}{L} = (\underbrace{\frac{K_A}{L_A} - \frac{K_B}{L_B}}_{>0,\text{ const.}})\,\alpha_A + \frac{K_B}{L_B} = (\underbrace{\frac{K_B}{L_B} - \frac{K_A}{L_A}}_{<0,\text{ const.}})\,\alpha_B + \frac{K_A}{L_A}$$

An increase in K/L implies α_A must rise and α_B fall. Hence

$$K \uparrow \implies L_A/L = \alpha_A \uparrow \implies K_A = \frac{K_A}{L_A} \text{ const.} \implies L_A \uparrow, K_A \uparrow$$
$$K \uparrow \implies L_B/L = \alpha_B \downarrow \implies K_B = \frac{K_B}{L_B} \text{ const.} \implies L_B \downarrow, K_B \downarrow.$$

Thus both capital and labor shift from the labor-intensive sector B to the capital-intensive sector A, raising output of A and reducing output of B. □

Role in the Heckscher–Ohlin Model

- In a **small open economy**, showing how changes in factor endowments affect outputs
- The result can be seen as a bench-mark for a larger model with endogenous price ratio
- To prove the *Heckscher-Ohlin-Theorem*, the result is used like:

  For a **given relative price**, the **relative supply** of the (labor-) capital-intensive good is **higher** in the (labor-) capital-**abundant** country than in the (capital-) labour-abundant one

Theorem: Heckscher and Ohlin

The labor-abundant country exports the labor intensive good, the capital abundant country exports the capital-intensive good; i.e. each country exports the good which uses the country's abundant factor intensively. By Stolper and Samuelson Theorem, those price changes in the beginning of the international trade express **distribution effects of starting trade**!