Parangat Bindal

IT

11912084.

Date :

Page :

DS Assignment

Q1 $t_{n_{10}} = 5$ sec.

$t_{n_{25}} = ?$, $kn^2 = t$

$k \times 100 = 5$

$k = 1/20$

$t_{n_{50}} = \frac{1}{20} \times 50 \times 50 = 125$ sec.

$= 12.5$ sec Ans

Q2 $T_A = n^3$, $T_B = 2n^2$

break pt $n^3 = 2n^2$

$\boxed{n = 2}$

Q3 $f(n) = n2^n$, $g(n) = 4^n$

Applying rule

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n2^n}{4^n}$$

$$\lim_{n\to\infty} \frac{n2^n}{2^{2n}} = \lim_{n\to\infty} \frac{n\cancel{2^n}}{2^n \cdot \cancel{2^n}}$$

Date :
Page :

applying L'Hospital rule.

$$\lim_{n\to\infty} \frac{1}{2^n \log 2}$$

Hence $f(n)$ is $O\,g(n)$ Ans

Q4 let poly no. fn $P(n)$ f and $\log(n)$ fn.

using limit rule, $\lim_{n\to\infty} \frac{\log(n)}{P(n)} = \left[\frac{\infty}{\infty}\right]$

Applying L-Hospital rule.

$$\lim_{n\to\infty} \frac{1}{nP(n)}$$

$\Rightarrow 0$

thus, $\log(n)$ grows slower than all fn.

as let 2 log function i.e. $\log_a n$ and $\log_b (n+1)$

apply limit rule.

$$\lim_{n\to\infty} \frac{\log n}{\log(n+1)} \Rightarrow \lim_{n\to\infty} \frac{\log n}{\log n + \log(1+1/n)}$$

$$\lim_{n\to\infty} \frac{\log_a n}{\log_b n}$$

Date :
Page :

$\lim_{n\to 0} \frac{\log b}{\log a}$. thus proved.

Q5 **Average Case (Θ)** | **Worst case (O)**

Average Case (Θ)	Worst case (O)
① performs avg. no. of steps.	performs max. no. of steps.
② Averaged over all possible inputs	Input is arbitrary same.
③ Eg: Θ(n log(n)) quick sort	eg (O(n²).) quick sort

Q6 $f(n) = n^4 + \log n + 11$, $g(n) = n^4$

$\lim_{n\to\infty} f(n) = \lim_{n\to\infty} \frac{n^4 + \log n + 11}{n^4}$

$\lim_{n\to\infty} \frac{4n^3 + 1/n}{4n^3}$

$\lim_{n\to\infty} \frac{\cancel{4n^3}}{\cancel{4n^3}} + \cancel{\frac{1}{4n^4}} \to 0.$

= Constant.

Hence fn is $\Theta(gn)$

Date :
Page :

Q7 a) $k = 1$

$k = 2$

$k = 3$

$O(n-1)$

$\approx O(n)$

$k = n-1$

(b) for $i = 1$, $j = 2, 3 - - - - n$ -- $(n-1)$

$i = 2$, $j = 3, 4 - - - - n$ -- $(n-2)$

$i = n + - - -$

$\therefore (n-1) + (n-2) + (n-3) + - - + - - 3 \cdot 2 \cdot 1$

$$\frac{n(n-1)}{2}$$

$= O(n^2)$

Q8 Quadratic fn. $g(n) = n^2$

$n * n = n^2$

$$1+2+3 \cdot - - - + (n-1) + n = \frac{n(n+1)}{2}.$$

Ans

Date :
Page :

Q9 $T_A = 100^n$

$T_B = n^4$.

using limit rule

$$\lim_{n\to\infty} \frac{n^4}{100^n}$$

$$= \lim_{n\to\infty} \frac{4n^3}{100^n \log(100)} = 0$$

Hence T_A grows faster, when $n \to \infty$.

Q10 $f(n) = n\log_2 n$, $g(n) = \log(n!)$

$\log(1 \cdot 2 \cdot 3 \cdot 4 \cdot ---- (n-1) \cdot n)$

$= \log\left(\frac{n^{n/2}}{2}\right)$

$\frac{n}{2}\log\left(\frac{n}{2}\right)$

$\frac{n}{2}\log n - \frac{n}{2}\log 2$

$n/2(\log n - \cancel{n} \log 2) \approx \theta$([illegible])

Hence $f(n) \in \Theta\, g(n)$

Q11 (a) $2^{n-1} + 4^{n+1}$

For θ = $c_1 g(n) \le f(n) \le c_2 g(n)$

$2^{n-1} + 2^{2n+2}$

$\frac{2^n}{2} + 4 \cdot 2^{2n}$

$2^{2n}\left(4 + \frac{1}{2^{n+1}}\right)$

Highest order is 2^{2n} = $\theta(4^n)$

(b) $(n^2+6)^8$

highest order are = $(n^2)^8$

= n^{16}

$\theta(n^{16})$

Q12 $T_A = n^2$

$T_B = n+1$

Because ft

$n^2 = n+2$

$n^2 - n - 2 = 0$

$n(n-2) + 1(n-2) = 0$

$(n-2)(n+1) = 0$

$\boxed{n = 2}$ Ans